| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 1 | Option A is correct | To identify the rule that describes the transformation (change of a shape using a rotation (circular movement), reflection (flip), translation (slide), or dilation (resize)), the student should have recognized that when a shape is translated 4 units to the left, each $x$-coordinate (horizontal position from 0 ) is decreased by 4 units, which is represented by the expression $x-4$. When a shape is translated 5 units up, each $y$-coordinate (vertical position from 0 ) is increased by 5 units, which is represented by the expression $y+5$. Therefore the rule $(x, y) \rightarrow(x-4, y+5)$ describes this transformation. |
|  | Option B is incorrect | The student likely reversed the relationship between the effect on $x$ and $y$ and identified the rule for translating the figure 4 units to the right and 5 units down. The student needs to focus on writing rules for translations and explaining the effect. |
|  | Option C is incorrect | The student likely reversed the relationship of the effect on $x$ and translated the figure to the right rather than to the left and identified the rule for translating the figure 4 units to the right and 5 units up. The student needs to focus on writing rules for translations and explaining the effect. |
|  | Option D is incorrect | The student likely reversed the relationship of the effect on $y$ and translated the figure down rather than up and identified the rule for translating the figure 4 units to the left and 5 units down. The student needs to focus on writing rules for translations and explaining the effect. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 2 | Option H is correct | To identify the representation of $y$ as a function of $x$, the student should have determined that in order for the representation to show a function (a relationship where each input (value put into an equation, $x$ ) has a single output (value that comes out of the equation, $y$ )), each value of $x$ can only be paired with one value of $y$. The mapping shows each of the four values of $x$ mapped to a single value of $y$. Therefore this representation shows $y$ as a function of $x$. |
|  | Option F is incorrect | The student likely did not recognize that the $x$-values of -1 and 1 have two $y$-values, and therefore the graph does not represent a function. The student needs to focus on the definition of a function and on applying it to determine whether a set of ordered pairs that are part of a graphed line represents a function. |
|  | Option G is incorrect | The student likely did not recognize that the $x$-values of -3 and -2 have two $y$-values, and therefore the graph does not represent a function. The student needs to focus on the definition of a function and on applying it to determine whether a set of ordered pairs that are part of a graphed line represents a function. |
|  | Option J is incorrect | The student likely confused which variable (symbol used to represent an unknown number) was the input of the function and which variable was the output of the function. The student needs to focus on how to determine whether any of the given representations show a function by determining whether each value of $x$ is paired with a single value of $y$. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 3 | Option B is correct | To determine the volume (amount of three-dimensional space taken up) of the bowling ball shaped like a sphere in cubic centimeters, the student should have used the formula for the volume of a sphere, $V=\frac{4}{3} \pi r^{3}$. To determine the value of $r$, the radius (distance from the center to the circumference of a circle), the student should have divided the diameter (straight line going through the center of a circle connecting two points on the circumference) by 2 . The student should have then evaluated $V=\frac{4}{3} \pi(10.8)^{3}$, which is approximately equal to $5,276.7$. |
|  | Option A is incorrect | The student likely used the length of the diameter in place of the radius in the volume formula and squared the diameter instead of cubing the diameter. The student needs to focus on understanding and properly applying the formula for determining the volume of a sphere. |
|  | Option C is incorrect | The student likely used the correct formula to calculate the volume of the sphere but squared the radius instead of cubing the radius. The student needs to focus on understanding and properly applying the formula for determining the volume of a sphere. |
|  | Option D is incorrect | The student likely used the length of the diameter in place of the radius in the volume formula. The student needs to focus on understanding and properly applying the formula for determining the volume of a sphere. |


| Item\# |  | Rationale |
| :---: | :---: | :---: |
| 4 | Option F is correct | To determine the loan option that would allow the customer to pay the least amount of interest, the student should have first calculated the amount of interest for each option using the simple interest formula $I=P r t$, in which $P$ represents the principal (initial loan amount), $r$ represents the rate (in decimal form), and $t$ represents the length of time in years. To determine the amount of interest for a 4-year loan with a $5.2 \%$ annual simple interest rate (option A), the student should have calculated $I=(\$ 12,000)(0.052)(4)=\$ 2,496$. To determine the amount of interest for a 5 -year loan with a $4.2 \%$ annual simple interest rate (option B), the student should have calculated $I=(\$ 12,000)(0.042)(5)=\$ 2,520$. To determine the amount of interest for a 6 -year loan with a $4.7 \%$ annual simple interest rate (option $C$ ), the student should have calculated $I=(\$ 12,000)(0.047)(6)=\$ 3,384$. To determine the amount of interest for a 3-year loan with an 8.4\% annual simple interest rate (option D), the student should have calculated $I=(\$ 12,000)(0.084)(3)=\$ 3,024$. Finally the student should have determined that a 4-year loan with a $5.2 \%$ annual simple interest rate (option $A$ ) results in the least amount of interest. |
|  | Option G is incorrect | The student likely thought the lowest interest rate (4.2\%) would result in the least amount of interest. The student needs to focus on attending to the details of the question in problems. |
|  | Option H is incorrect | The student likely thought the question was asking for the greatest amount of interest instead of the least amount of interest. The student needs to focus on attending to the details of the question in problems. |
|  | Option J is incorrect | The student likely thought the shortest length of time (3 years) would result in the least amount of interest. The student needs to focus on attending to the details of the question in problems. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 5 | Option D is correct | To determine which table appears to show a non-proportional relationship between $x$ and $y$, the student should have recognized that in a non-proportional relationship, the $y$-value is not equal to 0 when the corresponding (paired) $x$-value is 0 . In this table, when the $x$-value is equal to 0 , the corresponding $y$-value would equal -4 . |
|  | Option A is incorrect | The student likely identified that the given $x$-values do not increase at a constant rate but likely did not use the pattern in the table to determine that when $x=0, y=0$. The student needs to focus on understanding the definition of a proportional relationship and on applying it to determine whether a table represents a proportional relationship or a non-proportional relationship. |
|  | Option B is incorrect | The student identified a table that has two $x$-values that are negative ( -17 and -5 ) and could have thought the relationship was non-proportional as a result of the negative $x$-values. The student likely did not use the pattern in the table to determine that when $x=0, y=0$. The student needs to focus on understanding the definition of a proportional relationship and on applying it to determine whether a table represents a proportional relationship or a non-proportional relationship. |
|  | Option C is incorrect | The student likely reversed the meaning of a proportional relationship and a non-proportional relationship. The table shows a proportional relationship since the pattern in the table can be extended to show when $x=0, y=0$. The student needs to focus on understanding the definition of a proportional relationship and on applying it to determine whether a table represents a proportional relationship or a non-proportional relationship. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 6 | Option G is correct | To determine the ordered pair that best represents the location of point $R^{\prime}$, the student should have determined that when a figure is dilated (enlarged or reduced in size), its measurements increase or decrease based on the scale factor (ratio of the length of a side of one figure to the length of the corresponding (paired) side of a similar figure). A scale factor of $n$, with the origin (point where the $x$-axis (horizontal) and $y$-axis (vertical) on a coordinate grid intersect (cross); also, the point represented by the ordered pair $(0,0)$ ) as the center of dilation means that each point on the dilated figure will be $n$ times as far from the origin as it was on the original figure. If the location of point $R$ was originally represented by $(-3,-5)$, then the location of the corresponding point on the dilated figure, point $R^{\prime}$, will be $(-3 n,-5 n)$. |
|  | Option F is incorrect | The student divided the $x$ - and $y$-coordinates by the scale factor instead of multiplying them by $n$. The student needs to focus on the effect a dilation by a given scale factor has on a figure and how to determine the rule to explain the effect. |
|  | Option H is incorrect | The student added the scale factor instead of multiplying by $n$. The student needs to focus on understanding how the scale factor affects the dilation. |
|  | Option J is incorrect | The student subtracted the scale factor instead of multiplying by $n$. The student needs to focus on understanding how the scale factor affects the dilation. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 7 | Option C is correct | To determine the amount of interest the account earned at the end of 15 years, the student should have used the formula $I=P r t$, in which $P$ represents the principal (initial loan amount), $r$ represents the interest rate (in decimal form), and $t$ represents the length of time in years. Using the given information, the student should have written the equation as $I=(\$ 5,000)(0.0485)(15)$, which results in $\$ 3,637.50$. |
|  | Option A is incorrect | The student likely calculated the amount of interest using the formula for compound interest, $A=P(1+r)^{t}$, first calculating $A=(\$ 5,000)(1+0.0485)^{15}=\$ 10,174.11$. Then subtracting the principal from the total, calculating $\$ 10,174.11-\$ 5,000=\$ 5,147.11$. The student needs to focus on understanding the difference between simple and compound interest, the formula for each, and how to apply them. |
|  | Option B is incorrect | The student likely calculated the balance in the account using the formula for compound interest, $A=P(1+r)^{t}$, calculating $A=(\$ 5,000)(1+0.0485)^{15}=\$ 10,174.11$. The student needs to focus on understanding the difference between simple and compound interest, the formula for each, and how to apply them. |
|  | Option D is incorrect | The student likely calculated the balance in the account. The student used the correct formula but added the interest to the principal, calculating $\$ 3,637.50+\$ 5,000=\$ 8,637.50$. The student needs to focus on attending to the details of the question in problems. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 8 | Option J is correct | To determine the statement that is true, the student should have determined that because a rotation (circular movement) preserves congruence (same shape and size), triangle $X^{\prime} Y^{\prime} Z^{\prime}$ will be congruent to triangle $X Y Z$, which means the area of triangle $X Y Z$ is equal to the area of triangle $X^{\prime} Y^{\prime} Z^{\prime}$. |
|  | Option F is incorrect | The student likely misunderstood the effect a rotation has on the angle measures of a figure and thought that the sum (total) of the angle measures of triangle $X^{\prime} Y^{\prime} Z^{\prime}$ would be $90^{\circ}$ greater than the sum of the angle measures of triangle $X Y Z$. The student needs to focus on the effect a rotation has on a figure and on the fact that a rotation preserves congruence. |
|  | Option G is incorrect | The student likely misunderstood the effect a rotation has on the angle measures of a figure and thought that the corresponding (paired) angle measures of triangle $X^{\prime} Y^{\prime} Z^{\prime}$ would be greater than the corresponding angle measures of triangle $X Y Z$. The student needs to focus on the effect a rotation has on a figure and on the fact that a rotation preserves congruence. |
|  | Option H is incorrect | The student likely misunderstood the effect a rotation has on the congruency of a figure and thought that triangle $X^{\prime} Y^{\prime} Z^{\prime}$ would not be congruent to triangle $X Y Z$. The student needs to focus on the effect a rotation has on a figure and on the fact that a rotation preserves congruence. |

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| :---: | :--- | :--- |
| 9 | 5 and any equivalent <br> values are correct | To determine the number of groups that Classroom A sent to Classroom B, the student could have <br> solved the given equation for $x$. The student could have first added $4 x$ to both sides of the equation, <br> resulting in $70=30+8 x$. The student then could have subtracted 30 from both sides of the equation, <br> resulting in $40=8 x . ~ F i n a l l y, ~ t h e ~ s t u d e n t ~ c o u l d ~ h a v e ~ d i v i d e d ~ b o t h ~ s i d e s ~ o f ~ t h e ~ e q u a t i o n ~ b y ~$ <br> in $x=5$. resulting <br> the problem correctly. |


| Item\# |  | Rationale |
| :---: | :---: | :---: |
| 10 | Option G is correct | To determine which function (equation) represents $t$, the total number of gallons of water in the tank after $w$ weeks, the student could have determined the values of $m$ and $b$ in the slope-intercept form of the line, $y=m x+b$. The $m$ represents the rate of change (also referred to as slope, which is the steepness of a straight line when graphed on a coordinate grid; $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$ ). The $b$ represents the $y$-intercept (value where the line crosses the $y$-axis (vertical)) and the initial value in the scenario. In this scenario the student should have thought of the slope-intercept form of line $y=m x+b$, replacing $y$ with $t$ and $x$ with $w$, resulting in $t=m w+b$. The student then should have determined "the amount of water in the tank will decrease at a constant rate of 15 gallons per week" represents the rate of change $(m)$ and "currently contains 275 gallons of water" represents the $y$-intercept ( $b$ ). The student should have substituted these values into the slope-intercept form of the line for this scenario, resulting in $t=-15 w+275$. |
|  | Option F is incorrect | The student likely reversed the sign (positive and negative value) of $m$ when substituting into the slope-intercept form of the line for this scenario, resulting in $t=15 w-275$. The student needs to focus on paying attention to detail when identifying the value for the rate of change and initial value and using these values to write the function representing the situation. |
|  | Option H is incorrect | The student likely reversed the values of $m$ and $b$ when substituting into the slope-intercept form of the line for this scenario, resulting in $t=275 w-15$. The student needs to focus on paying attention to detail when identifying the value for the rate of change and initial value and using these values to write the function representing the situation. |
|  | Option J is incorrect | The student likely reversed the values of $m$ and $b$ when substituting into the slope-intercept form of the line for this scenario and reversed the signs of 275 and -15 , resulting in $t=-275 w+15$. The student needs to focus on paying attention to detail when identifying the value for the rate of change and initial value and using these values to write the function representing the situation. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 11 | Option D is correct | To determine the best prediction of the height in centimeters of a student with a weight of 64 kilograms using a scatterplot (a graph of plotted points that shows the relationship between two sets of data), the student could have drawn a line that closely follows the pattern formed by the points on the graph by keeping about half of the points above the line and the other half below the line. A good line for this scatterplot would pass slightly above the point at $(135,29)$ but very close to the point at $(135.5,33)$ and below the point at $(152,62)$, passing above the point at $(155,59)$, very close to $(155,61)$. The student could have then identified where the grid line marked 64 (representing a weight of 64 kilograms) intersects (crosses over) the line the student drew and determined that the height in centimeters corresponding to that point is about 156 centimeters. This is an efficient way to solve the problem; however, other methods could be used to solve the problem correctly. |
|  | Option A is incorrect | The student likely chose the approximate height of the closest data point to 64 kilograms $[(152,62)]$, resulting in 152. The student needs to focus on drawing a line as close as possible to all points with a similar number of points above and below the line. |
|  | Option B is incorrect | The student likely chose the points at $(135,29)$ and $(138,33)$ to draw the line to make the prediction. However, this line has 17 points above and 0 points below. The student needs to focus on drawing a line as close as possible to all points with a similar number of points above and below the line. |
|  | Option C is incorrect | The student likely chose the greatest (tallest) height based on the increments of 4 shown on the horizontal axis, resulting in 162. The student needs to focus on drawing a line as close as possible to all points with a similar number of points above and below the line. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 12 | Option H is correct | To determine the transformation applied to a figure on a coordinate grid that does NOT preserve congruence (same size and shape), the student should have recognized that a dilation will enlarge or reduce the size of the figure by a scale factor (relationship between the length of a side of one figure to the length of the corresponding (paired) side of a similar figure). The dilated figure will be similar (two figures with corresponding angles equal and corresponding sides proportional) to the original figure but NOT congruent. |
|  | Option F is incorrect | The student likely recalled that a rotation (circular movement) of $270^{\circ}$ counterclockwise may result in negative numbers being used as coordinates for the figure. The student may not have recognized that the congruence of the figure is preserved with a rotation. The student needs to focus on differentiating the characteristics of a rotation from the characteristics of a dilation. |
|  | Option G is incorrect | The student likely recalled that a reflection (flip) across the $y$-axis (vertical) may result in negative numbers being used as coordinates for the figure. The student may not have recognized that the congruence of the figure is preserved with a reflection. The student needs to focus on differentiating the characteristics of a reflection from the characteristics of a dilation. |
|  | Option J is incorrect | The student likely recalled that a translation (slide) 50 units down may result in negative numbers being used as coordinates for the figure. The student may not have recognized that the congruence of the figure is preserved with a translation. The student needs to focus on differentiating the characteristics of a translation from the characteristics of a dilation. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 13 | Option A is correct | To determine which table shows the relationship between $x$, the number of sets of markers bought by the teacher, and $y$, the total cost of the markers, the student should have recognized that because the teacher had a $\$ 2$-off coupon to use toward the total cost of the markers bought, the $y$-intercept (value where the line crosses the $y$-axis (vertical)) and the initial (beginning) value of $y$ is negative \$2. The student also should have recognized that because the teacher bought the markers at a cost of $\$ 3$ per set, the rate of change (constant increase or decrease) is 3 . The student should have recognized that each corresponding (paired) $x$ - and $y$-value in the table should fit the equation $y=3 x-2$. This is an efficient way to solve the problem; however, other methods could be used to solve the problem correctly. |
|  | Option B is incorrect | The student likely determined that the corresponding $x$ - and $y$-values in the table fit the equation $y=\frac{1}{3} x-2$. The student needs to focus on understanding and interpreting the initial value and the rate of change from a verbal situation to identify the table with the same relationship. |
|  | Option C is incorrect | The student likely determined that the corresponding $x$ - and $y$-values in the table fit the equation $y=3 x+2$. The student needs to focus on understanding and interpreting the initial value and the rate of change from a verbal situation to identify the table with the same relationship. |
|  | Option D is incorrect | The student likely determined that the corresponding $x$ - and $y$-values in the table fit the equation $y=\frac{1}{3} x+2$. The student needs to focus on understanding and interpreting the initial value and the rate of change from a verbal situation to identify the table with the same relationship. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 14 | Option H is correct | To determine the volume (amount of three-dimensional space taken up) of the cylinder, the student should have used the formula $V=\pi r^{2} h$, in which $r$ represents the radius (distance from the center to the circumference of a circle) and $h$ represents the height (vertical distance from top to bottom) of the cylinder. The student should have calculated the length of the radius by dividing the length of the diameter (straight line going through the center of a circle connecting two points on the circumference) by $2(2.5 \div 2=1.25)$. Substituting $r=1.25$ and $h=7.5$ into the formula results in $V=\pi(1.25)^{2}(7.5)$ |
|  | Option F is incorrect | The student likely used the length of the diameter in place of the length of the radius in the formula. The student needs to focus on understanding the formula for determining the volume of a cylinder. |
|  | Option G is incorrect | The student likely used the length of the diameter in place of the length of the radius and reversed the placement of the values in the formula. The student needs to focus on understanding the formula for determining the volume of a cylinder. |
|  | Option J is incorrect | The student likely divided the length of the height by 2 instead of the length of the diameter and reversed the placement of the values in the formula. The student needs to focus on understanding the formula for determining the volume of a cylinder. |

Option A is correct
To determine the proportion that can be used to show that the slope (steepness of a straight line when graphed on a coordinate grid; $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$ ) of $\overline{K M}$ is equal to the slope of $\overline{P S}$, the student should have determined that two of the sides of each triangle can be used to show the slope using the formula for slope. For $\overline{K M}$, the two sides of triangle $K L M$ can be used to show the slope, where $y_{2}-y_{1}$ can be represented by the difference in the vertical values, which is the same value as the length of side $K L$, and $x_{2}-x_{1}$ can be represented by the difference in the horizontal values, which is the same value as the length of side $L M$. Plugging into the formula, the slope of $\overline{K M}$ can be represented by $m=\frac{K L}{L M}$. For $\overline{P S}$, the two sides of triangle $P R S$ can be used to show the slope where $y_{2}-y_{1}$ can be represented by the difference in the vertical values, which is the same value as the length of side $P R$, and $x_{2}-x_{1}$ can be represented by the difference in the horizontal values, which is the same value as the length of side $R S$. Plugging into the formula, the slope of $\overline{P S}$ can be represented by $m=\frac{P R}{R S}$. Therefore the proportion $\frac{K L}{L M}=\frac{P R}{R S}$ shows the slopes of $\overline{K M}$ and $\overline{P S}$ are equal.

Option B is incorrect
The student likely used the horizontal side of each triangle over the longest side of each triangle, resulting in $\frac{R S}{S P}=\frac{L M}{M K}$. The student needs to focus on correctly using the formula for the slope of a line and the correct sides of the triangle to show the slopes are equal.

Option C is incorrect

The student likely reversed the formula for $\overline{K M}$, using $m=\frac{x_{2}-x_{1}}{y_{2}-y_{1}}$ instead of $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$, and for $\overline{P S}$ the student likely used the longest side of the triangle over the vertical side of the triangle, resulting in $\frac{M L}{L K}=\frac{S P}{R P}$. The student needs to focus on correctly using the formula for the slope of a line and the correct sides of the triangle to show the slopes are equal.

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|  | Option D is incorrect | The student likely used the longest side of each triangle over the vertical side of each triangle, <br> resulting in $\frac{P S}{P R}=\frac{K M}{K L}$. <br> line and the correct sides of the triangle to show the slopes are equal. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 16 | Option G is correct | To determine the TV screen that has the smallest diagonal length in feet, the student should have ordered the diagonal lengths of the TV screens in the table from the shortest length to the longest length (smallest diagonal size to largest diagonal size) and could have then converted each number to a fraction with the same denominator (bottom number) (12, since the denominator of $\sqrt{144}$ is equal to 12 ). The diagonal length of Screen $W(\sqrt{12.25})$ results in the decimal 3.5 , which converts to the fraction $\frac{42}{12}$. The diagonal length of Screen $X\left(2 \frac{1}{6}\right)$ results in the fraction $\frac{13}{6}$, which converts to the fraction $\frac{26}{12}$. The diagonal length of Screen $Y\left(\frac{37}{\sqrt{144}}\right)$ converts to the fraction $\frac{37}{12}$. The diagonal length of Screen $Z\left(\frac{8}{3}\right)$ converts to the fraction $\frac{32}{12}$. This results in fractions of $\frac{42}{12}, \frac{26}{12}, \frac{37}{12}$, and $\frac{32}{12}$. The student could then have ordered the diagonal lengths by comparing the fractions. The correct order of the fractions is $\frac{26}{12}, \frac{32}{12}, \frac{37}{12}$, and $\frac{42}{12}$, which means the shortest diagonal length is $\frac{26}{12}$, which is the diagonal length for Screen $X$. This is an efficient way to solve the problem; however, other methods could be used to solve the problem correctly. |
|  | Option F is incorrect | The student likely chose the largest (longest) diagonal length instead of the smallest diagonal length. The student needs to focus on attending to the details of the question in problems that involve ordering different representations of real numbers correctly. |
|  | Option H is incorrect | The student likely did not take the square root of 144 in the denominator for the diagonal length of Screen $Y$ and therefore compared the fraction $\frac{37}{144}$ against the other fractions, which would be the smallest diagonal length. The student needs to focus on attending to the details of the question in problems that involve ordering different representations of real numbers correctly. |
|  | Option J is incorrect | The student likely compared Screen $Z$ as the fraction $\frac{3}{8}$ instead of $\frac{8}{3}$ against the other fractions, which would be the smallest diagonal length. The student needs to focus on attending to the details of the question in problems that involve ordering different representations of real numbers correctly. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 17 | Option D is correct | To determine which scatterplot (a graph of plotted points that shows the relationship between two sets of data) supports the teacher's observation (as the number of hours students studied their multiplication facts increased, the number of errors the students made on their multiplication tests decreased), the student should have chosen the graph that shows the following association: as the study time (independent variable, $x$ ) increases, the number of errors (dependent variable, $y$ ) decreases. In this scenario the points on the correct graph represent a negative slope (the steepness of a straight line when graphed on a coordinate grid; $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$. |
|  | Option A is incorrect | The student likely reversed the association based on the observation, resulting in a graph with a positive slope. The student needs to focus on choosing a scatterplot based on a description of the association. |
|  | Option B is incorrect | The student chose a graph that shows a negative slope from the middle to the right side of the graph. The student needs to focus on choosing a scatterplot based on a description of the association. |
|  | Option C is incorrect | The student likely focused on the words "increased" and "decreased" thinking the data points would be scattered randomly about the graph and therefore chose a graph that shows no association between the study time and number of errors. The student needs to focus on choosing a scatterplot based on a description of the association. |


| Item\# |  | Rationale |
| :---: | :---: | :---: |
| 18 | Option H is correct | To determine the radius (distance from the center to the edge of a circle) of the cone, the student should have used the formula for the volume (amount of three-dimensional space taken up) of a cone, $V=\frac{1}{3} B h$, where $B$ represents the area (amount of space covered by the surface) of the base of the cone (circle), which is also the formula for the area of a circle, $A=\pi r^{2}$. Combining the formulas results in the formula $V=\frac{1}{3} \pi r^{2} h$. Substituting the values into the formula results in $\begin{aligned} & (1.5 \pi)=\frac{1}{3} \pi r^{2}(4.5) . \text { Solving the equation for } r, r^{2}=(1.5 \pi) \div\left(\frac{1}{3} \pi(4.5)\right) \rightarrow \\ & r=\sqrt{(1.5 \pi) \div\left(\frac{1}{3} \pi(4.5)\right)}=\sqrt{(1.5 \pi) \div(1.5 \pi)}=\sqrt{1} \text {, results in } 1.0 . \end{aligned}$ |
|  | Option F is incorrect | The student likely did not include $\frac{1}{3}$ in the formula and solved $(1.5 \pi)=\pi r^{2}(4.5)$ for $r$, resulting in approximately 0.6 . The student needs to focus on paying attention to detail to write the correct formula for the task. |
|  | Option G is incorrect | The student likely used the correct formula but used half of the height in the calculation ( 2.25 instead of 4.5 ) and solved $(1.5 \pi)=\frac{1}{3} \pi^{2}(2.25)$ for $r$, resulting in approximately 1.4 . The student needs to focus on paying attention to detail to use the correct values in the formula for the task. |
|  | Option J is incorrect | The student likely used $\frac{4}{3}$ instead of $\frac{1}{3}$ in the formula and solved $(1.5 \pi)=\frac{4}{3} \pi r^{2}(4.5)$ for $r$, resulting in 0.5 . The student needs to focus on paying attention to detail to write the correct formula for the task. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 19 | Option B is correct | The student should have determined that in order for a set of ordered pairs to represent a function (relationship in which each input (value put into an equation, $x$ ) has a single output (value that comes out of the equation, $y$ )), each value of $x$ can only be paired with one value of $y$. The set of ordered pairs contains only one $y$-value for each $x$-value. |
|  | Option A is incorrect | The student likely did not realize that the last two ordered pairs in the set have the same $x$-value (input value, -5.5 ) paired with different $y$-values (output values, 5 and -6 ). The student needs to focus on the definition of a function and on applying it to determine whether a set of ordered pairs represents a function. |
|  | Option C is incorrect | The student likely confused which variable (symbol used to represent an unknown number) was the input for the function and which variable was the output for the function. The set has one $x$-value (input value, 9.3) paired with two different $y$-values (output values, -1 and 1) and another $x$-value (7.3) paired with two different $y$-values ( -2 and 2 ). The student needs to focus on the definition of a function and on applying it to determine whether a set of ordered pairs represents a function. |
|  | Option D is incorrect | The student likely did not realize that the second and last ordered pairs in the set have the same $x$-value (input value, 9.1) paired with different $y$-values (output values, 10 and 8 ). The student needs to focus on the definition of a function and on applying it to determine whether a set of ordered pairs represents a function. |


| Item\# |  | Rationale |
| :---: | :---: | :---: |
| 20 | Option J is correct | To determine the rule that best represents the dilation, the student should have determined that when a figure is dilated (enlarged or reduced in size), its measurements increase or decrease based on the scale factor (ratio of the length of a side of one figure to the length of the corresponding (paired) side of a similar figure). A scale factor with the origin (point where the $x$-axis (horizontal) and $y$-axis (vertical) on a coordinate grid intersect (cross); also, the point represented by the ordered pair $(0,0)$ ) as the center of dilation means that each point on the dilated figure will be a certain number of times as far from the origin as it was on the original figure. Since the location of vertex $Q$ was originally represented at $(6,2)$, and the location of the corresponding vertex (vertex $Q^{\prime}$ ) on the dilated figure is at $(21,7)$, the scale factor of the dilation can be determined by dividing the value of each corresponding coordinate of vertex $Q^{\prime}$ by the value of each corresponding coordinate of vertex $Q$ $\left(21 \div 6=\frac{7}{2}\right.$ and $\left.7 \div 2=\frac{7}{2}\right)$, so the rule $(x, y) \rightarrow\left(\frac{7}{2} x, \frac{7}{2} y\right)$ represents the dilation. |
|  | Option F is incorrect | The student likely divided the corresponding values of the original vertex (vertex $Q$ ) by the corresponding values of the dilated vertex (vertex $\left.Q^{\prime}\right)\left(6 \div 21=\frac{2}{7}\right.$ and $\left.2 \div 7=\frac{2}{7}\right)$ and therefore chose the rule $(x, y) \rightarrow\left(\frac{2}{7} x, \frac{2}{7} y\right)$. The student needs to focus on the effect a dilation by a given scale factor has on a figure and how to determine the rule to explain the effect. |
|  | Option G is incorrect | The student likely thought of a translation (slide) and added different values to the coordinates of the original vertex (vertex $Q$ ) to get to vertex $Q^{\prime}$, instead of multiplying each coordinate by a single scale factor, choosing the rule $(x, y) \rightarrow(x+15, y+5)$. The student needs to focus on understanding how the scale factor affects the dilation. |
|  | Option H is incorrect | The student likely thought of a translation (slide) and subtracted different values from the coordinates of vertex $Q^{\prime}$ to get to the original vertex (vertex $Q$ ), instead of multiplying each coordinate by a single scale factor, choosing the rule $(x, y) \rightarrow(x-15, y-5)$. The student needs to focus on understanding how the scale factor affects the dilation. |


| Item\# |  | Rationale |
| :---: | :---: | :---: |
| 21 | Option C is correct | To determine which function (equation) is represented by the graph (line) passing through the points $\left(-1,-\frac{1}{4}\right)$ and $\left(1,-\frac{3}{4}\right)$, the student should have determined the values of $m$ and $b$ in the slope-intercept form of the line, $y=m x+b$. The $m$ represents the slope (steepness of a straight line when graphed on a coordinate grid; $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$ ) and the $b$ represents the $y$-intercept (value where a line crosses the $y$-axis (vertical)) of the line. The student should have determined the slope using the given points; $m=\frac{-\frac{3}{4}-\left(-\frac{1}{4}\right)}{1-(-1)}=\frac{-\frac{1}{2}}{2}=-\frac{1}{2} \cdot \frac{1}{2}=-\frac{1}{4}$. The student could have then substituted the value of the slope and the point $\left(1,-\frac{3}{4}\right)$ into the slope-intercept form equation, $\left(-\frac{3}{4}\right)=\left(-\frac{1}{4}\right)(1)+b$, to solve for $b ; b=\left(-\frac{3}{4}\right)+\left(\frac{1}{4}\right)=-\frac{1}{2}$. Substituting $m$ with $-\frac{1}{4}$ and $b$ with $-\frac{1}{2}$, the function that represents the line is $y=-\frac{1}{4} x-\frac{1}{2}$. This is an efficient way to solve the problem; however, other methods could be used to solve the problem correctly. |
|  | Option A is incorrect | The student likely reversed the placement of the slope and the $y$-intercept in the slope-intercept form of the equation, resulting in $y=-\frac{1}{2} x-\frac{1}{4}$. The student needs to focus on attending to detail when substituting values into the formula to find the slope and when identifying the $y$-intercept in order to write the function that passes through two points. |
|  | Option B is incorrect | The student likely reversed the placement of the slope and the $y$-intercept in the slope-intercept form of the equation and reversed the sign (positive and negative value) of each, resulting in $y=\frac{1}{2} x+\frac{1}{4}$. The student needs to focus on attending to detail when substituting values into the formula to find the slope and when identifying the $y$-intercept in order to write the function that passes through two points. |

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| Item\# | Rationale |  |
| :--- | :--- | :--- |
|  | Option D is incorrect | The student likely miscalculated the slope and the $y$-intercept by reversing the sign (positive and <br> negative value) of each, resulting in $y=\frac{1}{4} x+\frac{1}{2}$. <br> when substituting values into the formula to find the slope and when identifying the $y$-intercept in <br> order to write the function that passes through two points. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 22 | Option F is correct | Scientific notation is used to express very large and very small numbers. Instead of writing <br> 0.00000001 , this value can be expressed as a multiple of $10: \frac{1}{100,000,000}=\frac{1}{10^{8}}=10^{-8}$. The shorthand method of translating from standard form to scientific notation is to move the decimal point left (when the number is greater than 1) or right (when the number is less than 1) until there is only 1 non-zero number before the decimal point. Any extra zeros prior to that number are not included within the scientific notation. The number of decimal places the point moves determines the value of the exponent (number raised to a power). The direction the decimal point is moved determines whether the exponent is positive or negative. Moving the decimal point to the right creates a negative exponent. Moving the decimal point to the left creates a positive exponent. Since the decimal point moves to the left 6 places, the exponent is 6 , resulting in $4.951 \times 10^{6}$. |
|  | Option G is incorrect | The student likely only counted the number of places the decimal point moved and did not consider which direction the decimal point moved. The student needs to focus on understanding how to determine the exponent based on the place value when converting standard form to scientific notation. |
|  | Option H is incorrect | The student likely counted the number of zeros before the decimal point (not shown in the original number) as the exponent. The student needs to focus on understanding how to determine the exponent based on the place value when converting standard form to scientific notation. |
|  | Option J is incorrect | The student likely counted the number of zeros before the decimal point (not shown in the original number) as the exponent and did not consider which direction the decimal point moved. The student needs to focus on understanding how to determine the exponent based on the place value when converting standard form to scientific notation. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 23 | 736 and any equivalent values are correct | To determine the total surface area (total area of the surfaces of a three-dimensional figure) of the prism, the student could have used the formula $S=P h+2 B$, where $P$ represents the perimeter of the base, $h$ represents the height of the prism, and $B$ represents the area of the base. To determine the perimeter of the rectangular base, the student should have added the four sides of the rectangle, $P=16+16+8+8=48$. To determine the area of the rectangular base, the student should have multiplied 16 by 8 , resulting in 128. The student should have identified $h$ as 10 from the diagram and evaluated $S=(48)(10)+2(128)$, which is equal to 736 . This is an efficient way to solve the problem; however, other methods could be used to solve the problem correctly. |


| Item\# |  | Rationale |
| :---: | :---: | :---: |
| 24 | Option H is correct | To determine which graph has a slope (steepness of a straight line when graphed on a coordinate grid; $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$ ) that best represents the number of customers that were served per hour at the restaurant, the student should have identified that 600 customers served during a 10 -hour period of time is the same as 60 customers served in 1 hour. The student could have identified that there were 0 customers served in 0 hours. Then the student could have determined that the graph appears to pass through the points located at $(0,0)$ and $(1,60)$ and the slope between these points is represented by the equation $m=\frac{60-0}{1-0}=\frac{60}{1}$, which is equal to 60 . This is an efficient way to solve the problem; however, other methods could be used to solve the problem correctly. |
|  | Option F is incorrect | The student likely calculated the slope as 600 and chose the graph in which the $y$-value is always equal to 600 . The student needs to focus on understanding the meaning of the value of a slope and identifying the graph that correctly represents the slope. |
|  | Option G is incorrect | The student likely calculated the slope correctly as 60 but chose the graph in which the $y$-value is always equal to 60 . The student needs to focus on understanding the meaning of the value of a slope and identifying the graph that correctly represents the slope. |
|  | Option J is incorrect | The student likely thought the line must pass through $(0,0)$ and through the grid line marked 600 (representing 600 customers) without regard to the corresponding (paired) $x$-value of the intersecting (crossing over) grid line. The student needs to focus on understanding the meaning of the value of a slope and identifying the graph that correctly represents the slope. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 25 | Option B is correct | To determine the difference between the two savings accounts with different types of interest, the student should have calculated the amount of interest in each account and then subtracted the two amounts. To determine the amount of interest in Account I, the student should have used the formula for simple interest, $I=P r t$, in which $P$ represents the principal (initial loan amount), $r$ represents the interest rate (in decimal form), and $t$ represents the length of time in years, $I=(\$ 1,750)(0.0275)(2)=\$ 96.25$. To determine the balance in Account II, the student should have used the formula for compound interest, $A=P(1+r)^{t}$, in which $A$ represents the balance amount (principal and interest combined), $A=(\$ 1,750)(1+0.0275)^{2}=\$ 1,847.57$. The student should have then subtracted the principal from the balance amount, $\$ 1,847.57-\$ 1,750=\$ 97.57$. Finally the student should have subtracted (\$97.57-\$96.25) and determined that the difference between the interest of the two accounts at the end of 2 years was $\$ 1.32$. |
|  | Option A is incorrect | The student likely determined only the amount of the simple interest, resulting in $\$ 96.25$. The student needs to focus on attending to the details of the question in problems. |
|  | Option C is incorrect | The student likely determined only the amount of the compound interest, resulting in \$97.57. The student needs to focus on attending to the details of the question in problems. |
|  | Option D is incorrect | The student likely added the amount of interest from each account (\$97.57+\$96.25), resulting in $\$ 193.82$. The student needs to focus on attending to the details of the question in problems. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 26 | Option F is correct | To determine the $y$-intercept (value where a line crosses the $y$-axis) of the graph of the function, the student could have identified the point on the graph where the line appears to pass through the $y$-axis (vertical), which is at 5 . This is an efficient way to solve the problem; however, other methods could be used to solve the problem correctly. |
|  | Option G is incorrect | The student likely confused the $y$-intercept by finding the approximate point on the graph where the line would pass through the $x$-axis (horizontal). The student needs to focus on paying attention to details when identifying the $y$-intercept of a linear function. |
|  | Option H is incorrect | The student likely confused the $y$-intercept with the slope and used the formula $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$ when determining the slope, resulting in $m=\frac{(-2)-5}{8-0}=-\frac{7}{8}$. The student needs to focus on paying attention to details when identifying the $y$-intercept of a linear function. |
|  | Option J is incorrect | The student likely confused the $y$-intercept with the slope and used the incorrect formula, $m=\frac{x_{2}-x_{1}}{y_{2}-y_{1}}$, when determining the slope, resulting in $m=\frac{8-0}{(-2)-5}=-\frac{8}{7}$. The student needs to focus on paying attention to details when using the formula to find the slope of a linear function and when identifying the $y$-intercept of a linear function. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 27 | Option D is correct | To determine which statement about $x$ is true, the student should have determined that the intersection (crossing) of transversal (line) $t$ with the two parallel lines ( $g$ and $k$ ) produced pairs of angles that are congruent (same measure) and pairs of angles that are supplementary (have a sum of $180^{\circ}$ ). The student should have determined that the angles labeled $x^{\circ}$ and $136^{\circ}$ are congruent. |
|  | Option A is incorrect | The student most likely determined that the intersection of transversal $t$ with the two parallel lines produced pairs of angles that are congruent and pairs of angles that are supplementary. The student then determined that the angle labeled $x^{\circ}=180$, or $180^{\circ}$. The student needs to focus on the relationship between angles when parallel lines are intersected by a transversal. |
|  | Option B is incorrect | The student most likely determined that the intersection of transversal $t$ with the two parallel lines produced pairs of angles that are congruent and pairs of angles that are complimentary (have a sum of $90^{\circ}$ ) instead of supplementary. The student then determined that the angle labeled $x^{\circ}=90$, or $90^{\circ}$. The student needs to focus on the relationship between angles when parallel lines are intersected by a transversal. |
|  | Option C is incorrect | The student most likely determined that the intersection of transversal $t$ with the two parallel lines produced pairs of angles that are congruent and pairs of angles that are supplementary. The student then determined that the angles labeled $x^{\circ}$ and $136^{\circ}$ are supplementary and determined that $x^{\circ}=180-136$, or $44^{\circ}$. The student needs to focus on the relationship between angles when parallel lines are intersected by a transversal. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 28 | Option H is correct | To determine the value of $x$, the student could have written and solved an equation such as $2 x+(4 x-2)+2(x+7)=4(2.5 x)$, where $x$ is a fixed length in yards. The student could have distributed the values in front of the parentheses to the values inside the parentheses and combined $x$ terms, resulting in the equation $8 x+12=10 x$. The student then could have solved for $x$ by subtracting $8 x$ from both sides of the equation, resulting in $12=2 x$. Finally the student could have solved for $x$ by dividing both sides of the equation by 2 , resulting in $x=6$. This is an efficient way to solve the problem; however, other methods could be used to solve the problem correctly. |
|  | Option F is incorrect | The student likely set up the equation incorrectly as $2 x+(4 x-2)+2(x+7)=2(2.5 x)$. The student then likely rewrote the equation as $8 x+12=5 x$. The student then likely solved for $x$ by subtracting $8 x$ from both sides of the equation, resulting in $-3 x=12$. To solve for $x$, the student likely divided both sides of the equation by -3 , resulting in $x=-4$. The student needs to focus on writing the correct equation for a word problem. |
|  | Option G is incorrect | The student likely set up the correct equation and rewrote the equation as $8 x+12=10 x$ but added $8 x$ to both sides of the equation instead of subtracting, resulting in $12=18 x$. To solve for $x$, the student likely divided both sides of the equation by 18 , resulting in $x=\frac{2}{3}$. The student needs to focus on using the proper steps to solve an equation. |
|  | Option J is incorrect | The student likely did not distribute 2 correctly to both $x$ and 7 in the term $2(x+7)$ and set up the equation incorrectly as $2 x+(4 x-2)+(2 x+7)=4(2.5 x)$. The student then likely rewrote the equation as $8 x+5=10 x$. The student then likely solved for $x$ by subtracting $8 x$ from both sides of the equation, resulting in $2 x=5$. To solve for $x$, the student likely divided both sides of the equation by 2 , resulting in $x=\frac{5}{2}$. The student needs to focus on writing the correct equation for a word problem. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 29 | Option B is correct | To determine why the balance in the account has grown, the student should have used the table to determine that money is being deposited into the account each year ( $\$ 500.00$ ) and interest is being earned on the account balance each year ( $\$ 32.00, \$ 66.05$, and $\$ 102.28$ ). This means that the balance in the account is growing due to the deposits made each year and the interest earned being added each year. |
|  | Option A is incorrect | The student likely understood that deposits being made to the account increase the balance in the account but did not realize that the balance is also increasing by the amount of interest earned. The student needs to focus on understanding the vocabulary used for showing how an account balance may grow over time. |
|  | Option C is incorrect | The student likely understood that interest earned increases the balance in the account but did not realize that the balance is also increasing by the amount being deposited. The student needs to focus on understanding the vocabulary used for showing how an account balance may grow over time. |
|  | Option D is incorrect | The student likely misunderstood the meaning of the terms "amount deposited," "interest earned," or "ending balance." The student needs to focus on understanding the vocabulary used for showing how an account balance may grow over time. |

## Rationale

Option J is correct
To identify the phrase that describes the transformation (change of a shape using a rotation (circular movement), reflection (flip), translation (slide), or dilation (resize)), the student should have determined that the transformation used was a rotation $180^{\circ}$ clockwise about the origin (point where the $x$-axis (horizontal) and the $y$-axis (vertical) on a coordinate grid intersect (cross); also, the point represented by the ordered pair $(0,0))$ because of the rule $(x, y) \rightarrow(-x,-y)$. The student made this determination based on how the $x$-value in the ordered pair changed to the opposite sign and the $y$-value in the ordered pair changed to the opposite sign. Therefore the phrase "A $180^{\circ}$ clockwise rotation about the origin" describes the transformation.
Option F is incorrect

The student likely identified the transformation as a reflection of $(x, y)$ across the $x$-axis but did not realize that the sign of the $x$-value would have remained the same instead of changing to the opposite sign. The student needs to focus on correctly identifying the type of transformation that was used and using the rule to explain the effect on $(x, y)$ when graphed on a coordinate grid.

Option G is incorrect The student likely identified the transformation as a reflection of $(x, y)$ across the $y$-axis but did not realize that the sign of the $y$-value would have remained the same instead of changing to the opposite sign. The student needs to focus on correctly identifying the type of transformation that was used and using the rule to explain the effect on $(x, y)$ when graphed on a coordinate grid.

Option H is incorrect

The student likely identified the transformation as a rotation of $(x, y)$ but did not realize that $(x, y)$ would have used the rule $(x, y) \rightarrow(y,-x)$ instead of $(x, y) \rightarrow(-x,-y)$ for a $90^{\circ}$ clockwise rotation about the origin. The student needs to focus on correctly identifying the type of transformation that was used and using the rule to explain the effect on $(x, y)$ when graphed on a coordinate grid.

| Item\# |  | Rationale |
| :---: | :---: | :---: |
| 31 | Option A is correct | To determine which graph shows the relationship between $y$, the number of customers who preferred the new spaghetti sauce, and $x$, the total number of customers, the student could have determined that for every 10 customers, 7 prefer the new sauce, so the unit rate is $\frac{7}{10}$, graphing the ordered pairs $(0,0)$ and $(10,7)$. The student should have then identified the graph that shows a line that passes through those points (a survey of 0 customers shows 0 customers prefer the new sauce, and a survey of 10 customers shows 7 customers prefer the new sauce). The line in this graph appears to pass through these points. |
|  | Option B is incorrect | The student likely inverted (flipped) the unit rate using $\frac{10}{7}$ instead of $\frac{7}{10}$, graphing the ordered pairs $(0,0)$ and $(7,10)$. The student needs to focus on determining the unit rate and how that translates to the graph. |
|  | Option C is incorrect | The student likely used 7 customers who preferred the sauce as the $y$-intercept (value where the line intersects the $y$-axis (vertical)) along with the correct unit rate, graphing the ordered pairs $(0,7)$ and $(10,14)$. The student needs to focus on determining the $y$-intercept of the graph. |
|  | Option D is incorrect | The student likely used 7 customers who preferred the sauce as the $x$-intercept (value where the line intersects the $x$-axis (horizontal)) along with the inverted (flipped) unit rate of $\frac{10}{7}$ instead of $\frac{7}{10}$, graphing the ordered pairs $(7,0)$ and $(14,10)$. The student needs to focus on determining the unit rate and the $y$-intercept of the graph. |


| Item\# |  | Rationale |
| :---: | :---: | :---: |
| 32 | Option G is correct | To determine which proportion (comparison of two ratios) is true for similar figures (two figures with corresponding angles that are equal and corresponding sides that are proportional), the student should have determined that the angles in triangle $A B C$ and triangle $D E F$ are corresponding (paired) and are equal, which means that the sides of the figures forming those equal angles are also corresponding and their lengths are proportional. The student then should have determined that the ratio (comparison of two or more values) $\frac{4}{6}$ represents the ratio of the length in inches of the left side in triangle $D E F$ to the length in inches of the left side in triangle $A B C$. The ratio $\frac{x}{7}$ represents the ratio of the length in inches of the bottom side in triangle $D E F$ to the length in inches of the bottom side in triangle $A B C$. |
|  | Option F is incorrect | The student likely reversed the proportions on the right side of the equation; $\frac{7}{x}$ equals $\frac{6}{4}$ instead of $\frac{4}{6}$. The student needs to focus on paying attention to detail when identifying the correspondence between the sides of similar figures. |
|  | Option H is incorrect | The student likely reversed the proportions on the right side of the equation; $\frac{6}{4}$ equals $\frac{7}{x}$ instead of $\frac{x}{7}$. The student needs to focus on paying attention to detail when identifying the correspondence between the sides of similar figures. |
|  | Option J is incorrect | The student likely reversed the proportions on the right side of the equation; $\frac{4}{x}$ equals $\frac{6}{7}$ instead of $\frac{7}{6}$. The student needs to focus on paying attention to detail when identifying the correspondence between the sides of similar figures. |


| Item\# |  | Rationale |
| :---: | :---: | :---: |
| 33 | Option A is correct | To determine the order of the numbers from least (smallest) to greatest (largest), the student could have converted each value to the same form so a comparison could be made. The student could have converted $\sqrt{38}$ to approximately $6.16,63 \%$ to 0.63 , and $-\frac{19}{3}$ to approximately -6.33 . Then the student should have ordered the decimal values ( $6.16,-6.35,0.63,-6.33$ ) from least to greatest, which means the correct order of the given values from least to greatest would be $-6.35,-\frac{19}{3}, 63 \%$, $\sqrt{38}$. This is an efficient way to solve the problem; however, other methods could be used to solve the problem correctly. |
|  | Option B is incorrect | The student likely ignored the negative signs. The student needs to focus on attending to the details of the question in problems. |
|  | Option C is incorrect | The student likely converted $63 \%$ to 6.3 . The student needs to focus on converting percentages into decimals correctly in order to make comparisons. |
|  | Option D is incorrect | The student likely reversed the order of the negative values thinking that -6.35 is greater than $-\frac{19}{3}$. The student needs to focus on comparing negative values correctly. |

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| Item\# | Rationale |  |  |
| :---: | :--- | :--- | :---: |
| 34 | 0.5 and any equivalent <br> values are correct | To determine the slope (steepness of a straight line when graphed on a coordinate grid; $\left.m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}\right)$, <br> the student could have calculated the slope of the line that passes through the points located at |  |
|  |  | $(-6,-5)$ and $(6,1)$ using $m=\frac{1-(-5)}{6-(-6)}=\frac{6}{12}=\frac{1}{2}$, which is equal to 0.5. This is an efficient way to <br> solve the problem; however, other methods could be used to solve the problem correctly. |  |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 35 | Option B is correct | To determine which inequality represents the situation, the student could have used the information from the problem that the length of $\overline{A B}(9 x-16$ units) is greater than the length of $\overline{A D}(1.5 x+42$ units), resulting in $9 x-16>1.5 x+42$. This is an efficient way to solve the problem; however, other methods could be used to solve the problem correctly. |
|  | Option A is incorrect | The student likely misinterpreted the inequality symbol. The student needs to focus on using the correct symbols to write inequalities. |
|  | Option C is incorrect | The student likely misinterpreted the inequality symbol and created the inequality $9 x-16<1.5 x+42$ but moved $1.5 x$ to the left side of the inequality and moved -16 to the right side of the inequality, resulting in $10.5 x<26$. The student needs to focus on using the correct symbols to write inequalities and needs to focus on accurately writing algebraic expressions that represent verbal descriptions. |
|  | Option D is incorrect | The student likely created the inequality $9 x-16>1.5 x+42$ but moved $1.5 x$ to the left side of the inequality and moved -16 to the right side of the inequality, resulting in $10.5 x>26$. The student needs to focus on accurately writing algebraic expressions that represent verbal descriptions. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 36 | Option J is correct | To determine which measurement is closest to the length of $\overline{X Y}$ in units, the student should have determined that the line segment represents the hypotenuse (longest side) of a right triangle (a closed figure with three sides and one 90-degree angle). The lengths in units of the two legs of the right triangle can be represented by subtracting the $x$-coordinates (horizontal position from 0 ) of the two endpoints of the line segment (the part of a line connecting two endpoints) and by subtracting the $y$-coordinates (vertical position from 0 ) of the two endpoints of the line segment. This means that the lengths in units of the legs are $5-(-5)=10$ and $6-0=6$. Using the Pythagorean theorem (in a right triangle, the square of the hypotenuse (longest side) is equal to the sum (total) of the squares of the other two sides; $a^{2}+b^{2}=c^{2}$ or $c=\sqrt{a^{2}+b^{2}}$ ), the length in units of line segment $X Y$ can be represented by the expression $\sqrt{10^{2}+6^{2}}$, or $\sqrt{136}$, which has a value of approximately 11.7 units. Therefore the approximate length of line segment $X Y$ is 11.7 units. This is an efficient way to solve the problem; however, other methods could be used to solve the problem correctly. |
|  | Option F is incorrect | The student likely used the coordinates of point $X(5,6)$ to identify the lengths of the sides of the right triangle as 5 and 6 (using only the values of those coordinates) and used the Pythagorean theorem, solving $\sqrt{5^{2}+6^{2}}$, or $\sqrt{61}$, resulting in a value of approximately 7.8 units. The student needs to focus on how to properly apply the Pythagorean theorem using the given information. |
|  | Option G is incorrect | The student likely identified the lengths of the sides of the right triangle (10 and 6) but added them instead of using the Pythagorean theorem. The student needs to focus on recognizing when the Pythagorean theorem should be used and how to properly apply it with the given information. |
|  | Option H is incorrect | The student likely counted the lengths of the sides of the right triangle incorrectly as 11 and 7 instead of 10 and 6 and used the Pythagorean theorem, solving $\sqrt{11^{2}+7^{2}}$, or $\sqrt{170}$, resulting in a value of approximately 13.0 units. The student needs to focus on how to properly apply the Pythagorean theorem using the given information. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 37 | Option B is correct | To determine which statement is true, the student should have recognized that in a non-proportional relationship, the $y$-value is not equal to 0 when the corresponding (paired) $x$-value is 0 . In this graph, when the $x$-value is equal to 0 , the corresponding $y$-value would equal 7 , resulting in the statement that the graph shows a non-proportional relationship because the graph does not contain the origin (point where the $x$-axis (horizontal) and the $y$-axis (vertical) on a coordinate grid intersect (cross); also, the point represented by the ordered pair $(0,0)$ ). |
|  | Option A is incorrect | The student likely identified the slope of the line as a negative slope (steepness of a straight line when graphed on a coordinate grid) and interpreted the negative slope as meaning the line showed a non-proportional relationship. The student needs to focus on the definition of a proportional relationship and on applying it to determine whether a graph represents a proportional relationship. |
|  | Option C is incorrect | The student likely identified that the $x$-intercept (value where the line intersects the $x$-axis (horizontal)) and the $y$-intercept (value where the line crosses the $y$-axis (vertical)) of the line both contain 7 and interpreted these intercepts as meaning the line showed a proportional relationship. The student needs to focus on the definition of a proportional relationship and on applying it to determine whether a graph represents a proportional relationship. |
|  | Option D is incorrect | The student likely identified that the line passes through three quadrants and interpreted the passing through three quadrants as meaning the line showed a proportional relationship, instead of recognizing that a proportional relationship can only be in two quadrants. The student needs to focus on the definition of a proportional relationship and on applying it to determine whether a graph represents a proportional relationship. |

## 2021 STAAR Grade 8 Mathematics Rationales

| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 38 | 61 and any equivalent values are correct | To determine the length of the ramp in feet, the student should have determined that because the diagram represents a right triangle (a closed figure with three sides and one 90-degree angle) with legs of 60 feet and 11 feet, the Pythagorean theorem (in a right triangle, the square of the hypotenuse (longest side) is equal to the sum (total) of the squares of the other two sides, $a^{2}+b^{2}=c^{2}$ or $c=\sqrt{a^{2}+b^{2}}$ ) can be used to calculate this distance. The distance in feet is equal to $\sqrt{60^{2}+11^{2}}$. When the two values under the radical symbol $(\sqrt{ })$ are squared, the result is $\sqrt{3,600+121}$, which equals $\sqrt{3,721}$. The value of $\sqrt{3,721}$ is 61 , which means the distance of the ramp is 61 feet. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 39 | Option D is correct | To determine which measurement is closest to $x$, the length of the diagonal of the square in centimeters, the student could have determined that $x^{2}=132$ can be solved for $x$ by taking the square root of both sides of the equation $\left(\sqrt{x^{2}}=\sqrt{132}\right.$ or $\left.x=\sqrt{132}\right)$, resulting in 11 centimeters being closest to the measurement of $x$. This is an efficient way to solve the problem; however, other methods could be used to solve the problem correctly. |
|  | Option A is incorrect | The student likely divided 132 by 2 instead of taking the square root, resulting in 66 centimeters. The student needs to focus on understanding how to find the square root of a number. |
|  | Option B is incorrect | The student likely divided 132 by 3 instead of taking the square root, resulting in 44 centimeters. The student needs to focus on understanding how to find the square root of a number. |
|  | Option C is incorrect | The student likely divided 132 by 4 instead of taking the square root, resulting in 33 centimeters. The student needs to focus on understanding how to find the square root of a number. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 40 | Option F is correct | To determine which two cylinders have the same lateral surface area (total area of the surfaces of a three-dimensional figure, not including the area of the bases), the student could have used the formula for the lateral surface area of a cylinder, $S=2 \pi r h$, where $r$ represents the radius (distance from the center to the circumference of a circle) of the circular base and $h$ represents the height (vertical distance from top to bottom) of the cylinder. To determine the lateral surface area for Cylinder W, the student could have substituted the given values and an approximate value for $\pi$ in the formula for lateral surface area and evaluated $S=(2)(3.14)(3)(9)$, which is approximately equal to 169.56 square inches. To determine the lateral surface area for Cylinder $X$, the student could have substituted the given values and an approximate value for $\pi$ in the formula for lateral surface area and evaluated $S=(2)(3.14)(4)(2)$, which is approximately equal to 50.24 square inches. To determine the lateral surface area for Cylinder $Y$, the student could have substituted the given values and an approximate value for $\pi$ in the formula for lateral surface area and evaluated $S=(2)(3.14)(4.5)(6)$, which is approximately equal to 169.56 square inches. The student then could have compared the values, resulting in Cylinder W and Cylinder $Y$ having the same lateral surface area. This is an efficient way to solve the problem; however, other methods could be used to solve the problem correctly. |
|  | Option G is incorrect | The student likely noticed that the sum (total) of each radius for Cylinders $W$ and $X$ is $7(3+4=7)$ and the difference of each height for Cylinders $W$ and $X$ is $7(9-2=7)$ and therefore assumed both cylinders had the same lateral surface area. The student needs to focus on understanding and properly applying the formula for determining the lateral surface area of cylinders. |
|  | Option H is incorrect | The student likely noticed that the sum (total) of the radius and the height for Cylinder X is 6 $(4+2=6)$ and the height for Cylinder $Y$ is 6 and therefore assumed both cylinders had the same lateral surface area. The student needs to focus on understanding and properly applying the formula for determining the lateral surface area of cylinders. |
|  | Option J is incorrect | The correct answer (Cylinders W and Y ) was presented in one of the other answer options. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 41 | Option C is correct | To determine the best prediction of the high temperature in degrees Fahrenheit on a day when the low temperature is $71^{\circ}$ Fahrenheit using a scatterplot (a graph of plotted points that shows the relationship between two sets of data), the student could have drawn a line that closely follows the pattern formed by the points on the graph by keeping about half of the points above the line and the other half below the line. A good line for this scatterplot would pass above the point at (73,90), slightly above the point at $(74,92)$, and slightly above the point at $(79,99)$, but passing below the point $(79,100)$. The student could have then identified where the grid line marked 71 (representing a low temperature of $71^{\circ}$ Fahrenheit) intersects (crosses over) the line the student drew and determined that the high temperature in degrees Fahrenheit corresponding to that point is about $88^{\circ}$ Fahrenheit. This is an efficient way to solve the problem; however, other methods could be used to solve the problem correctly. |
|  | Option A is incorrect | The student likely chose the points at $(76,96)$ and $(77,97)$ to draw the line to make the prediction. However, this line has five points above and seven points below. The student needs to focus on drawing a line as close as possible to all points with a similar number of points above and below the line. |
|  | Option B is incorrect | The student likely drew a line that closely follows the pattern formed by the points on the graph by keeping about half of the points above the line and the other half below the line but could have identified where the grid line marked 80 (representing a low temperature of $80^{\circ}$ Fahrenheit) intersects the line the student drew and determined that the high temperature in degrees Fahrenheit corresponding to that point is about $101^{\circ}$ Fahrenheit. The student needs to focus on attending to the details of the question in problems. |
|  | Option D is incorrect | The student likely focused on the highest dot that is the furthest to the right located at $(79,100)$ and chose the high temperature at that point, resulting in $100^{\circ}$ Fahrenheit. The student needs to focus on drawing a line as close as possible to all points with a similar number of points above and below the line. |


| Item\# | Rationale |  |
| :---: | :---: | :---: |
| 42 | Option F is correct | To identify the graph with a slope (steepness of a straight line when graphed on a coordinate grid; $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$ ) that best represents the cost of each pencil at the store, the student could have determined that the cost of 30 cents for 6 pencils is equivalent to a cost of 5 cents for each pencil ( $30 \div 6=5$ ). Then the student could have determined that the graph appears to pass through the points located at $(0,0)$ and $(6,30)$ and the slope between these points is represented by the equation $m=\frac{30-0}{6-0}=\frac{30}{6}=\frac{5}{1}$, which is equal to 5 . This is an efficient way to solve the problem; however, other methods could be used to solve the problem correctly. |
|  | Option G is incorrect | The student likely misinterpreted 6 pencils as 1 pencil, resulting in a slope of 30 . The student needs to focus on using the formula to find the value of the slope and identifying the graph that represents the slope. |
|  | Option H is incorrect | The student likely calculated the slope correctly as 5 but chose the graph in which the $x$-value and the $y$-value are each equal to 5 . The student needs to focus on understanding the meaning of the value of a slope and identifying the graph that correctly represents the slope. |
|  | Option J is incorrect | The student likely calculated the slope as 6 and chose the graph in which the $x$-value is always equal to 6 . The student needs to focus on understanding the meaning of the value of a slope and identifying the graph that correctly represents the slope. |


[^0]:    Texas Education Agency Student Assessment Division

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