Approved Innovative Course

Course: Modern Geometry
PEIMS Code: N1110019
Abbreviation: MODGEO
Grade Level(s): 9-12
Number of Credits: 1.0

Course description:

This course is designed to explore concepts and development of non-Euclidean geometry, including projective, spherical, and hyperbolic geometries. The course will lead the student through the stages of discovery experienced by mathematicians during the development of non-Euclidean geometry. The student will:

- explore concepts of Euclidean geometry, including Euclid's Elements and the mystery of the Parallel Postulate;
- develop concepts of projective geometry through the study of pre- and post-17th century art and the idea of perspective; and
- discover the foundations of new, valid geometries – including spherical and hyperbolic geometries – via hands-on experimentation including dynamic geometry software.

Specifically, this course would offer students an opportunity to connect and apply their knowledge of art and perspective drawing to rigorous mathematical concepts.

Essential knowledge and skills:

(a) General requirements. Students can be awarded one credit for successful completion of the course. Prerequisites: Algebra II, Geometry.

(b) Introduction.

(1) In Modern Geometry, students will use Algebra II and Geometry concepts to explore geometric concepts beyond the Euclidean plane. The study of non-Euclidean geometry develops appreciation for the prominent and precise role of definitions in the study of mathematics, and allows the student to gain new mathematical perspectives while strengthening Euclidean geometry concepts. Non-Euclidean geometry continues to pave the way for innovative scientific discoveries. Exposure to alternate geometries illuminates geometry as a dynamic field of study that continues to develop and advance.

(2) Students will use problem solving and technology to “discover” and justify (prove) non-Euclidean geometry concepts, including concepts of projective geometry, spherical geometry, and hyperbolic geometry. The students will explore the historical development and impact of modern geometry, and make connections to art and science.

(c) Knowledge and skills.

(1) Euclidean geometry. The student interprets the definitions and postulates outlined in Euclid’s Elements, and uses these concepts to explore constructions with the straight edge and compass. The student is expected to:

(A) demonstrate an understanding of the historical context of Euclid’s Elements, including the importance of the Parallel Postulate and its role in the development of non-Euclidean geometry;

(B) interpret various propositions in Euclidean geometry using the definitions and postulates in Euclid’s Elements; and

(C) analyze geometric constructions and propositions using only the straight edge and compass.

(2) Projective geometry. The student uses an understanding of Euclidean geometry to develop axioms of projective geometry, understand the historical context of projective geometry, and connect concepts of projective geometry to perspective art. The student is expected to:

(A) compare and contrast axioms of Euclidean geometry to the axioms of projective geometry, including the Parallel Postulate and the projective axiom;

(B) understand the historical context of projective geometry and its connection to the development of art and perspective drawing during the Renaissance;

(C) create and analyze projective constructions by applying the notion that parallel lines meet at a point at infinity;

(D) explore the principle of duality by replacing the word point in a theorem by the word line and proving the validity of the theorem;

(E) justify classic theorems of projective geometry such as Pappus’ Theorem, Desargues’ Theorem, and Pascal’s theorem using dynamic geometry software; and

(F) use concepts of projective geometry to explore conic sections.

(3) Spherical geometry. The student translates geometric concepts from Euclidean geometry onto the sphere and formulates axioms of spherical geometry. The student is expected to:

(A) develop an understanding of key definitions such as point, line, antipodal point, and lune using spherical models;

(B) make conjectures about parallel and perpendicular lines as they exist on the sphere;

(C) define the Parallel Postulate as it exists in spherical geometry and justify why parallel lines do not exist on the sphere;

(D) find the area of figures on the surface of a sphere, including lunes and triangles;

(E) derive the area formulas for lunes and spherical triangles using a constructionist approach and an algebraic approach; and

(F) determine that the angle sum of a spherical triangle is greater than 180 degrees using models.

(4) Hyperbolic geometry. The student translates geometric concepts from Euclidean geometry onto the hyperbolic plane and formulates axioms of hyperbolic geometry. The student is expected to:

(A) compare and contrast axioms of Euclidean geometry to the axioms of hyperbolic geometry, including the Parallel Postulate and the hyperbolic axiom;

(B) develop an understanding of key definitions such as point, line, parallel lines, and perpendicular lines using various models of hyperbolic geometry, including the Poincare Disc Model;

(C) analyze the properties and historical context of the Saccheri Quadrilateral; and

(D) analyze properties of hyperbolic triangles using models.

Description of specific student needs this course is designed to meet:

This course provides an opportunity for students to directly connect concepts of art and perspective to advanced mathematical ideas, thus offering relevance for and promoting interest in mathematical concepts.

The study of modern geometry will expose students to new ways of thinking about mathematical concepts while reinforcing traditional concepts in geometry, with the hope that exposure to modern geometry may encourage and inspire students to pursue further studies in mathematics.

Major resources and materials:

(1) ***Experiencing Geometry, 3/E***
David W. Henderson, Cornell University
Daina Taimina, Cornell University

ISBN-10: 0131437488
ISBN-13: 9780131437487

Publisher: Prentice Hall
Copyright: 2005
Format: Paper; 432 pp
Published: 07/28/2004

(2) ***Euclid's Elements*** *(available on-line at http://cs.clarku.edu/~djoyce/java/elements/elements.html)*

(3) ***Survey of Classical and Modern Geometries, A: With Computer Activities***
Arthur Baragar, University of Nevada, Las Vegas

ISBN-10: 0130143189
ISBN-13: 9780130143181

Publisher: Prentice Hall
Copyright: 2001
Format: Paper; 370 pp
Available on Demand

Other Resources:

Geometer's Sketchpad (or other dynamic geometry software)

Artwork: Images of artwork drawn without perspective (pre-Renaissance) and with perspective (Renaissance – present day).

Manipulatives: Lenart Spheres, plastic balls (such as beach balls), string, rubber bands, markers, etc.

Suggested course activities:

The student will engage in explorations of non-Euclidean geometry concepts via Geometer's Sketchpad and pencil/paper. The student will formulate explanations/justifications for observed geometric concepts.

Suggested methods for evaluating student outcomes:

The students will be evaluated via:

- Geometer's Sketchpad Activities – The student will use Geometer's Sketchpad to explore ideas of non- Euclidean geometry and create images/models. The students will include written explanations/"proofs" of computer-generated images.
- Projects – The student will work in groups to explore ideas of non-Euclidean geometry using hands-on manipulatives (i.e. plastic balls, string, rubber bands, pencil and paper models, etc.). The student will include written explanations/"proofs" to justify observations made during explorations.
- Homework/Exams/Quizzes – The students will demonstrate knowledge of modern geometry concepts via homework, exams and quizzes.

Teacher qualifications:

secondary teaching certificate in mathematics

Recommended: Masters in Mathematics and/or three credits from a Modern Geometry course.

Additional information: