May 31, 2011

The Commissioner and his staff should be congratulated on the excellent work they have done, and the high standard they have set. These draft standards are a definite improvement over earlier TEKS, and are better than the Common Core Standards in many respects. There was a good representation of interest groups among the document's authors. I urge the Board and the review committees to avoid the temptation to re-engineer what is a very good document, and to avoid making wholesale changes to the draft. That said, I do have some suggestions for further improvement.

My content-related comments are mostly centered around three topics: algorithmic thinking and computer science, the use of statistics, and the section on mathematical modelling. I also have comments that center on use of appropriate technology, supplementary materials for the standards document, an emphasis on the variety of mathematically valid approaches, and the high school curriculum. I have tried to organize each of these into sections; they appear in rough priority order. At the end, I have a few assorted remarks, and some appendices.

My only wish is that I had more time to review the proposal in detail. I hope that my comments can serve as a good starting point for a conversation about the standards. I will continue to review the revisions. Should any of the members of the Board or of the review committees wish to contact me, they may do so by email at jim@rath.us or by phone at 512-554-1229. I welcome interactions with y'all.

Supplementary materials

There are several sections that should be added to the document that would help expand on and clarify the standards. I recommend adding the following prior to adoption: an introduction that explains the philosophy and overall intent, a section with definitions of terms, a timeline of topical focal points as they span grade levels, and a statement on equity. There is also material that that can come later, after adoption, such as a framework or progressions, with more explanatory material and examples. All of these give the opportunity to clarify intent, and move away from a document that is merely a list of skills.

A good start to the document would be an overview, one where the guiding principles of the standards are explicitly laid out. The overview should emphasize concepts as chief among all else, and give some organization to and motivation for them. It should lay out ideas central to mathematics such as abstraction, logic and reasoning, algorithmic thinking, and problem solving. Organization of and relationships between key concepts could be shown through a diagram or graph or web of topics (a schematic of how key ideas are related). Another could show the interrelations or connections between process and content. Motivation for the standards could be set out by describing philosophy and principles behind the standards. The 2000 Massachusetts standards are a good example. They describe problem solving, communicating, reasoning and proof, multiple representations, and making connections as central to mathematics, and detail how they are important. The overview should be a chance to emphasize concepts over skills and competencies, and to emphasize connecting representations among symbols, words, and diagrams. We don't want to lose focus on conceptual understanding, problem solving, and higher-level cognitive processes. These are harder to test, but they are more important for kids to take away.

Another section to add is one with definitions of curricular terms. For example, what are "mathematical and real-world problems"? What are the differences between the two? (The Common Core does not define these either, another chance to improve on their work.) When a student is asked to "explain", what is acceptable as an explanation? When a student is required to "justify" their answer, make sure to ask them to check their answer. "Does my answer make sense?" "How else could you arrive at the same answer?" (The TEKS previously included language such as "determine the reasonableness of a solution to a problem".) Defining or expanding on these terms will help avoid confusion. It will also be useful to curriculum and assessment writers, and help them write materials better targeted to the standards' intent.

A related section is a glossary of mathematical terms. By necessity, it would be brief and incomplete, but at least it can reduce confusion over common terms. For instance, in the Advanced Quantitative Reasoning section, there appears to be confusion over "iteration" and "recursion" in AQRA06 -- even experts can slip up occasionally. A glossary could easily help avoid such mistakes.

Each of these sections appears in one form or another in other standards documents. For instance, they appear in the Common Core Standards, the 2007 Florida standards, and the 2000 Massachusetts standards. The Florida standards also have additional examples and further details for each standard listed. (Though such additional material might better be incorporated in a set of progressions for each grade.)

Another feature common to both the Florida and Massachusetts standards is what might be described as a grade timeline, one that lines up strands or focal points across grades. A timeline would be useful in seeing connections across grades and concepts, and in setting a pacing guide.

Be sensitive to the diversity of Texas' students. For instance, Massachusetts' standards include a statement on equity. I think theirs is a good example to follow, and it sits well with the fundamental right to a free public education written into the Texas Constitution as "a general diffusion of knowledge being essential to the preservation of the liberties and rights of the people." The Texas Constitution also assures equal rights and treatment. The standards document should use this section to make a statement of non-discrimination, a caution to be proactive about avoiding language that has a disparate impact on other group or another. Texas spans everything from the most rural to the very urban; even using a word as innocuous as "elevator" can cause a problem: is it a grain elevator or a people elevator? Texas has vast geographic differences; some students have never experienced snow while others have never seen a palm tree. This is a challenge, to be sure, but far from impossible especially with aid from the Board and the TEA.

There is a vast body of empirical research on young children's learning. I was surprised to see few references to primary sources in the literature on mathematics education.[1] The TEKS should be connected to current research in mathematics education. That is, it is not enough to compare our standards to others, or to try to develop mathematics logically from its definitions. Time permitting, I

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[1] I find myself in the odd position of a mathematician calling for math education research. I am always mystified by other research mathematicians who tell math education specialists what to do. I suppose it's like a line backer telling a wide receiver what to do; one may know something about the other's job -- and isn't just some fool off the street, an armchair quarterback -- but it's not quite the same thing.

hope the review committees track down research on students' thinking and learning that supports each standard within each grade. These results can be summarized in a more extensive bibliography.

There should be a description of the relationship between the standards and assessments. This need not be its own section; perhaps it could be incorporated in the philosophy of the standards. I know it may be premature, but I would like to suggest that any future assessments include free response problems -- say similar to the AP tests but grade-level appropriate -- and not just multiple-choice problems.

There is more connective material that can be added later. Other standards documents include progressions or frameworks or trajectories within years.[2] That is, the standards that students are expected to meet are endpoints for the year without the scaffolding to get there. There is also the opportunity to add further examples for particular skills.

The Dana Center at the University of Texas did work on the previous TEKS. They produced instructional materials for the Mathematics TEKS Toolkit. These can be found online at http://www.utdanacenter.org/mathtoolkit/instruction/. There are drafts underway for the Common Core Standards; these can be found at http://ime.math.arizona.edu/progressions/. Several people currently advising Texas are contributing to this effort as well. Perhaps the Board or the TEA can commission a new set of progressions after adoption but before implementation is required. That way, the materials will be available to writers as they produce new curricula.

Use of appropriate technology

"Technology is an essential tool in a mathematics education."
2000 Massachusetts math curriculum standards, Guiding Principle #3.

Missing from both the draft TEKS and the Common Core Standards is an emphasis on the use of appropriate technology in the classroom. Technology is ubiquitous. Students ought to be able to answer: How do I use it? It's not voodoo; how does it work? The central concepts students should be introduced to are: knowing when and how to use technology, knowing how it works and its limitations, using technology to tackle more complex, engaging examples not possible with pen and paper. Ideally students should begin using technology from a very young age; it brings with it a familiarity and confidence that is difficult to regain later in life.

Teachers should use appropriate technology: appropriate to the skill level of the students, and appropriate to the skill being taught. There is no one silver bullet, no one gadget or program that can do everything. There is a veritable smorgasbord of available technologies -- everything from maps to learning games to computer algebra systems to graphing programs and more -- and not nearly all of them expensive. No one in their right mind would suggest relying on technology to simply provide answers to problems. However, students should know how to use technology, need to understand how it operates, and should appreciate its limits.

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[2] I apologize for using what to some are words with a precise, technical meaning; I use them in a more everyday, general sense here.

These are skills that are essential to the workforce, and living in and engaging with the modern world. When I have worked with petroleum engineers, they speak of the oilfield of the future. It is filled with instrumentation and simulation tools. They may only need a few researchers to create those tools, but they need a great many other careers and personnel that interact with those tools and are essential to the companies' function. A general familiarity and appreciation of technology goes a long way. It is best not to tie it to any one specific use. Students should comfortable incorporating it in everyday situations; they should feel free to play with it, and figure out how it works.

The standards can incorporate technology in the supplementary materials, in the process standards, and in the content standards. Massachusetts' guiding principles provide a good example of the first. The supplementary materials can expand on what "technology" means, and state that it should be appropriate to the situation and grade-level. Technology already indirectly appears in the draft in the third process standard. However, the process standard could be rephrased. For instance, the standard asks student to "select tools" and goes on to list paper and pencil separate from "technology". But paper and pencil are techologies. The Common Core Standards make the same mistake (a protractor is a technology too); here is a chance for Texas to improve on their mark. A better phrasing might be: "Select appropriate tools and technology resources to [accomplish a variety of tasks and] solve problems." Or the process standard could read more simply: "Use appropriate tools strategically to solve problems."

As for content standards, a couple of examples caught my attention. There are many standards like KG07 that mention "drawings". The standards (or supplementary material) can emphasize a variety of media for drawing: paper with pen, pencil, marker or crayon; chalkboard with chalk; and digital tools. Among the latter, the types of software are almost boundless, and hardware from cell phones to tablets and laptops to desktops can be used as teachers see fit. Another standard, 6A04, asks students to demonstrate representations of problems. There are many tools available to help create concept maps (graphs or schematics), and to display words, equations, tables, and graphs.

I would also suggest the use of educational mathematics games. Though I am rather dated, I remember from my own schooling such games as Tesselmania! from the former Minnesota Educational Computing Consortium (MECC), and The King's Rule from Sunburst Communications. The last one can help kids create or extend patterns, justify predictions from patterns, and check and test their results -- even from very early grades. A simple web search turns up hundreds of like software titles. There is a role here for the TEA in evaluating them or finding reviews, and disseminating a list of free, high-quality resources on the web.

Instruction in the use of technology should include safe use and ethical use. For instance, in interacting with the world wide web, students should be encouraged to use question and answer forums for exploration and group work, but should be cautioned against sharing too much personal information with strangers. Conversely, an ethical use would be to share information and ideas widely, and as in Texas, there should be a free public education for all comers.

Computer science

I recommend adding computer science topics to the draft in kindergarten through grade eight. Algorithmic thinking and digital representations are two central concepts in computer science, and are shared principles with mathematics.

Mathematics and computer science are intimately related and have common roots. Indeed, the first computer science department was not formed until 1962, and was composed largely of people who previously worked in mathematics departments. To this day, many mathematicians and computer scientists are housed in the same department at smaller schools.

As computer science has branched off from mathematics, there comes a point in schooling where it rightly stands as its own subject. Two professional computer science organizations, the Association for Computing Machinery (ACM) and the Computer Science Teachers Association (CSTA), have developed a model K-12 computer science curriculum. In it they provide standalone courses at the high school level, but in earlier grades topics are mixed with the mathematics curriculum (among others). Though I am not an expert in education in early grades, I do have some general suggestions about where computer science dovetails with the mathematics standards. Fortunately, computational ideas already appear in some places in the standards (such as KM03, 1N05, 1M05, 3N04); for that, the draft deserves praise. However, there are other elements that should be added.

One way to add algorithmic thinking to the curriculum is using communication to exercise procedural thinking. For instance, there could be a seventh process standard: "Describe a problem-solving process in a step-by-step procedure. Use words, symbols, diagrams, and other constructions at a grade-level appropriate precision and sophistication to explain and justify the procedure in a way compelling to a classmate." This can begin as early as kindergarten. For instance, standard KN13 could be modified to read, "Explain the solution process in a way understandable to one's peers to all the above sorts of problems (KN01-KN12). Use spoken words, objects, pictorial models, symbols, or number sentences as needed." In KG07, add to the end, "and be able to describe the steps involved in doing so." The standard 1N15 is much like KN13, but the word "process" should be reinserted. These types of standards serve the dual purpose of encouraging algorithmic thinking, and creating the deeper understanding that comes with trying to teach another a skill. There are repeated instances in the early grades where such phrasing is -- or could be inserted -- in the standards.

Another process standard that should be added at early grades is: "be able to sort a list of objects into a useful order without a computer. For instance, a list of names in a cell phone, or an array of objects by size, shape, or color (people, geometric tiles, ...)." One more is: "Give a step-by-step process for (1) how to get home from school (what turns? what bus number? how far? navigate by landmarks, timing, or distance); (2) how to tie one's shoes; (3) a cooking recipe such a PB&J, potato and egg taquitos, or a favorite dish; and (4) making a line-drawing of a house, animal, or any favorite or everyday object." This last exercise can serve a useful introduction to programming with LOGO. The grade one Common Core also adds, "Order three objects by length; compare the lengths of two objects indirectly by using a third object." One can even tie into object-oriented programming with such games as 20 questions. What defines an object? What are its attributes and methods? (What does it look like, and what does it do?) This also relates to top-down programming, breaking a complicated problem down to more basic steps.

A process standard for later grades should be, "understand a simple algorithm -- the process and how and why it works." Sorting is a common example. There are basic operations involved: labelling or indexing a list, comparing two list elements, and swapping two list elements. With regards to labelling, there is the concept that the location where data is stored is not the same as the data stored there. Sorting can be done by a variety of procedures. As a basic step, students should be able to describe one in detail. Getting more advanced, they should be able to note tradeoffs in space, time, parallelism, and code complexity; they can begin to make comparisons between algorithms' performance. Last, these are activities that do not require technology. This is an important concept: the principles and ideas are independent of the technology platform used. It also means that even though resources available in the classroom vary widely from one school system to another, students can learn the same ideas, and be just as prepared.

My wording here is sometimes exercise-oriented. Though these do not belong in the content standards, I hope they are useful in spurring thinking about where these concepts fit in the curriculum. Such exercises can be at a sophistication appropriate to grade level with increasing abstraction as students mature. The ACM/CSTA model curriculum suggests "the concept of algorithm is often used only to tech students the steps of arithmetic (addition, subtraction, multiplication, division) and other basic mathematical ideas." We can do much better.

The standards should also include the concept of digital (or discrete) representations of data. For instance, one standard could be, "understand how numbers can be used to represent information such as alphabets and words, images, or directed graphs." For images, one can ask questions like "When looking at a black-and-white photograph, what part of the scene is brighter? What part is darker?" There are also exercises such as color by numbers, and creating low-fi graphics and pixellated images (Figure 1 was made using only black, white, green, and blue). Another exercise is encoding alphabets or symbols as numbers (numbered lists) then using those number codes along with encryption to transmit secret sentences.

Last, to emphasize the central concepts of digital representations and algorithmic thinking, add them to the list in the fourth process standard: "Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, words, digital representations, and algorithms."

I have added a few words about what computer science is -- and what it is not -- in an appendix. I have also given a working definition for "algorithm".

The ACM/CSTA model curriculum can be found online at http://csta.acm.org/Curriculum/sub/ACMK12CSModel.html. Their "Level I" for K-8 topics is especially appropriate. Further resources are Bell, Witten, and Fellows' "Computer Science Unplugged" from 2002 (online at http://csunplugged.org/); Papert's 1980 classic "Mindstorms: Children, Computers, and Powerful Ideas (it and much other information can be found at http://el.media.mit.edu/logo-foundation/); and Barr and Stephenson's "Bringing Computational Thinking to K-12" in ACM Inroads from March 2011.

The zoo of mathematically valid approaches

In mathematics, there is rarely one right way to approach a problem. For instance, even the venerable Pythagorean theorem has dozens of independent proofs. The proofs draw on ideas from pictorial constructions -- unique dissections, relying on symmetries and similarities involving circles, squares, parallelograms, and trapezoids (one even discovered by a former President of the United States!) --

algebra, and trigonometry. The theorem is even tightly tied to physics (mechanics) by the principle of conversation of angular momentum.[3]

There is enormous variety and creativity in mathematics. It is wild, yes, but mostly under control. In any local school, no doubt there will be a selective slice of the world. At best, it will be representative of the mathematical world, and tailored to the students' learning. It will use neutral language, and be mindful of the differences of experiences between urban and rural, rich or poor, black or white, woman or man. I think the Commissioner's draft should be praised for largely avoiding these sorts of problems. I hope this continues, and that supplementary materials to the skill lists will emphasize the variety of mathematically valid approaches, emphasize the connections between multiple representations of concepts, and urge the use of setting appropriate problems. As they say, the devil is in the details, and it would be worthwhile to urge caution in unintentionally tripping over exclusionary wording. We should beware disparate impacts, and try to broaden participation in mathematics -- we can use all the talent we can get!

To that end, I have a number of constructive suggestions for improving the draft. Some of the standards have examples or additional explanations added to them. I would suggest carefully checking them all looking for three things: a variety in the examples; an emphasis on choice of approach, algorithm, units, or whatnot; and making sure the examples are not limiting. I will try to provide examples myself of each of these; I think it would be wise for TEKS supplementary materials to also include these cautions. Additionally, I would warn against examples that are tied to a specific technology; I did not note any in the current draft, but it is worth another pair of eyes to check.

Check to make sure examples are explanatory and not limiting. For instance, KM02 could be changed to read: "... temperature, color) using contrasting language." In KM04, the sample question is good, but should be added to. For example, "How many more yellow shoes than green shoes do you see?" In 1G02, the samples could be added to using interior angles, lengths of sides or edges, area or shape of faces. There is also a factual error: a solid with rectangular faces is not always a rectangular prism; see, for example, figures 2a and 2b. This could be corrected by adding the word "regular", but that is a rather technical concept for a first grader. Perhaps it would be better to leave off the word "regular" and instead use this as an exercise to broaden students' thinking. Ask, "What kinds of solids have only rectangular faces?" These difficulties appear throughout the document from kindergarten (KM04) to modelling (MMAD09). As time permits, I will build a more comprehensive list, and I hope others take up this endeavor to comb through the entire document. Last, in constructing examples, beware the "elevator" problem (the "elevator" problem mentioned earlier in my comments).

In the same vein of examples that constrain understanding, I would especially like to highlight the use of units of measurement. A mix of units -- both appropriate to the problem and familiar to the student -- usually works best in assisting understanding, and emphasizing that the math is the same regardless of units. There are English and metric units, there are domain-specific units, and also everyday or comparative units. Some domain-specific units are knots for speed in the navy, cups and tablespoons for volume in cooking, or barrels for volume in petroleum engineering. Some comparative units are the

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[3] See, for instance, http://mathforum.org/library/drmath/view/62539.html, http://www.cut-the-knot.org/pythagoras/index.shtml, or the book "The pythagorean proposition" by Elisha Scott Loomis in 1907.

length of a football or soccer field, the breadth of a hair, or the diameter of the Earth. One approach to units taken in the draft is to simply leave them off (KM02); this is good. Even better, in some places the draft states that any "appropriate units" are acceptable (3M04), or offers a variety of units (2M02). Another good example is one of the standards in the eighth grade Common Core; it offers both the use of "appropriate units" and gives a more world-expanding example: "choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading)." One example that is not-so-good: 2M01 requires the use of "standard" units; the text would read just as well without this word: "... using concrete models for units of length."

Last, I would suggest rephrasing the sixth process standard (and the fourth along with it). Incorporate the broader way mathematics is communicated -- using words, symbols, pictorial means, and formal logic and proof. We don't want to impose a narrow view of mathematics. "Precise mathematical language" is a narrow view of mathematics; mathematical communication uses words and language, yes, but it also uses symbols and diagrams, informal sketches of concepts and proofs with appeal to intuition and formal logical deductions. I would suggest striking "language" and substituting "thinking", and leaving "communication" to the fourth process standard; change "or" to "and" among explaining, displaying, and justifying. Reorder to put mathematical thinking first; the goal of the standard is for students to demonstrate understanding to others. Last, the process standards are repeated verbatim at all grade levels, but the phrasing isn't always appropriate. For instance, kindergartners cannot write, and demanding precision of language is probably too much. Perhaps a better phrasing would be "Communicate at a level understandable to a student's peers (or compelling to a classmate), using a precision of language appropriate to the skill level". Ideally, the wording should be grade appropriate (for example, kindergarten above, as writing isn't assessed until grade four), and section appropriate (for example, three additional process standards for MMA suggested below). My suggestions are summarized in these phrasings:

IV. Communicate mathematical ideas, reasoning, and their implications with grade-level appropriate skills. This includes using words and language; symbols; diagrams, graphs, and other pictorial means; formal logic and proof; algorithms; and digital representations.

VI. Use precise mathematical thinking to explain, display, and justify mathematical ideas and arguments at a level understandable to a student's peers (or compelling to a classmate).

I also have one bit of prospective advice: please avoid references to "the" algorithm or "standard" algorithms. The Commissioner's draft deserves praise for avoiding their use, and I hope the adopted standards document keeps it that way. I think the notion of "the" standard algorithm is just nutty; there is no such thing. Further, trying to force a one-size-fits-all approach disadvantages anyone who better understands another equally valid method (in procedure or notation). I strongly feel this is a mistake that the Common Core Standards make. It is a place where Texas should improve on their work and set a better standard.

As an example, how would you subtract 3,008 from 5,000? One method is to brute force it: line up the two numbers by place, borrow from the thousands to the hundreds to the tens to the ones, 10 minus 8 is 2, and so on. But another way is to group the thousands together and subtract them first; for instance, 5000-3008=(5000-3000)-8=2000-8=1992. For quick mental arithmetic, this makes more sense. This way also requires the student to connect multiple representations of the numbers involved (as opposed to mindlessly following a fixed procedure). It emphasizes efficiency, fluency, and flexibility; it requires anticipatory and relational thinking.

I have two other quick related points. I noticed in the draft in the kindergarten focal areas, addition is referred to as combining, and subtraction as separation. For instance, subtraction can be thought of as comparison (how much less, how much more), equalizing (how many more are needed to make these equal), and separation (how much remaining).[4]

Texas schools are quite a mix of students. Among them are a lot of non-Texans: students that move here from other U.S. states, along with English language learners from other countries. Folks from other places don't always do things the Texas way, and I think it would be a mistake to force them to relearn something they already know by requiring "the" standard algorithm for a mathematics problem.

Numeracy and statistics

The draft's Q&A says, "The [standards] underscore that mathematics of everyday life is different in this century than it was in the last. The increased use of data and statistics in everyday life makes it more important than ever for all students to know how to interpret data and make informed decisions using data." I heartily agree, and the draft standards are much better than the Common Core in this regard. The standards should give this statement more prominence; perhaps put it in the overview or statement of purpose.

It is good to see data, numerical literacy, and number sense appear in early grades. And the standards deserve praise for so many standards related to financial literacy and consumer applications. Money could be introduced earlier; for instance, the second grade Common Core has: "Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using dollar and cent symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?" Money is not otherwise mentioned until grade four in draft.

The standards should include other types of numeracy, especially as regards to understanding news from the popular media. I would rewrite the first process standard: "Engage everyday situations with mathematics, and be able to critically assess mathematical content in an article from the popular media." There are TV reports, newspaper articles, web pages, and more; topics could include news, elections (polling and voting), finances, engineering disasters, agriculture, government budgets, and many others. I feel strongly that this should appear everywhere from K to 12. Especially appropriate was a standard previously included in the grade six TEKS: "recognize misuses of graphical or numerical information and evaluate predictions and conclusions based on data analysis." However, this standard does not appear in the draft.

The use of estimation is essential to developing intuition about scale, to evolving quick mental arithmetic ability, and as an independent means of checking one's solution to a problem. Estimation as a content standard was repeated in several places in the previous TEKS. For instance, as early as grade three the standards read: "use strategies including rounding and compatible numbers to estimate solutions to multiplication and division problems." It also appeared as: "The student estimates to determine reasonable results. The student is expected to: (A) round whole numbers to the nearest ten or hundred to approximate reasonable results in problem situations; and (B) use strategies including

[4] This particular phrasing is from the Massachusetts mathematics curriculum standards.

rounding and compatible numbers to estimate solutions to addition and subtraction problems." However, in the draft estimation only appears in 5N20 in grade five. Though it does also appear in the third process standard, it is worthwhile setting out skills that develop this process.

Last, the previous grade eight TEKS had a standard: "select and use an appropriate representation for presenting and displaying relationships among collected data, including line plots, line graphs, stem and leaf plots, circle graphs, bar graphs, box and whisker plots, histograms, and Venn diagrams, with and without the use of technology." This would be a worthwhile readdition.

In statistics, I have comments in three areas: Bayesian versus frequentist statistics, the use of simulation, and the concept of central tendency.

In the mathematics community, there has been considerable discussion about the use of Bayesian versus frequentist statistical methods. It would be best if the standards avoided preferencing one approach over the other. The standards should be praised for using the term "statistical inference" over the more specific "hypothesis testing" (such as MMAD02). However, in A2D06 uses the term "statistical significance" but this is an idea tied to hypothesis testing and frequentist statistics. A better phrasing might ask the student to "decide if the observed differences are significant; that is, whether they are meaningful or simply due to chance." Another possible phrasing might as students to "use simulations to examine the differences between treatments." The standards' introduction should include a statement cautioning about examples that assume one approach or the other. Though I did not find any examples in the draft that worried me, I recently came across a school test problem worded something like this: "A coin is flipped 20 times and comes up heads every time. What is the probability that it will come up heads on the next flip?" I think they were trying to get at the idea of a fair coin and a memoryless process (with independent trials). But even assuming the latter, the word "fair" did not appear in the problem. I can think of at least four valid answers: 1, 1/2, 21/22, and "any number between 0 and 1, inclusive". Cautionary words about such assumptions in the standards' overview or supplementary materials would help assessment writers avoid such problems.

Simulation only appears in the draft's standards for grade eight and Algebra II. The standards could benefit from employing the use of simulation more often; it would allow students to explore statistics more thoroughly and in more realistic settings than available with theoretical calculations alone. There should be an emphasis on the use appropriate technology for simulation: everything from push-button simulated variates on a TI to visualizing a distribution of simulated variates, seeing a cluster, and examing the effect on the mean when an outlier is introduced.

In earlier grades, it might be appropriate to introduce the concept of the "middle" or "center" of a collection of attributes, and examine how the mean, median, and mode attempt to describe a central tendency. Later, one could ask: what does the mean mean? Describe the concept and relate it to an equal share among partitions, the center of mass of an object, and many other related physical ideas. Also: what happens to the mean (or median or mode) in the presence of an outlier?

High school curriculum

I have several comments about the high school curriculum in two areas: its overall organization, and the content of the Advanced Quantitative Reasoning (AQR) section and the Mathematical Models with Applications (MMA) section.

Overall organization of the high school curriculum.

I have a philosophical question for the Commissioner and the Board: how should the high school curriculum be organized? This is a difficult question, and the Commissioner's draft perhaps reflects this difficulty. High school mathematics is something of a hybrid or transition from elementary to university style subject classes. There is a traditional sequence of classes -- Algebra I, Geometry, Algebra II, and Pre-calculus -- that has much to recommend to it. There are also more integrated styles that match concepts more appropriately to a progression of learning. Finally, there is a great deal invested in the traditional approach, and it would take a considerable amount of work to change.

As written in the draft, there is an abrupt transition into high school level mathematics. The standards proceed from Kingergarten to Grade 1 to Grade 2 and so on up to Grade 8. But then it suddenly switches to Algebra I, Geometry, Algebra II, Pre-calculus, Advanced Quantitative Reasoning, and Mathematical Models with Applications. These high school titles look almost like distinct classes: the 4 traditional plus 2 more. Why not have an integrated, grade-level oriented curriculum like for K-8? Are these intended to be 6 one-year courses? There is overlap among them, especially shared whole sections between AQR and MMA. At a more basic level, why the separation between AQR and MMA? Will schools be given explicit reign to integrate them (or any and all the high school sections)?

Is there wisdom in tracking every student at the high school level along the same path, where some students take more math, some less, but all follow the same progression? As an alternative, the standard should require every student take four years of math but offer a choice of directions: entering the workforce, college bound (both science and engineering majors and others), or future career mathematicians. To the question, "Should consideration be given toward adding other courses at the high school level to provide more options for students," I strongly believe the answer is yes. The standards should answer: is there a core set of concepts and skills that every high school student should know and have? What other conceptual units can be added on -- units that are more bite-sized and can be done a la carte? Meanwhile, these options can fulfill four years of mathematics course requirements; there can be suggested sequences or combinations. This would be a little more of a hybrid between the grade school and university approach.

Be careful of setting a uniform expectation for all students: those entering the workforce immediately after high school versus those attending college (among them are math majors, science and engineering majors, and everyone else). Even that is too coarse a description. If we only aim at potential future math majors, we lose the opportunity to reach many more.

As another example, the Common Core Standards' strategy lays out concepts, topics, and skills without assigning them to a prescribed course sequence. It is more subject-area oriented. This allows schools that adopt the CCS the flexibility to implement either a traditional course sequence (Algebra I to Pre-calculus) or a more integrated one. The Common Core also gives a section specially devoted to introducing the high school standards.

Last, I have two comments about calculus. First, calculus is missing from the draft whereas it was mentioned in the previous TEKS (referenced to the AP curriculum). I support this choice with reservations. It keeps the standards focused on core content, and gets back to my earlier question about what high school students should be required to study over four years. Second, I would also like to voice the opinion that calculus is not the pinnacle of mathematics. It is a useful and practical subject, but there is a broad swath of mathematics that has little or nothing to do with it. There is no sequence of classes that build toward calculus without being extremely limiting in coverage. By focusing on calculus as a target, there is much mathematics that is missed that could be both engaging and useful to high school students.

Content of MMA and AQR sections.

Modelling is a central concept in mathematics, and it deserves a prominent place in the standards. Applications are more central to mathematics than some care to admit; they very often provide the motivation for theoretical study. I recommend adding an eighth process standard: "Model with mathematics." The CCS has been praised for its recognition of mathematical modeling as an important mathematical practice. Texas would do well to follow the supporting text in the Common Core's introduction expounding on modelling.

With regards to the MMA content, I have several suggestions. First, put the three standards from the MMAN section into the process standards or focal points for MMA. They are not so much related to "numeric reasoning" as they more enunciate core principles of mathematical modelling. Toward that end, I would add "Pick a topic or problem, and analyze it in depth. What mathematics is applicable?" There are a whole world of examples that teachers and students can choose from. Gravitational lensing for planet detection, futures contracts and derivatives, properties of social networks, and the psychology of sound and image perception and their relation to digital audio and video compression -- these are but a few contemporary topics that can be made accessible to a high school audience. The Moody's Mega Math Challenge uses a number of wonderful problems as well; see http://m3challenge.siam.org/. All of these can help enliven the subject, avoid limiting examples and dated material.

In the MMAA section, all but one of the standards are financially related topics. This is good in a way; financial mathematics offers a wonderful set of interesting mathematics. Simply change the label for the section from "algebraic reasoning" to "financial modelling". (And break out MMAA10 into its own section on physics.) I would also combine MMAA05 and MMAA06 and add a little: "use amortization models to (1) investigate home financing and compare buying versus renting a home, (2) investigate auto financing and compare buying versus leasing a vehicle, or (3) investigate any other capital purchase such as farm equipment, seed, cattle, or cell phones."

Applications of geometry are not limited to music, art and architecture, and periodic motion. Astronomy is a historical application. A more contemporary example is surface reconstruction in the medical imaging of organs; another is the rendering of scenes in computer games or movies. My comments on the AQRG section (below) apply equally here. (As a minor point, the example of population growth and decay in MMAG01 seems to confuse geometric population growth with the geometry of population growth.)

The MMAD section is a mix of things. Standards MMAD01, MMAD03, and MMAD06 are not particular to probability and statistics; they should be separated in their own section, "analyzing graphical data". In MMAD01, maps and other 2-D data presentations should be added to the list; maps are ubiquitous, and should be included in the curriculum. Edward Tufte's series of books offer many good examples of presenting and analyzing data. In the same vein, I would suggest adding to MMAD03: "analyze maps, graphs, data, and other figures from journals, newspapers, and other sources to determine the validity of stated arguments (their wording and reasoning)." In MMAD04, periodic functions should be added to the list; they are commonly used in analysis, and are ubiquitous in nature. (This is related to my suggestion for AQRA03 below.) In MMAD09, the examples are limiting. I would suggest instead to "use experiments to determine the reasonableness of using a theoretical distribution to model a problem; include distributions such as the binomial (yes/no) distribution, the normal (bell curve) distribution, and Zipf's Law." If the list is to be larger, I would suggest the uniform, Poisson, geometric, and Pareto distributions. Last, I would suggest covering the use of simulations to explore probabilistic models; perhaps introduce Monte Carlo methods and their ilk.

There should be a section -- either here in MMA or in AQR -- on logical fallacies. For instance, a student should know that even if a theorem is true, its converse is not necessarily true. This is a oft seen mistake, and there are many like it. Students should learn about missteps in reasoning, and practice detecting them. (This is also a wonderful opportunity to employ the practice standard I suggested earlier: critically assessing popular media.)

I had a difficult time organizing my thoughts on the modelling section. This is the subject I feel I have the most expertise in, but I have not before been challenged to articulate guiding principles for an introductory class. I think the draft represents a good start on the subject, but I also think it could use further work. Like with the rest of my comments on the draft, I wish I had more time to craft a better vision. I will continue to work on this and search for other resources to help develop a better guideline.

I have several suggestions for the AQR content standards.

Add a concept: precision in comparison to accuracy. "Analyze numerical data" is selective, and could use a wider variety of examples. Perhaps a menu of topics that curriculum writers can choose from would be more appropriate. In AQRN08, add an example of situations where rankings are used (sports, universities, and so on). In AQRN09, change the ending "... determine an appropriate method for different types of elections (at-large elections, bullet voting, run offs and plurality versus majority voting, and so on)." In AQRN11, add "including conditionals, loops, iteration, and recursion."

Add an AQRA02.5: "identify how uncorrelated variables may or may not be unrelated, and identify situations where correlated variables do or do not indicate a cause and effect relationship." Students (and adults) often trip over these. For AQRA03, periodic functions are much more commonly used in modelling, and would serve students better; rephrase as "determine or analyze an appropriate growth or decay model for problem situations; models relationships may include linear, exponential, and periodic functions, among others." In AQRA06, recursion is not the same as iteration, and recursive models do not all have exponential growth. Recursion is the idea of breaking down a given problem to one(s) already solved (or less complex). An iterative process tells me how to step from where I am to the next adjacent step; for example, it is used in approximation algorithms when an initial approximation is given along with a refinement procedure. In the standard, either trim it to simply "solve problems using recursion or iteration", or add many more examples. Add an AQRA09.5 or swap for another: models involving purchases, and buying versus leasing (for example, cars and common cell phone plans). Finally, is the AQRA section intended to be separate from the MMA standards? Perhaps the standards should combine them, and present an integrated core.

The standard AQRG01 is very specific compared to the other three skills in the AQRG section. Was the author thinking of the use of projections? Or route planning? Perhaps it would be better to untie the concept or skill from the example. The AQRG section is noticeably thinner than the others. Additional skills could include the use of maps, graphs, and route optimization. (Here I mean an abstract graph, not the graph of a function.) Another possibility is a discussion of simplices and their use in discretization or digitalization of real-world geometric objects and surfaces. This AQRG section is very similar to the corresponding MMAG section; here is another opportunity to combine sections and eliminate duplication.

The standard AQRD01 would be a good place to introduce Bayes' theorem, and the idea that reversing the events in the conditional gives very different probabilities. In AQRD03 I would suggest adding more examples; these are too limited. To AQRD05 I would add the question: "Is a given ranking or categorization statistically meaningful?" I would also add another standard AQRD11.5: "clearly identify the assumptions in a statistical analysis, and identify how and when they might be violated."

Other remarks

I always emphasize to my students that practice is important. You gotta get the basics right, and know 'em cold. I tell my students that even T. J. Ford gets down on the court and practices free throws. Colt McCoy runs drills day and night. Lee Treviño spends time out on the driving range and putting green. But that's not all they do.

When visiting the United States to observe our schools, a recent Singapore Minister of Education said, "We both have meritocracies. Yours is a talent meritocracy, ours is an exam meritocracy. We know how to train people to take exams. You know how to use people's talents to the fullest. Both are important, but there are some parts of the intellect that we are not able to test well --- like creativity, curiosity, a sense of adventure, ambition. Most of all, America has a culture of learning that challenges conventional wisdom, even if it means challenging authority." Since then, I've found it odd hearing comments that we are "behind" countries like Singapore or that we set lower standards. It flies in the face of common sense for them to visit us, looking to us as a model for how to improve their schools. I urge the Board and the review committees to avoid a rush to "catch up" and pushing our students too far, too fast with too many tests.

That said, it would be wise to ask: how does Texas' curriculum standards compare to others? For example, there are the NGA's Common Core Standards, Massachusetts' and other states' standards, and other OECD countries' standards. By observing what others do, we can take their best practices and adopt them for ourselves. We can avoid duplication of effort, and save money.

Part of the Governor's rationale for rejecting the Common Core approach was Texas' then recent revision of the TEKS and corresponding TAKS. He didn't want to waste money on changing horses midstream. But if we set standards parallel to the Common Core now, we have a chance to take advantage of the other states' work and save money on creating TAKS/STAAR in the future. We lost an opportunity then; we should grab it now.

Related to pushing students too far too early, I would urge the Board and the review committees to avoid shaping the curriculum to fit students who will be future math PhDs. Please resist the pressure to tailor the curriculum only to students who will be future mathematicians. I write this as a research mathematician myself. I would suggest aiming for excellence and high standards without sacrificing inclusiveness. Recognize the reality that not everyone has the same skills or the same challenges in background. Leave no child behind.

APPENDICES

Figures

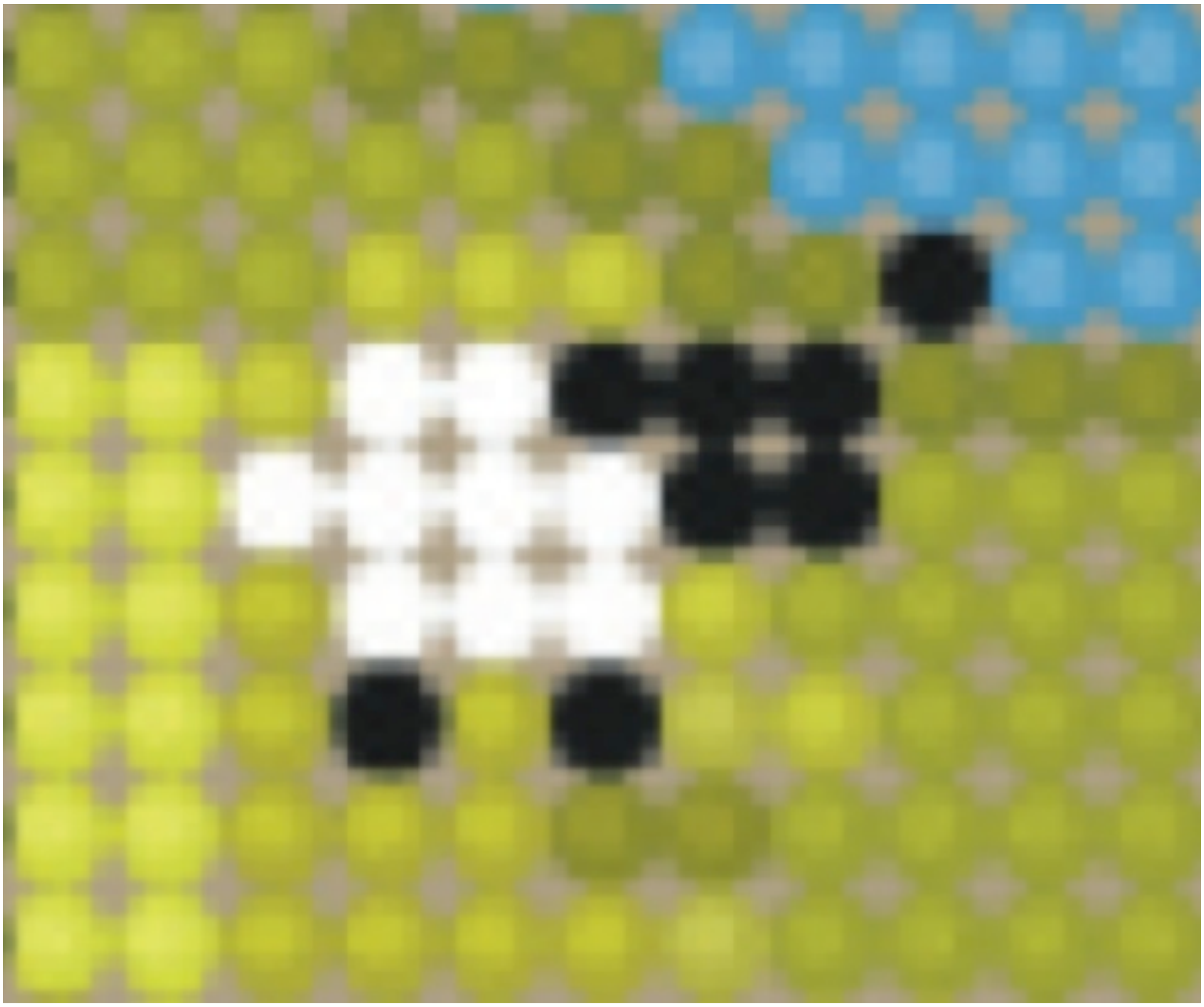

Figure 1. Pixellated image adopted from a work by Bob Sabiston.

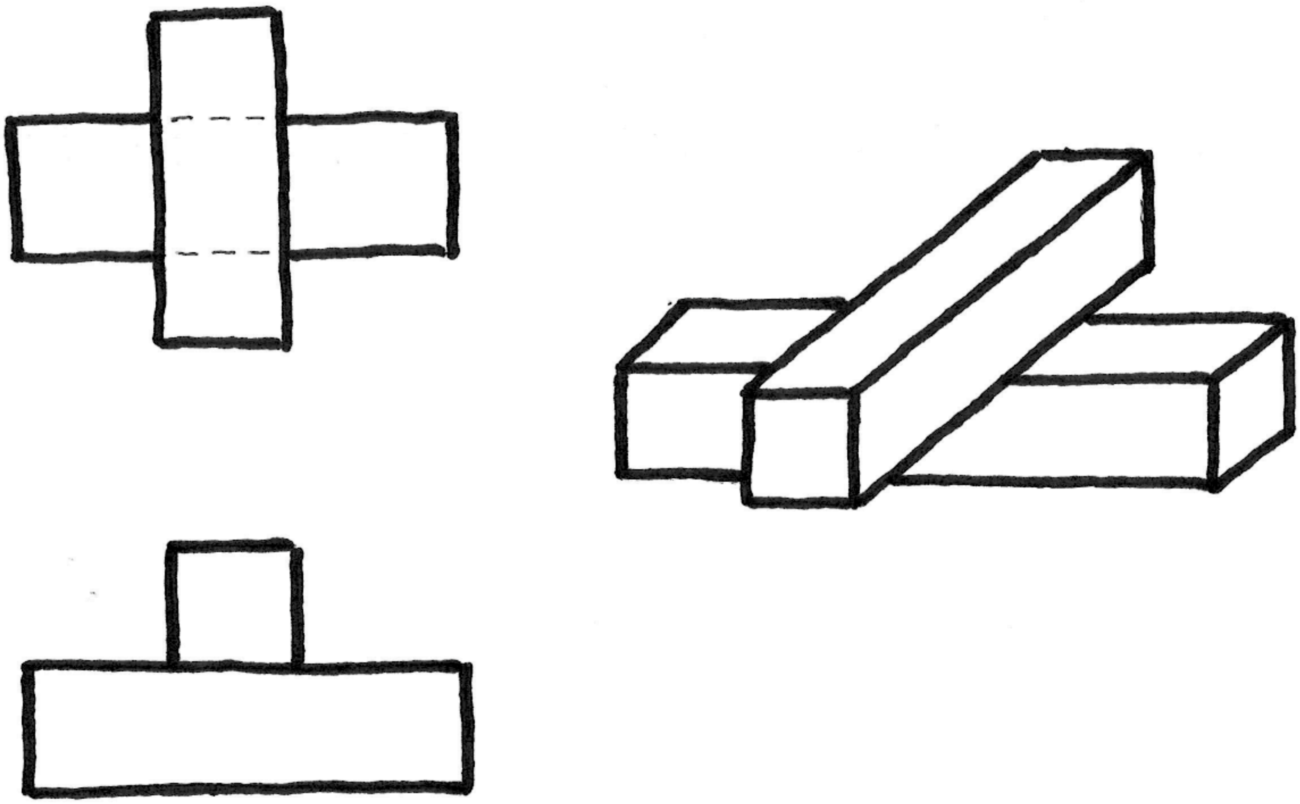

Figure 2a. A plus-like shape formed from two stacked crossbars.

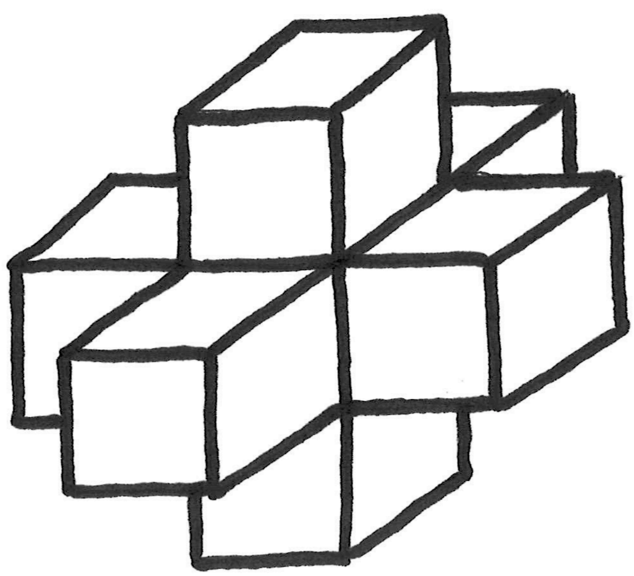

Figure 2b. A star-like object formed from seven glued cubes.

What is computer science? What is it not?

"Computer science is turning ideas into something a computer can do."
Michelle Hutton, President CSTA

Computer science is "the study of computers and algorithmic processes, including their principles, their hardware and software designs, their applications, and their impact on society."
ACM, Running on Empty, 2008

"Computer science and engineering is the systematic study of algorithmic processes --- their theory, analysis, design, efficiency, implementation, and application --- that describe and transform information."
Denning et al, "Computing as a discipline" in Communications of the ACM, 1989

"Where computer science education differs from basic technology literacy/IT goals is that it teaches fundamental concepts of computing, just as an academic course in physics will teach a student the fundamental laws of motion and energy."
ACM, Running on Empty, 2008

Computer science is not a vocational skill and should not be confused with information technology. It is not about programming nor is it about installing the latest software or using a gadget. There is certainly an element of programming in computer science, just as it is helpful to speak English in order to study English literature. However, "the focus of computer science is more on understanding the properties of the programs used to implement software such as games and web-browsers, and using that understanding to create new programs or improve existing ones." (From http://www.cl.cam.ac.uk/admissions/undergraduate/myths/.) Also, setting up and using a database, installing an operating system, and creating a PowerPoint presentation are all useful skills; but they are not computer science.

What is an algorithm?

A precise definition of "algorithm" is tricky, involves many subtleties, and is still the subject of active investigation. As a working definition, I would describe algorithmic thinking as laying out a process to achieve a well-defined goal. Included below are other definitions from a variety of sources.

An algorithm is "a procedure for solving a mathematical problem (as of finding the greatest common divisor) in a finite number of steps that frequently involves repetition of an operation; broadly, [it is] a step-by-step procedure for solving a problem or accomplishing some end especially by a computer."
Merriam-Webster's dictionary

An algorithm is "a set of rules that precisely defines a sequence of operations."
Stone, 1972, Introduction to Computer Organization and Data Structures

"In its simplest form, an algorithm is a method for solving a problem in a step-by-step manner. Children learn about algorithmic problem solving whenever they discover a collection of steps that can be carried

out to accomplish a task. These steps should accommodate unusual contingencies (using conditional or "if" statements) and repetitions (using loops or "while" statements)."
CSTA/ACM K-12 model curriculum

"Computational thinking (CT) is a problem-solving process that includes (but is not limited to) the following characteristics:
* formulating problems in a way that enables us to use a computer and other tools to help solve them
* logically organizing and analyzing data
* representing data through abstractions such as models and simulations
* automating solutions through algorithmic thinking (a series of ordered steps)
* identifying, analyzing, and implementing possible solutions with the goal of achieving the most efficient and effective combination of steps and resources, and
* generalizing and transferring this problem solving process to a wide variety of problems."
International Society for Technology in Education (ISTE)

Who am I?

Both my parents were high school math teachers. They have inspired my teaching, and I learned a lot from them. I was bit by the math bug myself in third grade. Roger, my teacher, taught me the joy of mathematics, and set me on a lifelong path as a mathematician.

I earned a bachelor's degree in mathematics from the Massachusetts Institute of Technology, and a doctorate in computational and applied mathematics from the University of Texas at Austin. I have served as a math and science consultant to industry and government. I have worked in such diverse fields as petroleum engineering, groundwater remediation, advertising, finance, electrical engineering, protein/DNA sequence alignment, among others. I have taught extensively at the university level, and have won awards for my teaching. I have also tutored hundreds of students at the high school level and have practice taught at the middle and high school levels.

In preparing my comments, I consulted with a wide variety of mathematics and education experts. I value their opinions, and tried to incorporate them with my thoughts although ultimately these comments are my own.

Please feel free to contact me with questions, suggestions, or just to engage in a friendly conversation about the draft.

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