Approved Innovative Course

Course: Multivariable Calculus
PEIMS Code: N1110018
Abbreviation: MULTCAL
Grade Level(s): 11-12
Number of Credits: 1.0

Course description:

Multivariable Calculus takes the concepts learned in the single variable calculus course and extends them to multiple dimensions. Topics discussed include: vector algebra; applications of the dot and cross product; equations of lines, planes, and surfaces in space; converting between rectangular, cylindrical, and spherical coordinates; continuity, differentiation, and integration of vector-valued functions; application of vector-valued functions such as curvature, arc length, speed, velocity, and acceleration; continuity, limits, and derivatives of multivariable functions, tangent planes and normal lines of surfaces; applying double and triple integrals to multivariable functions to find area, volume, surface area, mass, center of mass, and moments of inertia; vector fields; finding curl and divergence of vector fields; line integrals; conservative vector fields, conservation of energy; Green's Theorem; parametric surfaces, including normal vectors, tangent planes, and areas; orientation of a surface; Divergence Theorem; and Stokes's Theorem.

This course is designed as an additional math course for those students who have successfully completed AP Calculus BC and have an interest in continuing their mathematical studies while in high school.

Essential knowledge and skills:

(a) Introduction. In Multivariable Calculus, students will build on the knowledge and skills for mathematics in AP Calculus BC, which provides a foundation in derivatives, integrals, limits, approximation, application, and modeling along with connections among representations of functions. Students will study vectors, vector-value functions, functions of multiple variables, multiple integration, and vector analysis. Students will connect functions and their associated solutions in both mathematical and real-world situations. Students will use technology to model vectors and functions in a 3-D coordinate system.

(b) Knowledge and Skills

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Vectors and Vector Operations. The student applies the mathematical process standards to develop improved fluency with vector operations including vector proofs and the ability to translate back and forth among the various ways to describe geometric properties, namely, in pictures, in words, in vector notation, and in coordinate notation. The student is expected to:

(A) represent vectors algebraically and geometrically;

(B) add, subtract, and find scalar multiples of vectors and interpret the results geometrically;

(C) calculate the cross product of two vectors;

(D) find the projection of a vector onto another vector;

(E) use properties of vectors to solve real world application problems involving force and motion;

(G) write equations of lines and planes and represent them parametrically, implicitly, symmetrically, graphically in a three-dimensional coordinate system;

(H) find the distances between points, lines, and planes in space; and

(I) identify the angle of inclination of planes in space.

(3) Quadrics. The student applies the mathematical process standards to identify and analyze quadric surfaces. The student is expected to:

(A) identify and graph cylinders and quadric surfaces;

(B) use spherical coordinates to represent surfaces in space;

(C) identify the intersections of surfaces; and

(D) represent the equations of quadrics and cylinders in parametric, vector and graphical form.

(4) Vector-Valued Functions. The student applies the mathematical process standards to analyze vector – valued functions. The student is expected to:

(A) analyze and sketch a space curve given by a vector-valued function;

(B) extend the concepts of limits and continuity to vector-valued functions;

(C) differentiate a vector-valued function; and

(D) integrate a vector-valued function.

(5) Vector-Valued Functions. The student applies the mathematical process standards to recognize the relationship between vector-valued functions and position, velocity and acceleration of a particle moving through space. The student is expected to:

(A) describe the velocity and acceleration associated with a vector-valued function;

(B) use a vector-valued function to analyze projectile motion;

(C) find a unit tangent, unit normal and unit binormal vector at a point on a space curve;

(D) find the arc length of a space curve;

(E) equate the unit tangent, normal and binormal vectors to velocity, centripetal force and torque;

(F) use the arc length parameter to describe a plane curve or space curve; and

(G) find the curvature of a curve at a point on the curve.

(6) Introduction to Functions of Several Variables. The student applies the mathematical process standards to extend the properties of functions to those with more than one variable and examine the graphical properties of functions of multiple variables. The student is expected to:

(A) describe a function of several variables with correct notation;

(B) sketch the graph and level curves, contour maps, traces of a function of two variables; and

(C) use computer graphics to graph a function of two variables.

(7) Functions of Several Variables. The student applies the mathematical process standards to utilize the calculus of functions of several variables. The student is expected to:

(A) use the definition of the limit of a function of two variables;

(B) extend the concept of continuity to a function of two or three variables;

(C) find and use partial derivatives of functions of two or more variables;

(D) describe and extend the concepts of differentiability to functions of two variables; and

(E) apply and extend the Chain Rule to find the derivative of a compound function of two or more variables.

(8) Partial Derivatives. The student applies the mathematical process standards to utilize partial derivatives. The student is expected to:

(A) find and use the gradient function of two variables to solve application problems;

(B) find directional derivatives and compare gradients of F(x, y) and F(x, y, z);

(C) write equations of tangent planes and normal lines to surfaces;

(D) identify critical points and find relative extrema of a function of two variables, including using the second partials test;

(E) use single variable methods to find absolute extrema along a boundary; and

(F) solve optimization problems using the Lagrange multipliers.

(9) Multiple Integration. The student applies the mathematical process standards to find areas and volumes of regions. The student is expected to:

(A) evaluate an iterated integral and use it to find the area of a plane region;

(B) use a double integral to represent the volume of a solid region;

(C) use properties of double integrals;

(D) evaluate a double integral an iterated integral;

(E) solve problems involving applications of double integrals;

(F) use a triple integral to find volume of a solid region; and

(G) find the center of mass and moments of inertia of a solid region.

(10) Multiple Integration Conversions. The student applies the mathematical process standards to evaluate multiple integrals. The student is expected to:

(A) use polar coordinates to aid in the evaluation of a multiple integrals;

(B) use spherical coordinates to aid in the evaluation of a multiple integrals;

(C) convert from rectangular to cylindrical to spherical coordinates and apply conversions to solve iterated integrals;

(D) use a Jacobian to simplify the region a multiple integral is being evaluated over; and

(E) change the order of integration to allow for the integration of a multiple integral.

(11) Vector Analysis. The student applies the mathematical process standards to understand and explore vector calculus. The student is expected to:

(A) describe a vector field;

(B) determine whether a vector field is conservative;

(C) find the curl and divergence of a vector field;

(D) understand and apply the concept of a piecewise smooth curve; and

(E) find the potential function of a conservative vector field.

(12) Line and Surface Integrals. The student applies the mathematical process standards to evaluate line and surface integrals over a path. The student is expected to:

(A) write and evaluate a line integral and a line integral of a vector field;

(B) write and evaluate a line integral in differential form;

(C) understand and apply the Fundamental Theorem of Line Integrals;

(D) apply Green's Theorem and discover its application to finding the area inside the path given;

(E) find a set of parametric equations to represent a surface; and

(F) find a normal vector and a tangent plane to a parametric surface.

(13) Line and Surface Areas. The student applies the mathematical process standards to evaluate line and surface areas over a vector field. The student is expected to:

(A) find the area of a parametric surface;

(B) evaluate a surface integral as a double integral;

(C) evaluate a surface integral for a parametric surface;

(D) determine the orientation of a surface;

(E) understand the concept of a flux integral;

(F) understand and use Divergence Theorem;

(G) understand and apply Stoke's Theorem; and

(H) use curl to analyze the motion of a rotating liquid.

Description of specific student needs this course is designed to meet:

The Multivariable Calculus course was developed to address the needs of students who complete AP Calculus BC in their junior year or earlier. This course provides mathematical enrichment to students who would not otherwise be served.

Major resources and materials:

Cengage Calculus 10th Edition. 3-D graphing feature of the TI-Npire CX , TI 89, and software such as Winplot, Wolfram-Alpha, and Sage to model multivariable topics.

Suggested course activities:

Small group exploration projects will be provided based on students' future career interests.

Suggested methods for evaluating student outcomes:

Students participate in a lecture class at a rigorous pace. Students have three exams and multiple quizzes each grading period. In addition, students will submit five portfolio problems each grading period. These portfolio problems take advanced applications in the section and require the student to apply lessons and techniques learned in class to solve real-world application problems.

Teacher qualifications:

Mathematics Grades 9-12; Teacher selection should include an alignment with the instructor for AP Calculus BC, as this is the knowledge background of the students in the course.

Additional information: