

Lesson Proposal

3rd Grade, Math, Properties of Multiplication

November 1, 2016

Region 5, Port Neches-Groves ISD, Ridgewood Elementary School, Port Neches, Texas

Students: Tracie Billeaud's class, 22 students

Lesson observation instructor: Tracie Billeaud

Lesson proposal developed by: Tracie Billeaud, Angie Melancon, Paige Tobias

Lesson Title:

Distributive Property of Multiplication

TEKS Addressed:

3(4)(K) solve one-step and two-step problems involving multiplication and division within 100 using strategies based on objects; pictorial models, including arrays, area models, and equal groups; properties of operations; or recall of facts

Brief Description of the Lesson:

Students break an array apart to represent the sum of two multiplication facts, showing the distributive property.

Research Theme:

This is a theme created by the Texas Lesson Study (TXLS) group to guide their thinking throughout the Lesson Study process. It is an overarching goal that unifies the work of the campus and teachers.

Create students who listen intently in order to communicate effectively and gain a complete understanding of the skill being taught.

Goals of the Unit and Research Lesson:

Students will learn to do the following:

1. Recall multiplication facts from 0–10 with automaticity and understand multiplication using patterns.
2. Use the distributive property to break apart a large array into smaller arrays.

Relationship of the Unit to the TEKS:

As we looked at the vertical alignment document document available to teachers through the Texas Education Agency and regional TEKS resource pages, it became evident why students struggle with this standard. The skills and knowledge within this standard are not addressed in the second grade TEKS. Students are also required to use multiple skills to correctly calculate the answer using the distributive property. The chosen standard is identified as a third grade readiness standard, which means that the intent is to help students develop a deep understanding of how to effectively use these skills in mathematics. This deep understanding is desired to help students at the fourth grade level since the standard is identified as a supporting standard.

Read below to learn about the TEKS related to this unit. The highlighted student expectation(s) is the chosen focus for the Research Lesson.

Related TEKS Taught Prior to the Unit:

3(4)(D) determine the total number of objects when equally-sized groups of objects are combined or arranged in arrays up to 10 by 10

3(4)(E) represent multiplication facts by using a variety of approaches such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line, and skip counting

TEKS that will be taught for this unit:

3(1)(D) communicate mathematical ideas, reasoning, and their implications using multiple representations,

including symbols, diagrams, graphs, and language as appropriate

3(4)(F) recall facts to multiply up to 10 by 10 with automaticity and recall the corresponding division facts

3(4)(K) solve one-step and two-step problems involving multiplication and division within 100 using strategies based on objects; pictorial models, including arrays, area models, and equal groups; properties of operations; or recall of facts

Materials:

- Dry-erase sleeves
- Place value blocks (unit cubes)
- Popsicle sticks
- Bingo daubers
- [Focus page](#)
- [Pretest](#) and [posttest](#)
- Graphic Organizers
 - [“Distributive property graphic”](#) handout
 - [“Distributive property of multiplication with arrays”](#) handout
 - [“Distributive property of multiplication page without models”](#) handout
 - [“Distributive property practice”](#) handout

Assessment:

Students will take a six-question [pretest](#) and [posttest](#) that asks them to perform the operations learned during the research lesson. The open-ended questions will show students’ work and thus their knowledge of how to effectively calculate answers using the distributive property.

Background and Rationale:

This section explains the rationale for selecting the student expectation(s) (TEKS) and the instructional decisions made by the Texas Lesson Study (TXLS) group.

We selected this unit of study based on our observations of students struggling with learning the distributive property using a variety of models. Additionally, State of Texas Assessments of Academic Readiness (STAAR) scores indicated a need for further study. For 3(4)(K), third grade students at Ridgewood elementary school averaged 76 percent mastery on the 2015 STAAR exam and 70 percent mastery on the 2016 STAAR exam. This qualifies it as an area of concern.

Prior to this unit, in second grade, students explored unitizing, which requires counting objects by groups (e.g., four groups of three may be counted as 3, 6, 9, 12). Second-grade students also used repeated addition or skip counting (e.g., counting from 23 by 10s or counting by 5s from 20) and related these big ideas to models of equal-sized groups in contextual multiplication situations.

During this unit, students represent multiplication facts through the use of context. These contextual situations provide real-life experiences and enable students to construct multiplication models (concrete, pictorial, and area models), equal groups, arrays, strip diagrams, and equations in a relevant way. Students also explore the commutative (if the order of the factors are changed, the product remains the same), associative (if three or more factors are multiplied, they can be grouped in any order, and the product will remain the same), and distributive (if multiplying a number by a sum of numbers, the product will be the same as multiplying the number by each addend and then adding the products together) properties of multiplication in order to provide the foundation needed to learn, retain, and apply basic multiplication facts up to 10 x 10. By applying the commutative property of multiplication, the number of basic facts needing to be recalled is reduced by half. The distributive and associative properties of multiplication also aid in developing automatic recall of facts, or automaticity, by allowing students to decompose a factor(s), find known partial products, and compose the partial products to determine the total product (e.g., if a student knows that $2 \times 9 = 18$, then he or she can use the commutative property and doubling to find the product of 9×4). Although not expected to learn these properties of multiplication by name, students are expected to be able to analyze, describe, and represent these strategies. Students’ understanding of multiplication is strengthened by solving one-step problems, including problems involving area of rectangles, and making sense of meaningful, efficient representations and strategies.

After this unit, students will use familiar multiplication models along with their understanding of multiplication properties to develop an understanding of division.

Research:

The information below is a summary of the research findings by the TXLS group.

According to the Crosnoe et al. (2010), children enter elementary school with widely different skill levels in core subjects. Whatever the reason for differences in aptitude or preparedness, these initial skill differences often translate into systematic disparities in achievement over time. The authors offered three solutions for decreasing the differences: offer basic skills training, expose students to higher-order instruction, and provide socioemotional support. As we built the research lesson, we decided to focus on the basic skills of students. We wanted to use the basic skills that students possess and include higher-order instruction that focuses on including both the concrete and abstract in one lesson. Students are asked to take a minimal skill needed to complete the exercise and utilize higher-order instruction during the lesson to form a deeper understanding of the concept.

The Ding and Li (2014) study highlights the importance of cognitive development in Chinese textbooks. The authors found that the textbooks studied are designed in a manner in which students can learn to transition concepts from concrete to abstract. In addition to highlighting transitional skills needed to learn mathematics, the authors stress that the textbooks help students learn in a manner that is developmentally appropriate. We considered the results of the study as we planned the lesson. We considered age-appropriate activities that highlighted ways in which students could understand concrete information and show their comprehension using abstract models.

As we looked at the type of activities we should use in our research lesson, we reviewed the work of Skarr et al. (2014). This study looked at three students: a boy and a girl with normal learning patterns and one boy with learning disabilities. The students were taught math using normal techniques and games. The results indicated that each of the students benefited from the use of games. We considered the use of games in the lesson design. Students use bingo daubers to create visuals during the lesson.

References:

- Crosnoe, R., Morrison, F., Burchinal, M., Pianta, R., Keating, D., Friedman, S. L., & Clarke-Stewart, K. A. (2010). Instruction, teacher-student relations, and math achievement trajectories in elementary school. *Journal of Educational Psychology, 102*(2), 407–417. doi: 10.1037/a0017762
- Ding, M., & Li, X. (2014). Transition from concrete to abstract representations: The distributive property in a Chinese textbook series. *Educational Studies in Mathematics, 87*(1), 103–121. doi: 10.1007/s10649-014-9558-y
- Skarr, A., Zielinski, K., Ruwe, K., Sharp, H., Williams, R. L., & McLaughlin, T. F. (2014). The effects of direct instruction flashcard and math racetrack procedures on mastery of basic multiplication facts by three elementary school students. *Education and Treatment of Children, 37*(1), 77–93. doi: 10.1353/etc.2014.0007

Flow of the Unit:

The timeline below provides the following information about the unit:

- Learning objectives
- Students' understanding and common misconceptions of the objectives
- Estimated time to cover each learning objective

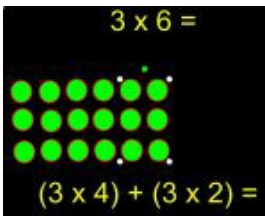
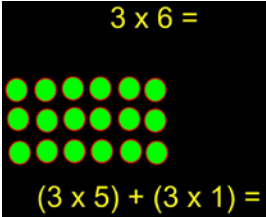
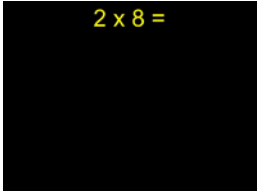
The dates may vary based on the Scope and Sequence used by the teachers.

Lesson	Learning Objective(s)	Students' Understanding and Misconceptions of Objective	# of Lesson Periods
1	Students will identify the identity and zero	Students might think that $4 \times 0 = 4$	1 x 15 min.

	properties of multiplication.	because they are confusing the identity (zero) property of addition with the zero property of multiplication.	
2	<p>Research Lesson</p> <p>Students will use the distributive property to solve problems involving multiplication and division within 100 using strategies based on pictorial models, including arrays. The distributive property recognizes that a multiplication fact can be broken into the sum of other multiplication facts.</p>	<p>Students may get confused as to which numbers to break apart when they move from concrete to abstract examples.</p> <p>Students may break apart both numbers instead of just one.</p>	1 x 60 min.
3	Students will learn about the associative (grouping) property of multiplication, which states that factors can be grouped in any way and the product will remain the same.	Students may not understand that each factor is used only once.	1 x 60 min.
4	Students will communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.	Students who struggle with multi-step problems may have problems finding the hidden question and solving it as a separate problem first. Approach this as breaking down a problem into steps.	1 x 60 min.

Plan of the Lesson:

Learning Activities, Teacher's Questions, and Expected Student Reactions	Teacher's Support	Points of Evaluation
<p>1. Introduction (10 minutes)</p> <p>Review the properties of multiplication.</p> <p>Break apart the number 6 into two addends using the projected lesson and focus page.</p> <p>Introduce the definition of the distributive property of multiplication.</p>	<p>Review the commutative, identity, and zero properties previously learned.</p> <p>Break apart the number 6 into two addends.</p>	<p>Did students demonstrate understanding and accuracy?</p>
<p>2. Posing the Task (45 minutes)</p> <p>Guided instruction: Solve different multiplication facts using the distributive property of multiplication.</p> <p>Have students use the "Distributive property graphic" handout in dry-erase sleeves to make arrays with unit cubes and break them apart with popsicle sticks into two smaller arrays. Have students work multiple ways to solve one problem as shown below.</p>	<p>Make sure the students know how to make an array correctly.</p> <p>Show the corresponding equations.</p> <p>Show all examples and answers on the interactive whiteboard.</p>	<p>Did students break factors apart correctly and make the correct array?</p> <p>Did students understand there is more than one way to break factors and arrays apart</p>

<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> $3 \times 6 =$  $(3 \times 4) + (3 \times 2) =$ </div> <div style="text-align: center;"> $3 \times 6 =$  $(3 \times 5) + (3 \times 1) =$ </div> </div> <p>Have students work on an example independently using unit cubes as shown below.</p> <div style="text-align: center; margin-top: 20px;"> $2 \times 8 =$  </div> <p>Have students complete two additional guided problems. They will use bingo daubers independently to break apart arrays on the “Distributive property of multiplication with arrays” handout and write the corresponding equations.</p> <p>Have students also complete the “Distributive property of multiplication without models” handout.</p>	<p>Circulate to check for student understanding throughout.</p> <p>Guide students on the interactive whiteboard through one way of breaking up an array. Circulate to check for student understanding while students complete the second way.</p>	<p>correctly?</p>
<p>3. Anticipated Student Responses (Note: S1 = Student 1)</p> <p>S1: Students may have difficulty breaking apart the arrays into sections to represent factors.</p> <p>S2: Students may break apart one factor incorrectly or construct the array incorrectly.</p>	<p>Facilitation of Student Responses (Note: T1 = Teacher 1)</p> <p>T1: Assist students as they break apart the first factor but focus on teaching them to break apart the second factor.</p> <p>T2: Monitor student responses and model the procedure again or provide feedback individually.</p>	
<p>4. Comparing and Discussing (15 minutes)</p> <p>Have students independently demonstrate what they learned by completing the “Distributive property of multiplication without models” handout and the “Multiplication practice” handout. To accomplish this, students will construct arrays and write the factors using models.</p> <p>Ask students to share their answers on the class whiteboard.</p>	<p>Observe students’ work for understanding. Specifically, look for the following:</p> <ul style="list-style-type: none"> • Students break arrays apart correctly and make the correct array. • Students understand there is more than one way to break arrays apart correctly. <p>Create anchor chart versions of student handouts, such as the multiplication practice page, so that students may share work with the class.</p>	<p>Did students communicate effectively to gain a complete understanding of the skill being taught?</p>
<p>5. Summing Up Lesson (5 minutes)</p>	<p>Look for the following answers:</p> <ul style="list-style-type: none"> • Distributive property of 	

<p>Ask students the following questions:</p> <ul style="list-style-type: none"> • What property did we learn today? • What is the distributive property of multiplication? • How does the distributive property of multiplication help us solve multiplication equations? 	<p>multiplication</p> <ul style="list-style-type: none"> • A multiplication fact can be broken apart into the sum of two smaller facts. • It helps you solve larger facts that you may not know. 	
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Board Plan:

The board plan is a record of the lesson and may include the lesson objective, essential questions, anchor charts, the proposed problem or task, and/or student work used in the whole-group discussion.

The board plan below is an example from the TXLS group.

Focus page, Number bonds (addends), Distributive Doctor, Distributive property of multiplication with arrays using 3 x 6, Distributive property of multiplication without models

The image displays four educational handouts. The first, 'Number Bonds', shows a grid of circles with lines connecting them to represent addition. The second, 'Distributive Doctor', features a large plus sign and empty boxes for numbers. The third and fourth handouts, both titled 'Distributive Property of Multiplication', use red dot arrays to illustrate multiplication facts and their decomposition into smaller facts, with plus signs and boxes for equations.

Supplemental Materials and Resources:

- textbook: Charles, R. I. (2009). *EnVision Math 3 Texas*. Glenview, IL: Pearson. (topic 5, lesson 6)
- Projected or poster versions of lesson handouts as needed
- [Guided distributive property handout with teacher notes](#)
- [Number bonds graphic](#)

Evaluation:

The TXLS teachers gave the following metrics to outside observers for the Lesson Observation. The outside observers, as well as the TXLS teachers, took data on the metrics and reported the findings in a debrief meeting.

- Were students able to explain the distributive property of multiplication?
- Could students explain how the distributive property of multiplication helps us solve larger multiplication equations?
- Did students master the skill from this lesson?

Post-Lesson Reflection:

Pretest and posttest data was collected from 58 students that were taught this research lesson, and that data shows it was a successful lesson. Average mastery for this group of students grew from 22 percent on the pretest to 89 percent on the posttest. More telling information is that the only three students that did not have a positive increase in scores were those that scored 100 percent on both the pretest and posttest.

The lesson was fast-paced and contained information that had not been previously taught to students in second grade. It was evident that the majority of the students eventually understood the concept of the distributive property. Although the students were authentically engaged during the lesson, they could have benefited from more small-group collaboration to assist the teacher during independent practice time. This small-group time would have allowed support from other students, rather than the teacher monitoring and providing feedback to every student.

Recommendations:

- Include a problem with numbers that third-grade students do not typically memorize, such as 3×13 . This will allow students to understand the true benefits of the distributive property.
- Group students during the guided practice portion so there is immediate support available to each student.