Chapter 111. Texas Essential Knowledge and Skills for Mathematics

Subchapter C. High School

Statutory Authority: The provisions of this Subchapter C issued under the Texas Education Code, §§7.102(c)(4), 28.002, 28.008, and 28.025, unless otherwise noted.

§111.31. Implementation of Texas Essential Knowledge and Skills for Mathematics, Grades 9-12.

The provisions of this subchapter shall be implemented beginning with the 2006-2007 school year. This implementation date shall supersede any other implementation dates found in this subchapter.

Source: The provisions of this §111.31 adopted to be effective September 1, 1996, 21 TexReg 7371; amended to be effective August 1, 2006, 30 TexReg 4479.

§111.32. Algebra I (One Credit).

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students will continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Algebraic thinking and symbolic reasoning. Symbolic reasoning plays a critical role in algebra; symbols provide powerful ways to represent mathematical situations and to express generalizations. Students use symbols in a variety of ways to study relationships among quantities.

(3) Function concepts. A function is a fundamental mathematical concept; it expresses a special kind of relationship between two quantities. Students use functions to determine one quantity from another, to represent and model problem situations, and to analyze and interpret relationships.

(4) Relationship between equations and functions. Equations and inequalities arise as a way of asking and answering questions involving functional relationships. Students work in many situations to set up equations and inequalities and use a variety of methods to solve them.

(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

(b) Knowledge and skills.

(1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. The student is expected to:

(A) describe independent and dependent quantities in functional relationships;

(B) gather and record data and use data sets to determine functional relationships between quantities;

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

(2) Foundations for functions. The student uses the properties and attributes of functions. The student is expected to:

(A) identify and sketch the general forms of linear ($y = x$) and quadratic ($y = x^2$) parent functions;

(B) identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;

(C) interpret situations in terms of given graphs or creates situations that fit given graphs; and

(D) collect and organize data, make and interpret scatterplots (including recognizing positive, negative, or no correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations.

(3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. The student is expected to:

(A) use symbols to represent unknowns and variables; and

(B) look for patterns and represent generalizations algebraically.

(4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. The student is expected to:

(A) find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations;

(B) use the commutative, associative, and distributive properties to simplify algebraic expressions; and

(C) connect equation notation with function notation, such as $y = x + 1$ and $f(x) = x + 1$.

(5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations. The student is expected to:

(A) determine whether or not given situations can be represented by linear functions;

(B) determine the domain and range for linear functions in given situations; and

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

(6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations. The student is expected to:

(A) develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations;

(B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(C) investigate, describe, and predict the effects of changes in m and b on the graph of $y = mx + b$;

(D) graph and write equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept;

(E) determine the intercepts of the graphs of linear functions and zeros of linear functions from graphs, tables, and algebraic representations;

(F) interpret and predict the effects of changing slope and y-intercept in applied situations; and

(G) relate direct variation to linear functions and solve problems involving proportional change.

(7) Linear functions. The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to:

(A) analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;

(B) investigate methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities; and

(C) interpret and determine the reasonableness of solutions to linear equations and inequalities.

(8) Linear functions. The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to:

(A) analyze situations and formulate systems of linear equations in two unknowns to solve problems;

(B) solve systems of linear equations using concrete models, graphs, tables, and algebraic methods; and

(C) interpret and determine the reasonableness of solutions to systems of linear equations.

(9) Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions. The student is expected to:

(A) determine the domain and range for quadratic functions in given situations;

(B) investigate, describe, and predict the effects of changes in a on the graph of $y = ax^2 + c$;

(C) investigate, describe, and predict the effects of changes in c on the graph of $y = ax^2 + c$; and

(D) analyze graphs of quadratic functions and draw conclusions.

(10) Quadratic and other nonlinear functions. The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods. The student is expected to:

(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) make connections among the solutions (roots) of quadratic equations, the zeros of their related functions, and the horizontal intercepts (x-intercepts) of the graph of the function.

(11) Quadratic and other nonlinear functions. The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations. The student is expected to:

(A) use patterns to generate the laws of exponents and apply them in problem-solving situations;

(B) analyze data and represent situations involving inverse variation using concrete models, tables, graphs, or algebraic methods; and

(C) analyze data and represent situations involving exponential growth and decay using concrete models, tables, graphs, or algebraic methods.

Source: The provisions of this §111.32 adopted to be effective September 1, 1996, 21 TexReg 7371; amended to be effective August 1, 2006, 30 TexReg 1931.

§111.33. Algebra II (One-Half to One Credit).

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Algebraic thinking and symbolic reasoning. Symbolic reasoning plays a critical role in algebra; symbols provide powerful ways to represent mathematical situations and to express generalizations. Students study algebraic concepts and the relationships among them to better understand the structure of algebra.

(3) Functions, equations, and their relationship. The study of functions, equations, and their relationship is central to all of mathematics. Students perceive functions and equations as means for analyzing and understanding a broad variety of relationships and as a useful tool for expressing generalizations.

(4) Relationship between algebra and geometry. Equations and functions are algebraic tools that can be used to represent geometric curves and figures; similarly, geometric figures can illustrate algebraic relationships. Students perceive the connections between algebra and geometry and use the tools of one to help solve problems in the other.

(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

(b) Knowledge and skills.

(1) Foundations for functions. The student uses properties and attributes of functions and applies functions to problem situations. The student is expected to:

(A) identify the mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations; and

(B) collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.

(2) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. The student is expected to:

(A) use tools including factoring and properties of exponents to simplify expressions and to transform and solve equations; and

(B) use complex numbers to describe the solutions of quadratic equations.

(3) Foundations for functions. The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations. The student is expected to:

(A) analyze situations and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems;

(B) use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities; and

(C) interpret and determine the reasonableness of solutions to systems of equations or inequalities for given contexts.

(4) Algebra and geometry. The student connects algebraic and geometric representations of functions. The student is expected to:

(A) identify and sketch graphs of parent functions, including linear ($f(x) = x$), quadratic ($f(x) = x^2$), exponential ($f(x) = a^x$), and logarithmic ($f(x) = \log_a x$) functions, absolute value of x ($f(x) = |x|$), square root of x ($f(x) = \sqrt{x}$), and reciprocal of x ($f(x) = 1/x$);

(B) extend parent functions with parameters such as a in $f(x) = a/x$ and describe the effects of the parameter changes on the graph of parent functions; and

(C) describe and analyze the relationship between a function and its inverse.

(5) Algebra and geometry. The student knows the relationship between the geometric and algebraic descriptions of conic sections. The student is expected to:

(A) describe a conic section as the intersection of a plane and a cone;

(B) sketch graphs of conic sections to relate simple parameter changes in the equation to corresponding changes in the graph;

(C) identify symmetries from graphs of conic sections;

(D) identify the conic section from a given equation; and

(E) use the method of completing the square.

(6) Quadratic and square root functions. The student understands that quadratic functions can be represented in different ways and translates among their various representations. The student is expected to:

(A) determine the reasonable domain and range values of quadratic functions, as well as interpret and determine the reasonableness of solutions to quadratic equations and inequalities;

(B) relate representations of quadratic functions, such as algebraic, tabular, graphical, and verbal descriptions; and

(C) determine a quadratic function from its roots (real and complex) or a graph.

(7) Quadratic and square root functions. The student interprets and describes the effects of changes in the parameters of quadratic functions in applied and mathematical situations. The student is expected to:

(A) use characteristics of the quadratic parent function to sketch the related graphs and connect between the $y = ax^2 + bx + c$ and the $y = a\ (x - h)^2 + k$ symbolic representations of quadratic functions; and

(B) use the parent function to investigate, describe, and predict the effects of changes in a, h, and k on the graphs of $y = a\ (x - h)^2 + k$ form of a function in applied and purely mathematical situations.

(8) Quadratic and square root functions. The student formulates equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to:

(A) analyze situations involving quadratic functions and formulate quadratic equations or inequalities to solve problems;

(B) analyze and interpret the solutions of quadratic equations using discriminants and solve quadratic equations using the quadratic formula;

(C) compare and translate between algebraic and graphical solutions of quadratic equations; and

(D) solve quadratic equations and inequalities using graphs, tables, and algebraic methods.

(9) Quadratic and square root functions. The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to:

(A) use the parent function to investigate, describe, and predict the effects of parameter changes on the graphs of square root functions and describe limitations on the domains and ranges;

(B) relate representations of square root functions, such as algebraic, tabular, graphical, and verbal descriptions;

(C) determine the reasonable domain and range values of square root functions, as well as interpret and determine the reasonableness of solutions to square root equations and inequalities;

(D) determine solutions of square root equations using graphs, tables, and algebraic methods;

(E) determine solutions of square root inequalities using graphs and tables;

(F) analyze situations modeled by square root functions, formulate equations or inequalities, select a method, and solve problems; and

(G) connect inverses of square root functions with quadratic functions.

(10) Rational functions. The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to:

(A) use quotients of polynomials to describe the graphs of rational functions, predict the effects of parameter changes, describe limitations on the domains and ranges, and examine asymptotic behavior;

(B) analyze various representations of rational functions with respect to problem situations;

(C) determine the reasonable domain and range values of rational functions, as well as interpret and determine the reasonableness of solutions to rational equations and inequalities;

(D) determine the solutions of rational equations using graphs, tables, and algebraic methods;

(E) determine solutions of rational inequalities using graphs and tables;

(F) analyze a situation modeled by a rational function, formulate an equation or inequality composed of a linear or quadratic function, and solve the problem; and

(G) use functions to model and make predictions in problem situations involving direct and inverse variation.

(11) Exponential and logarithmic functions. The student formulates equations and inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to:

(A) develop the definition of logarithms by exploring and describing the relationship between exponential functions and their inverses;

(B) use the parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions, describe limitations on the domains and ranges, and examine asymptotic behavior;

(C) determine the reasonable domain and range values of exponential and logarithmic functions, as well as interpret and determine the reasonableness of solutions to exponential and logarithmic equations and inequalities;

(D) determine solutions of exponential and logarithmic equations using graphs, tables, and algebraic methods;

(E) determine solutions of exponential and logarithmic inequalities using graphs and tables; and

(F) analyze a situation modeled by an exponential function, formulate an equation or inequality, and solve the problem.

Source: The provisions of this §111.33 adopted to be effective September 1, 1996, 21 TexReg 7371; amended to be effective August 1, 2006, 30 TexReg 1931; amended to be effective February 22, 2009, 34 TexReg 1056.

§111.34. Geometry (One Credit).

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Geometric thinking and spatial reasoning. Spatial reasoning plays a critical role in geometry; geometric figures provide powerful ways to represent mathematical situations and to express generalizations about space and spatial relationships. Students use geometric thinking to understand mathematical concepts and the relationships among them.

(3) Geometric figures and their properties. Geometry consists of the study of geometric figures of zero, one, two, and three dimensions and the relationships among them. Students study properties and relationships having to do with size, shape, location, direction, and orientation of these figures.

(4) The relationship between geometry, other mathematics, and other disciplines. Geometry can be used to model and represent many mathematical and real-world situations. Students perceive the connection between geometry and the real and mathematical worlds and use geometric ideas, relationships, and properties to solve problems.

(5) Tools for geometric thinking. Techniques for working with spatial figures and their properties are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers)

to solve meaningful problems by representing and transforming figures and analyzing relationships.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem solving contexts.

(b) Knowledge and skills.

(1) Geometric structure. The student understands the structure of, and relationships within, an axiomatic system. The student is expected to:

(A) develop an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems;

(B) recognize the historical development of geometric systems and know mathematics is developed for a variety of purposes; and

(C) compare and contrast the structures and implications of Euclidean and non-Euclidean geometries.

(2) Geometric structure. The student analyzes geometric relationships in order to make and verify conjectures. The student is expected to:

(A) use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships; and

(B) make conjectures about angles, lines, polygons, circles, and three-dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.

(3) Geometric structure. The student applies logical reasoning to justify and prove mathematical statements. The student is expected to:

(A) determine the validity of a conditional statement, its converse, inverse, and contrapositive;

(B) construct and justify statements about geometric figures and their properties;

(C) use logical reasoning to prove statements are true and find counter examples to disprove statements that are false;

(D) use inductive reasoning to formulate a conjecture; and

(E) use deductive reasoning to prove a statement.

(4) Geometric structure. The student uses a variety of representations to describe geometric relationships and solve problems. The student is expected to select an appropriate representation (concrete, pictorial, graphical, verbal, or symbolic) in order to solve problems.

(5) Geometric patterns. The student uses a variety of representations to describe geometric relationships and solve problems. The student is expected to:

(A) use numeric and geometric patterns to develop algebraic expressions representing geometric properties;

(B) use numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles;

(C) use properties of transformations and their compositions to make connections between mathematics and the real world, such as tessellations; and

(D) identify and apply patterns from right triangles to solve meaningful problems, including special right triangles (45-45-90 and 30-60-90) and triangles whose sides are Pythagorean triples.

(6) Dimensionality and the geometry of location. The student analyzes the relationship between three-dimensional geometric figures and related two-dimensional representations and uses these representations to solve problems. The student is expected to:

(A) describe and draw the intersection of a given plane with various three-dimensional geometric figures;

(B) use nets to represent and construct three-dimensional geometric figures; and

(C) use orthographic and isometric views of three-dimensional geometric figures to represent and construct three-dimensional geometric figures and solve problems.

(7) Dimensionality and the geometry of location. The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly. The student is expected to:

(A) use one- and two-dimensional coordinate systems to represent points, lines, rays, line segments, and figures;

(B) use slopes and equations of lines to investigate geometric relationships, including parallel lines, perpendicular lines, and special segments of triangles and other polygons; and

(C) derive and use formulas involving length, slope, and midpoint.

(8) Congruence and the geometry of size. The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations. The student is expected to:

(A) find areas of regular polygons, circles, and composite figures;

(B) find areas of sectors and arc lengths of circles using proportional reasoning;

(C) derive, extend, and use the Pythagorean Theorem;

(D) find surface areas and volumes of prisms, pyramids, spheres, cones, cylinders, and composites of these figures in problem situations;

(E) use area models to connect geometry to probability and statistics; and

(F) use conversions between measurement systems to solve problems in real-world situations.

(9) Congruence and the geometry of size. The student analyzes properties and describes relationships in geometric figures. The student is expected to:

(A) formulate and test conjectures about the properties of parallel and perpendicular lines based on explorations and concrete models;

(B) formulate and test conjectures about the properties and attributes of polygons and their component parts based on explorations and concrete models;

(C) formulate and test conjectures about the properties and attributes of circles and the lines that intersect them based on explorations and concrete models; and

(D) analyze the characteristics of polyhedra and other three-dimensional figures and their component parts based on explorations and concrete models.

(10) Congruence and the geometry of size. The student applies the concept of congruence to justify properties of figures and solve problems. The student is expected to:

(A) use congruence transformations to make conjectures and justify properties of geometric figures including figures represented on a coordinate plane; and

(B) justify and apply triangle congruence relationships.

(11) Similarity and the geometry of shape. The student applies the concepts of similarity to justify properties of figures and solve problems. The student is expected to:

(A) use and extend similarity properties and transformations to explore and justify conjectures about geometric figures;

(B) use ratios to solve problems involving similar figures;

(C) develop, apply, and justify triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples using a variety of methods; and

(D) describe the effect on perimeter, area, and volume when one or more dimensions of a figure are changed and apply this idea in solving problems.

Source: The provisions of this §111.34 adopted to be effective September 1, 1996, 21 TexReg 7371; amended to be effective August 1, 2006, 30 TexReg 1931; amended to be effective February 22, 2009, 34 TexReg 1056.

§111.35. Precalculus (One-Half to One Credit).

(a) General requirements. The provisions of this section shall be implemented beginning September 1, 1998, and at that time shall supersede §75.63(bb) of this title (relating to Mathematics). Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisites: Algebra II, Geometry.

(b) Introduction.

(1) In Precalculus, students continue to build on the K-8, Algebra I, Algebra II, and Geometry foundations as they expand their understanding through other mathematical experiences. Students use symbolic reasoning and analytical methods to represent mathematical situations, to express generalizations, and to study mathematical concepts and the relationships among them. Students use functions, equations, and limits as useful tools for expressing generalizations and as means for analyzing and understanding a broad variety of mathematical relationships. Students also use functions as well as symbolic reasoning to represent and connect ideas in geometry, probability, statistics, trigonometry, and calculus and to model physical situations. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model functions and equations and solve real-life problems.

(2) As students do mathematics, they continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

(c) Knowledge and skills.

(1) The student defines functions, describes characteristics of functions, and translates among verbal, numerical, graphical, and symbolic representations of functions, including polynomial, rational, power (including radical), exponential, logarithmic, trigonometric, and piecewise-defined functions. The student is expected to:

(A) describe parent functions symbolically and graphically, including $f(x) = xn$, $f(x) = 1n\ x$, $f(x) = \log a\ x$, $f(x) = 1/x$, $f(x) = ex$, $f(x) = |x|$, $f(x) = ax$, $f(x) = \sin x$, $f(x) = \arcsin x$, etc.;

(B) determine the domain and range of functions using graphs, tables, and symbols;

(C) describe symmetry of graphs of even and odd functions;

(D) recognize and use connections among significant values of a function (zeros, maximum values, minimum values, etc.), points on the graph of a function, and the symbolic representation of a function; and

(E) investigate the concepts of continuity, end behavior, asymptotes, and limits and connect these characteristics to functions represented graphically and numerically.

(2) The student interprets the meaning of the symbolic representations of functions and operations on functions to solve meaningful problems. The student is expected to:

(A) apply basic transformations, including $a \cdot f(x)$, $f(x) + d$, $f(x - c)$, $f(b \cdot x)$, and compositions with absolute value functions, including $|f(x)|$, and $f(|x|)$, to the parent functions;

(B) perform operations including composition on functions, find inverses, and describe these procedures and results verbally, numerically, symbolically, and graphically; and

(C) investigate identities graphically and verify them symbolically, including logarithmic properties, trigonometric identities, and exponential properties.

(3) The student uses functions and their properties, tools and technology, to model and solve meaningful problems. The student is expected to:

(A) investigate properties of trigonometric and polynomial functions;

(B) use functions such as logarithmic, exponential, trigonometric, polynomial, etc. to model real-life data;

(C) use regression to determine the appropriateness of a linear function to model real-life data (including using technology to determine the correlation coefficient);

(D) use properties of functions to analyze and solve problems and make predictions; and

(E) solve problems from physical situations using trigonometry, including the use of Law of Sines, Law of Cosines, and area formulas and incorporate radian measure where needed.

(4) The student uses sequences and series as well as tools and technology to represent, analyze, and solve real-life problems. The student is expected to:

(A) represent patterns using arithmetic and geometric sequences and series;

(B) use arithmetic, geometric, and other sequences and series to solve real-life problems;

(C) describe limits of sequences and apply their properties to investigate convergent and divergent series; and

(D) apply sequences and series to solve problems including sums and binomial expansion.

(5) The student uses conic sections, their properties, and parametric representations, as well as tools and technology, to model physical situations. The student is expected to:

(A) use conic sections to model motion, such as the graph of velocity vs. position of a pendulum and motions of planets;

(B) use properties of conic sections to describe physical phenomena such as the reflective properties of light and sound;

(C) convert between parametric and rectangular forms of functions and equations to graph them; and

(D) use parametric functions to simulate problems involving motion.

(6) The student uses vectors to model physical situations. The student is expected to:

(A) use the concept of vectors to model situations defined by magnitude and direction; and

(B) analyze and solve vector problems generated by real-life situations.

Source: The provisions of this §111.35 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective August 1, 2006, 30 TexReg 1931.

§111.36. Mathematical Models with Applications (One-Half to One Credit).

(a) General requirements. The provisions of this section shall be implemented beginning September 1, 1998. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisite: Algebra I.

(b) Introduction.

(1) In Mathematical Models with Applications, students continue to build on the K-8 and Algebra I foundations as they expand their understanding through other mathematical experiences. Students use algebraic, graphical, and geometric reasoning to recognize patterns and structure, to model information, and to solve problems from various disciplines. Students use mathematical methods to model and solve real-life applied problems involving money, data, chance, patterns, music, design, and science. Students use mathematical models from algebra, geometry, probability, and statistics and connections among these to solve problems from a wide variety of advanced applications in both mathematical and nonmathematical situations. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to link modeling techniques and purely mathematical concepts and to solve applied problems.

(2) As students do mathematics, they continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

(c) Knowledge and skills.

(1) The student uses a variety of strategies and approaches to solve both routine and non-routine problems. The student is expected to:

(A) compare and analyze various methods for solving a real-life problem;

(B) use multiple approaches (algebraic, graphical, and geometric methods) to solve problems from a variety of disciplines; and

(C) select a method to solve a problem, defend the method, and justify the reasonableness of the results.

(2) The student uses graphical and numerical techniques to study patterns and analyze data. The student is expected to:

(A) interpret information from various graphs, including line graphs, bar graphs, circle graphs, histograms, scatterplots, line plots, stem and leaf plots, and box and whisker plots to draw conclusions from the data;

(B) analyze numerical data using measures of central tendency, variability, and correlation in order to make inferences;

(C) analyze graphs from journals, newspapers, and other sources to determine the validity of stated arguments; and

(D) use regression methods available through technology to describe various models for data such as linear, quadratic, exponential, etc., select the most appropriate model, and use the model to interpret information.

(3) The student develops and implements a plan for collecting and analyzing data (qualitative and quantitative) in order to make decisions. The student is expected to:

(A) formulate a meaningful question, determine the data needed to answer the question, gather the appropriate data, analyze the data, and draw reasonable conclusions;

(B) communicate methods used, analyses conducted, and conclusions drawn for a data-analysis project by written report, visual display, oral report, or multi-media presentation; and

(C) determine the appropriateness of a model for making predictions from a given set of data.

(4) The student uses probability models to describe everyday situations involving chance. The student is expected to:

(A) compare theoretical and empirical probability; and

(B) use experiments to determine the reasonableness of a theoretical model such as binomial, geometric, etc.

(5) The student uses functional relationships to solve problems related to personal income. The student is expected to:

(A) use rates, linear functions, and direct variation to solve problems involving personal finance and budgeting, including compensations and deductions;

(B) solve problems involving personal taxes; and

(C) analyze data to make decisions about banking.

(6) The student uses algebraic formulas, graphs, and amortization models to solve problems involving credit. The student is expected to:

(A) analyze methods of payment available in retail purchasing and compare relative advantages and disadvantages of each option;

(B) use amortization models to investigate home financing and compare buying and renting a home; and

(C) use amortization models to investigate automobile financing and compare buying and leasing a vehicle.

(7) The student uses algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning. The student is expected to:

(A) analyze types of savings options involving simple and compound interest and compare relative advantages of these options;

(B) analyze and compare coverage options and rates in insurance; and

(C) investigate and compare investment options including stocks, bonds, annuities, and retirement plans.

(8) The student uses algebraic and geometric models to describe situations and solve problems. The student is expected to:

(A) use geometric models available through technology to model growth and decay in areas such as population, biology, and ecology;

(B) use trigonometric ratios and functions available through technology to calculate distances and model periodic motion; and

(C) use direct and inverse variation to describe physical laws such as Hook's, Newton's, and Boyle's laws.

(9) The student uses algebraic and geometric models to represent patterns and structures. The student is expected to:

(A) use geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and architecture; and

(B) use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music.

Source: The provisions of this §111.36 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective August 1, 2006, 30 TexReg 1931; amended to be effective February 22, 2009, 34 TexReg 1056.

§111.37. Advanced Quantitative Reasoning (One Credit).

(a) General requirements. Students shall be awarded one credit for successful completion of this course. Prerequisite: Algebra II.

(b) Introduction.

(1) In Advanced Quantitative Reasoning, students continue to build upon the K-8, Algebra I, Algebra II, and Geometry foundations as they expand their understanding through further mathematical experiences. Advanced Quantitative Reasoning includes the analysis of information using statistical methods and probability, modeling change and mathematical relationships, and spatial and geometric modeling for mathematical reasoning. Students learn to become critical consumers of real-world quantitative data, knowledgeable problem solvers who use logical reasoning, and mathematical thinkers who can use their quantitative skills to solve authentic problems. Students develop critical skills for success in college and careers, including investigation, research, collaboration, and both written and oral communication of their work, as they solve problems in many types of applied situations.

(2) As students work with these mathematical topics, they continually rely on mathematical processes, including problem-solving techniques, appropriate mathematical language and communication skills, connections within and outside mathematics, and reasoning. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

(c) Knowledge and skills.

(1) The student develops and applies skills used in college and careers, including reasoning, planning, and communication, to make decisions and solve problems in applied situations involving numerical reasoning, probability, statistical analysis, finance, mathematical selection, and modeling with algebra, geometry, trigonometry, and discrete mathematics. The student is expected to:

(A) gather data, conduct investigations, and apply mathematical concepts and models to solve problems in mathematics and other disciplines;

(B) demonstrate reasoning skills in developing, explaining, and justifying sound mathematical arguments, and analyze the soundness of mathematical arguments of others; and

(C) communicate with mathematics orally and in writing as part of independent and collaborative work, including making accurate and clear presentations of solutions to problems.

(2) The student analyzes real-world numerical data using a variety of quantitative measures and numerical processes. The student is expected to:

(A) apply, compare, and contrast ratios, rates, ratings, averages, weighted averages, or indices to make informed decisions;

(B) solve problems involving large quantities that are not easily measured;

(C) use arrays to efficiently manage large collections of data and add, subtract, and multiply matrices to solve applied problems; and

(D) apply algorithms and identify errors in recording and transmitting identification numbers.

(3) The student analyzes and evaluates risk and return in the context of real-world problems. The student is expected to:

(A) determine and interpret conditional probabilities and probabilities of compound events by constructing and analyzing representations, including tree diagrams, Venn diagrams, and area models, to make decisions in problem situations;

(B) use probabilities to make and justify decisions about risks in everyday life; and

(C) calculate expected value to analyze mathematical fairness, payoff, and risk.

(4) The student makes decisions based on understanding, analysis, and critique of reported statistical information and statistical summaries. The student is expected to:

(A) identify limitations or lack of information in studies reporting statistical information, including when studies are reported in condensed form;

(B) interpret and compare the results of polls, given a margin of error;

(C) identify uses and misuses of statistical analyses in studies reporting statistics or using statistics to justify particular conclusions; and

(D) describe strengths and weaknesses of sampling techniques, data and graphical displays, and interpretations of summary statistics or other results appearing in a study.

(5) The student applies statistical methods to design and conduct a study that addresses one or more particular question(s). The student is expected to:

(A) determine the purpose of a statistical investigation and what type of statistical analysis can be used to answer a specific question or set of questions;

(B) identify the population of interest, select an appropriate sampling technique, and collect data;

(C) identify the variables to be used in a study;

(D) determine possible sources of statistical bias in a study and how such bias may affect the ability to generalize the results;

(E) create data displays for given data sets to investigate, compare, and estimate center, shape, spread, and unusual features; and

(F) determine possible sources of variability of data, including those that can be controlled and those that cannot be controlled.

(6) The student communicates the results of reported and student-generated statistical studies. The student is expected to:

(A) report results of statistical studies, including selecting an appropriate presentation format, creating graphical data displays, and interpreting results in terms of the question studied;

(B) justify the design and the conclusion(s) of statistical studies, including the methods used for each; and

(C) communicate statistical results in both oral and written formats using appropriate statistical language.

(7) The student analyzes the mathematics behind various methods of ranking and selection. The student is expected to:

(A) apply, analyze, and compare various ranking algorithms to determine an appropriate method to solve a real-world problem; and

(B) analyze and compare various voting and selection processes to determine an appropriate method to solve a real-world problem.

(8) The student models data, makes predictions, and judges the validity of a prediction. The student is expected to:

(A) determine if there is a linear relationship in a set of bivariate data by finding the correlation coefficient for the data, and interpret the coefficient as a measure of the strength and direction of the linear relationship;

(B) collect numerical bivariate data; use the data to create a scatterplot; and select a function such as linear, exponential, logistic, or trigonometric to model the data; and

(C) justify the selection of a function to model data, and use the model to make predictions.

(9) The student uses mathematical models to represent, analyze, and solve real-world problems involving change. The student is expected to:

(A) analyze and determine appropriate growth or decay models, including linear, exponential, and logistic functions;

(B) analyze and determine an appropriate cyclical model that can be modeled with trigonometric functions;

(C) analyze and determine an appropriate piecewise model; and

(D) solve problems using recursion or iteration.

(10) The student creates and analyzes mathematical models to make decisions related to earning, investing, spending, and borrowing money to evaluate real-world situations. The student is expected to:

(A) determine, represent, and analyze mathematical models for various types of income calculations;

(B) determine, represent, and analyze mathematical models for expenditures, including those involving credit; and

(C) determine, represent, and analyze mathematical models for various types of loans and investments.

(11) The student uses a variety of network models represented graphically to organize data in quantitative situations, make informed decisions, and solve problems. The student is expected to:

(A) solve problems involving scheduling or routing situations that can be represented by methods such as a vertex-edge graph using critical paths, Euler paths, or minimal spanning trees; and

(B) construct, analyze, and interpret flow charts in order to develop and describe problem-solving procedures.

(12) The student uses a variety of tools and methods to represent and solve problems involving static and dynamic situations. The student is expected to:

(A) create and use two- and three-dimensional representations of authentic situations using paper techniques or dynamic geometric environments for computer-aided design and other applications;

(B) use vectors to represent and solve applied problems;

(C) use matrices to represent geometric transformations and solve applied problems; and

(D) solve geometric problems involving inaccessible distances such as those encountered when building a bridge, constructing a skyscraper, or mapping planetary distances.

Source: The provisions of this §111.37 adopted to be effective August 22, 2011, 36 TexReg 925.

§111.38. Implementation of Texas Essential Knowledge and Skills for Mathematics, High School, Adopted 2012.

(a) The provisions of §§111.39-111.45 of this subchapter shall be implemented by school districts.

(b) No later than June 30, 2015, the commissioner of education shall determine whether instructional materials funding has been made available to Texas public schools for materials that cover the essential knowledge and skills for mathematics as adopted in §§111.39-111.45 of this subchapter.

(c) If the commissioner makes the determination that instructional materials funding has been made available under subsection (b) of this section, §§111.39-111.45 of this subchapter shall be implemented beginning with the 2015-2016 school year and apply to the 2015-2016 and subsequent school years.

(d) If the commissioner does not make the determination that instructional materials funding has been made available under subsection (b) of this section, the commissioner shall determine no later than June 30 of each subsequent school year whether instructional materials funding has been made available. If the commissioner determines that instructional materials funding has been made available, the commissioner shall notify the State Board of Education and school districts that §§111.39-111.45 of this subchapter shall be implemented for the following school year.

(e) Sections 111.31-111.37 of this subchapter shall be superseded by the implementation of §§111.38-111.45 under this section.

Source: The provisions of this §111.38 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.39. Algebra I, Adopted 2012 (One Credit).

(a) General requirements. Students shall be awarded one credit for successful completion of this course. This course is recommended for students in Grade 8 or 9. Prerequisite: Mathematics, Grade 8 or its equivalent.

(b) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) In Algebra I, students will build on the knowledge and skills for mathematics in Grades 6-8, which provide a foundation in linear relationships, number and operations, and proportionality. Students will study linear, quadratic, and exponential functions and their related transformations, equations, and associated solutions. Students will connect functions and their associated solutions in both mathematical and real-world situations. Students will use technology to collect and explore data and analyze statistical relationships. In addition, students will study polynomials of degree one and two, radical expressions, sequences, and laws of exponents. Students will generate and solve linear systems with two equations and two variables and will create new functions through transformations.

(4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

(A) determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities;

(B) write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points;

(C) write linear equations in two variables given a table of values, a graph, and a verbal description;

(D) write and solve equations involving direct variation;

(E) write the equation of a line that contains a given point and is parallel to a given line;

(F) write the equation of a line that contains a given point and is perpendicular to a given line;

(G) write an equation of a line that is parallel or perpendicular to the X or Y axis and determine whether the slope of the line is zero or undefined;

(H) write linear inequalities in two variables given a table of values, a graph, and a verbal description; and

(I) write systems of two linear equations given a table of values, a graph, and a verbal description.

(3) Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

(A) determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$;

(B) calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical and real-world problems;

(C) graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems;

(D) graph the solution set of linear inequalities in two variables on the coordinate plane;

(E) determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of *a*, *b*, *c*, and *d*;

(F) graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist;

(G) estimate graphically the solutions to systems of two linear equations with two variables in real-world problems; and

(H) graph the solution set of systems of two linear inequalities in two variables on the coordinate plane.

(4) Linear functions, equations, and inequalities. The student applies the mathematical process standards to formulate statistical relationships and evaluate their reasonableness based on real-world data. The student is expected to:

(A) calculate, using technology, the correlation coefficient between two quantitative variables and interpret this quantity as a measure of the strength of the linear association;

(B) compare and contrast association and causation in real-world problems; and

(C) write, with and without technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.

(5) Linear functions, equations, and inequalities. The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:

(A) solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides;

(B) solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides; and

(C) solve systems of two linear equations with two variables for mathematical and real-world problems.

(6) Quadratic functions and equations. The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to:

(A) determine the domain and range of quadratic functions and represent the domain and range using inequalities;

(B) write equations of quadratic functions given the vertex and another point on the graph, write the equation in vertex form ($f(x) = a(x - h)^2 + k$), and rewrite the equation from vertex form to standard form ($f(x) = ax^2 + bx + c$); and

(C) write quadratic functions when given real solutions and graphs of their related equations.

(7) Quadratic functions and equations. The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:

(A) graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including *x*-intercept, *y*-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry;

(B) describe the relationship between the linear factors of quadratic expressions and the zeros of their associated quadratic functions; and

(C) determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d.

(8) Quadratic functions and equations. The student applies the mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:

(A) solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula; and

(B) write, using technology, quadratic functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.

(9) Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:

(A) determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities;

(B) interpret the meaning of the values of a and b in exponential functions of the form $f(x) = ab^x$ in real-world problems;

(C) write exponential functions in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay;

(D) graph exponential functions that model growth and decay and identify key features, including *y*-intercept and asymptote, in mathematical and real-world problems; and

(E) write, using technology, exponential functions that provide a reasonable fit to data and make predictions for real-world problems.

(10) Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:

(A) add and subtract polynomials of degree one and degree two;

(B) multiply polynomials of degree one and degree two;

(C) determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend;

(D) rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property;

(E) factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two; and

(F) decide if a binomial can be written as the difference of two squares and, if possible, use the structure of a difference of two squares to rewrite the binomial.

(11) Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite algebraic expressions into equivalent forms. The student is expected to:

(A) simplify numerical radical expressions involving square roots; and

(B) simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.

(12) Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:

(A) decide whether relations represented verbally, tabularly, graphically, and symbolically define a function;

(B) evaluate functions, expressed in function notation, given one or more elements in their domains;

(C) identify terms of arithmetic and geometric sequences when the sequences are given in function form using recursive processes;

(D) write a formula for the n^{th} term of arithmetic and geometric sequences, given the value of several of their terms; and

(E) solve mathematic and scientific formulas, and other literal equations, for a specified variable.

Source: The provisions of this §111.39 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.40. Algebra II, Adopted 2012 (One-Half to One Credit).

(a) General requirements. Students shall be awarded one-half to one credit for successful completion of this course. Prerequisite: Algebra I.

(b) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) In Algebra II, students will build on the knowledge and skills for mathematics in Kindergarten-Grade 8 and Algebra I. Students will broaden their knowledge of quadratic functions, exponential

functions, and systems of equations. Students will study logarithmic, square root, cubic, cube root, absolute value, rational functions, and their related equations. Students will connect functions to their inverses and associated equations and solutions in both mathematical and real-world situations. In addition, students will extend their knowledge of data analysis and numeric and algebraic methods.

(4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Attributes of functions and their inverses. The student applies mathematical processes to understand that functions have distinct key attributes and understand the relationship between a function and its inverse. The student is expected to:

(A) graph the functions $f(x)=\sqrt{x}$, $f(x)=1/x$, $f(x)=x^3$, $f(x)=\sqrt[3]{x}$, $f(x)=b^x$, $f(x)=|x|$, and $f(x)=\log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval;

(B) graph and write the inverse of a function using notation such as $f^{-1}(x)$;

(C) describe and analyze the relationship between a function and its inverse (quadratic and square root, logarithmic and exponential), including the restriction(s) on domain, which will restrict its range; and

(D) use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other.

(3) Systems of equations and inequalities. The student applies mathematical processes to formulate systems of equations and inequalities, use a variety of methods to solve, and analyze reasonableness of solutions. The student is expected to:

(A) formulate systems of equations, including systems consisting of three linear equations in three variables and systems consisting of two equations, the first linear and the second quadratic;

(B) solve systems of three linear equations in three variables by using Gaussian elimination, technology with matrices, and substitution;

(C) solve, algebraically, systems of two equations in two variables consisting of a linear equation and a quadratic equation;

(D) determine the reasonableness of solutions to systems of a linear equation and a quadratic equation in two variables;

(E) formulate systems of at least two linear inequalities in two variables;

(F) solve systems of two or more linear inequalities in two variables; and

(G) determine possible solutions in the solution set of systems of two or more linear inequalities in two variables.

(4) Quadratic and square root functions, equations, and inequalities. The student applies mathematical processes to understand that quadratic and square root functions, equations, and quadratic inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

(A) write the quadratic function given three specified points in the plane;

(B) write the equation of a parabola using given attributes, including vertex, focus, directrix, axis of symmetry, and direction of opening;

(C) determine the effect on the graph of $f(x) = \sqrt{x}$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(bx)$, and $f(x - c)$ for specific positive and negative values of a, b, c, and d;

(D) transform a quadratic function $f(x) = ax^2 + bx + c$ to the form $f(x) = a(x - h)^2 + k$ to identify the different attributes of $f(x)$;

(E) formulate quadratic and square root equations using technology given a table of data;

(F) solve quadratic and square root equations;

(G) identify extraneous solutions of square root equations; and

(H) solve quadratic inequalities.

(5) Exponential and logarithmic functions and equations. The student applies mathematical processes to understand that exponential and logarithmic functions can be used to model situations and solve problems. The student is expected to:

(A) determine the effects on the key attributes on the graphs of $f(x) = b^x$ and $f(x) = \log_b(x)$ where b is 2, 10, and e when $f(x)$ is replaced by $af(x)$, $f(x) + d$, and $f(x - c)$ for specific positive and negative real values of a, c, and d;

(B) formulate exponential and logarithmic equations that model real-world situations, including exponential relationships written in recursive notation;

(C) rewrite exponential equations as their corresponding logarithmic equations and logarithmic equations as their corresponding exponential equations;

(D) solve exponential equations of the form $y = ab^x$ where a is a nonzero real number and b is greater than zero and not equal to one and single logarithmic equations having real solutions; and

(E) determine the reasonableness of a solution to a logarithmic equation.

(6) Cubic, cube root, absolute value and rational functions, equations, and inequalities. The student applies mathematical processes to understand that cubic, cube root, absolute value and rational functions, equations, and inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

(A) analyze the effect on the graphs of $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d;

(B) solve cube root equations that have real roots;

(C) analyze the effect on the graphs of $f(x) = |x|$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x-c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d;

(D) formulate absolute value linear equations;

(E) solve absolute value linear equations;

(F) solve absolute value linear inequalities;

(G) analyze the effect on the graphs of f(x) = 1/x when *f(x)* is replaced by *af(x)*, *f(bx)*, *f(x-c)*, and *f(x) + d* for specific positive and negative real values of *a*, *b*, *c*, and *d*;

(H) formulate rational equations that model real-world situations;

(I) solve rational equations that have real solutions;

(J) determine the reasonableness of a solution to a rational equation;

(K) determine the asymptotic restrictions on the domain of a rational function and represent domain and range using interval notation, inequalities, and set notation; and

(L) formulate and solve equations involving inverse variation.

(7) Number and algebraic methods. The student applies mathematical processes to simplify and perform operations on expressions and to solve equations. The student is expected to:

(A) add, subtract, and multiply complex numbers;

(B) add, subtract, and multiply polynomials;

(C) determine the quotient of a polynomial of degree three and of degree four when divided by a polynomial of degree one and of degree two;

(D) determine the linear factors of a polynomial function of degree three and of degree four using algebraic methods;

(E) determine linear and quadratic factors of a polynomial expression of degree three and of degree four, including factoring the sum and difference of two cubes and factoring by grouping;

(F) determine the sum, difference, product, and quotient of rational expressions with integral exponents of degree one and of degree two;

(G) rewrite radical expressions that contain variables to equivalent forms;

(H) solve equations involving rational exponents; and

(I) write the domain and range of a function in interval notation, inequalities, and set notation.

(8) Data. The student applies mathematical processes to analyze data, select appropriate models, write corresponding functions, and make predictions. The student is expected to:

(A) analyze data to select the appropriate model from among linear, quadratic, and exponential models;

(B) use regression methods available through technology to write a linear function, a quadratic function, and an exponential function from a given set of data; and

(C) predict and make decisions and critical judgments from a given set of data using linear, quadratic, and exponential models.

Source: The provisions of this §111.40 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.41. Geometry, Adopted 2012 (One Credit).

(a) General requirements. Students shall be awarded one credit for successful completion of this course. Prerequisite: Algebra I.

(b) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) In Geometry, students will build on the knowledge and skills for mathematics in Kindergarten-Grade 8 and Algebra I to strengthen their mathematical reasoning skills in geometric contexts. Within the course, students will begin to focus on more precise terminology, symbolic representations, and the development of proofs. Students will explore concepts covering coordinate and transformational geometry; logical argument and constructions; proof and congruence; similarity, proof, and trigonometry; two- and three-dimensional figures; circles; and probability. Students will connect previous knowledge from Algebra I to Geometry through the coordinate and transformational geometry strand. In the logical arguments and constructions strand, students are expected to create formal constructions using a straight edge and compass. Though this course is primarily Euclidean geometry, students should complete the course with an understanding that non-Euclidean geometries exist. In proof and congruence, students will use deductive reasoning to justify, prove and apply theorems about geometric figures. Throughout the standards, the term "prove" means a formal proof to be shown in a paragraph, a flow chart, or two-column formats. Proportionality is the unifying component of the similarity, proof, and trigonometry strand. Students will use their proportional reasoning skills to prove and apply theorems and solve problems in this strand. The two- and three-dimensional figure strand focuses on the application of formulas in multi-step situations since students have developed background knowledge in two- and three-dimensional figures. Using patterns to identify geometric properties, students will apply theorems about circles to determine relationships between special segments and angles in circles. Due to the emphasis of probability and statistics in the college and career readiness standards, standards dealing with probability have been added to the geometry curriculum to ensure students have proper exposure to these topics before pursuing their post-secondary education.

(4) These standards are meant to provide clarity and specificity in regards to the content covered in the high school geometry course. These standards are not meant to limit the methodologies used to convey this knowledge to students. Though the standards are written in a particular order, they are not necessarily meant to be taught in the given order. In the standards, the phrase "to solve problems" includes both contextual and non-contextual problems unless specifically stated.

(5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Coordinate and transformational geometry. The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to:

(A) determine the coordinates of a point that is a given fractional distance less than one from one end of a line segment to the other in one- and two-dimensional coordinate systems, including finding the midpoint;

(B) derive and use the distance, slope, and midpoint formulas to verify geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of lines; and

(C) determine an equation of a line parallel or perpendicular to a given line that passes through a given point.

(3) Coordinate and transformational geometry. The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:

(A) describe and perform transformations of figures in a plane using coordinate notation;

(B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane;

(C) identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane; and

(D) identify and distinguish between reflectional and rotational symmetry in a plane figure.

(4) Logical argument and constructions. The student uses the process skills with deductive reasoning to understand geometric relationships. The student is expected to:

(A) distinguish between undefined terms, definitions, postulates, conjectures, and theorems;

(B) identify and determine the validity of the converse, inverse, and contrapositive of a conditional statement and recognize the connection between a biconditional statement and a true conditional statement with a true converse;

(C) verify that a conjecture is false using a counterexample; and

(D) compare geometric relationships between Euclidean and spherical geometries, including parallel lines and the sum of the angles in a triangle.

(5) Logical argument and constructions. The student uses constructions to validate conjectures about geometric figures. The student is expected to:

(A) investigate patterns to make conjectures about geometric relationships, including angles formed by parallel lines cut by a transversal, criteria required for triangle congruence, special segments of triangles, diagonals of quadrilaterals, interior and exterior angles of polygons, and special segments and angles of circles choosing from a variety of tools;

(B) construct congruent segments, congruent angles, a segment bisector, an angle bisector, perpendicular lines, the perpendicular bisector of a line segment, and a line parallel to a given line through a point not on a line using a compass and a straightedge;

(C) use the constructions of congruent segments, congruent angles, angle bisectors, and perpendicular bisectors to make conjectures about geometric relationships; and

(D) verify the Triangle Inequality theorem using constructions and apply the theorem to solve problems.

(6) Proof and congruence. The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to:

(A) verify theorems about angles formed by the intersection of lines and line segments, including vertical angles, and angles formed by parallel lines cut by a transversal and prove equidistance between the endpoints of a segment and points on its perpendicular bisector and apply these relationships to solve problems;

(B) prove two triangles are congruent by applying the Side-Angle-Side, Angle-Side-Angle, Side-Side-Side, Angle-Angle-Side, and Hypotenuse-Leg congruence conditions;

(C) apply the definition of congruence, in terms of rigid transformations, to identify congruent figures and their corresponding sides and angles;

(D) verify theorems about the relationships in triangles, including proof of the Pythagorean Theorem, the sum of interior angles, base angles of isosceles triangles, midsegments, and medians, and apply these relationships to solve problems; and

(E) prove a quadrilateral is a parallelogram, rectangle, square, or rhombus using opposite sides, opposite angles, or diagonals and apply these relationships to solve problems.

(7) Similarity, proof, and trigonometry. The student uses the process skills in applying similarity to solve problems. The student is expected to:

(A) apply the definition of similarity in terms of a dilation to identify similar figures and their proportional sides and the congruent corresponding angles; and

(B) apply the Angle-Angle criterion to verify similar triangles and apply the proportionality of the corresponding sides to solve problems.

(8) Similarity, proof, and trigonometry. The student uses the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. The student is expected to:

(A) prove theorems about similar triangles, including the Triangle Proportionality theorem, and apply these theorems to solve problems; and

(B) identify and apply the relationships that exist when an altitude is drawn to the hypotenuse of a right triangle, including the geometric mean, to solve problems.

(9) Similarity, proof, and trigonometry. The student uses the process skills to understand and apply relationships in right triangles. The student is expected to:

(A) determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems; and

(B) apply the relationships in special right triangles 30°-60°-90° and 45°-45°-90° and the Pythagorean theorem, including Pythagorean triples, to solve problems.

(10) Two-dimensional and three-dimensional figures. The student uses the process skills to recognize characteristics and dimensional changes of two- and three-dimensional figures. The student is expected to:

(A) identify the shapes of two-dimensional cross-sections of prisms, pyramids, cylinders, cones, and spheres and identify three-dimensional objects generated by rotations of two-dimensional shapes; and

(B) determine and describe how changes in the linear dimensions of a shape affect its perimeter, area, surface area, or volume, including proportional and non-proportional dimensional change.

(11) Two-dimensional and three-dimensional figures. The student uses the process skills in the application of formulas to determine measures of two- and three-dimensional figures. The student is expected to:

(A) apply the formula for the area of regular polygons to solve problems using appropriate units of measure;

(B) determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure;

(C) apply the formulas for the total and lateral surface area of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure; and

(D) apply the formulas for the volume of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres, and composite figures, to solve problems using appropriate units of measure.

(12) Circles. The student uses the process skills to understand geometric relationships and apply theorems and equations about circles. The student is expected to:

(A) apply theorems about circles, including relationships among angles, radii, chords, tangents, and secants, to solve non-contextual problems;

(B) apply the proportional relationship between the measure of an arc length of a circle and the circumference of the circle to solve problems;

(C) apply the proportional relationship between the measure of the area of a sector of a circle and the area of the circle to solve problems;

(D) describe radian measure of an angle as the ratio of the length of an arc intercepted by a central angle and the radius of the circle; and

(E) show that the equation of a circle with center at the origin and radius r is $x^2 + y^2 = r^2$ and determine the equation for the graph of a circle with radius r and center (h, k), $(x - h)^2 + (y - k)^2 = r^2$.

(13) Probability. The student uses the process skills to understand probability in real-world situations and how to apply independence and dependence of events. The student is expected to:

(A) develop strategies to use permutations and combinations to solve contextual problems;

(B) determine probabilities based on area to solve contextual problems;

(C) identify whether two events are independent and compute the probability of the two events occurring together with or without replacement;

(D) apply conditional probability in contextual problems; and

(E) apply independence in contextual problems.

Source: The provisions of this §111.41 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.42. Precalculus, Adopted 2012 (One-Half to One Credit).

(a) General requirements. Students shall be awarded one-half to one credit for successful completion of this course. Prerequisites: Algebra I, Geometry, and Algebra II.

(b) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) Precalculus is the preparation for calculus. The course approaches topics from a function point of view, where appropriate, and is designed to strengthen and enhance conceptual understanding and mathematical reasoning used when modeling and solving mathematical and real-world problems. Students systematically work with functions and their multiple representations. The study of Precalculus deepens students' mathematical understanding and fluency with algebra and trigonometry and extends their ability to make connections and apply concepts and procedures at higher levels. Students investigate and explore mathematical ideas, develop multiple strategies for analyzing complex situations, and use technology to build understanding, make connections between representations, and provide support in solving problems.

(4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Functions. The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to:

(A) use the composition of two functions to model and solve real-world problems;

(B) demonstrate that function composition is not always commutative;

(C) represent a given function as a composite function of two or more functions;

(D) describe symmetry of graphs of even and odd functions;

(E) determine an inverse function, when it exists, for a given function over its domain or a subset of its domain and represent the inverse using multiple representations;

(F) graph exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions;

(G) graph functions, including exponential, logarithmic, sine, cosine, rational, polynomial, and power functions and their transformations, including $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d, in mathematical and real-world problems;

(H) graph arcsin x and arccos x and describe the limitations on the domain;

(I) determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum, relative minimum, zeros, asymptotes, and intervals over which the function is increasing or decreasing;

(J) analyze and describe end behavior of functions, including exponential, logarithmic, rational, polynomial, and power functions, using infinity notation to communicate this characteristic in mathematical and real-world problems;

(K) analyze characteristics of rational functions and the behavior of the function around the asymptotes, including horizontal, vertical, and oblique asymptotes;

(L) determine various types of discontinuities in the interval $(-\infty, \infty)$ as they relate to functions and explore the limitations of the graphing calculator as it relates to the behavior of the function around discontinuities;

(M) describe the left-sided behavior and the right-sided behavior of the graph of a function around discontinuities;

(N) analyze situations modeled by functions, including exponential, logarithmic, rational, polynomial, and power functions, to solve real-world problems;

(O) develop and use a sinusoidal function that models a situation in mathematical and real-world problems; and

(P) determine the values of the trigonometric functions at the special angles and relate them in mathematical and real-world problems.

(3) Relations and geometric reasoning. The student uses the process standards in mathematics to model and make connections between algebraic and geometric relations. The student is expected to:

(A) graph a set of parametric equations;

(B) convert parametric equations into rectangular relations and convert rectangular relations into parametric equations;

(C) use parametric equations to model and solve mathematical and real-world problems;

(D) graph points in the polar coordinate system and convert between rectangular coordinates and polar coordinates;

(E) graph polar equations by plotting points and using technology;

(F) determine the conic section formed when a plane intersects a double-napped cone;

(G) make connections between the locus definition of conic sections and their equations in rectangular coordinates;

(H) use the characteristics of an ellipse to write the equation of an ellipse with center (h, k); and

(I) use the characteristics of a hyperbola to write the equation of a hyperbola with center (h, k).

(4) Number and measure. The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. The student is expected to:

(A) determine the relationship between the unit circle and the definition of a periodic function to evaluate trigonometric functions in mathematical and real-world problems;

(B) describe the relationship between degree and radian measure on the unit circle;

(C) represent angles in radians or degrees based on the concept of rotation and find the measure of reference angles and angles in standard position;

(D) represent angles in radians or degrees based on the concept of rotation in mathematical and real-world problems, including linear and angular velocity;

(E) determine the value of trigonometric ratios of angles and solve problems involving trigonometric ratios in mathematical and real-world problems;

(F) use trigonometry in mathematical and real-world problems, including directional bearing;

(G) use the Law of Sines in mathematical and real-world problems;

(H) use the Law of Cosines in mathematical and real-world problems;

(I) use vectors to model situations involving magnitude and direction;

(J) represent the addition of vectors and the multiplication of a vector by a scalar geometrically and symbolically; and

(K) apply vector addition and multiplication of a vector by a scalar in mathematical and real-world problems.

(5) Algebraic reasoning. The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:

(A) evaluate finite sums and geometric series, when possible, written in sigma notation;

(B) represent arithmetic sequences and geometric sequences using recursive formulas;

(C) calculate the n^{th} term and the n^{th} partial sum of an arithmetic series in mathematical and real-world problems;

(D) represent arithmetic series and geometric series using sigma notation;

(E) calculate the n^{th} term of a geometric series, the n^{th} partial sum of a geometric series, and sum of an infinite geometric series when it exists;

(F) apply the Binomial Theorem for the expansion of $(a + b)^n$ in powers of a and b for a positive integer n, where a and b are any numbers;

(G) use the properties of logarithms to evaluate or transform logarithmic expressions;

(H) generate and solve logarithmic equations in mathematical and real-world problems;

(I) generate and solve exponential equations in mathematical and real-world problems;

(J) solve polynomial equations with real coefficients by applying a variety of techniques in mathematical and real-world problems;

(K) solve polynomial inequalities with real coefficients by applying a variety of techniques and write the solution set of the polynomial inequality in interval notation in mathematical and real-world problems;

(L) solve rational inequalities with real coefficients by applying a variety of techniques and write the solution set of the rational inequality in interval notation in mathematical and real-world problems;

(M) use trigonometric identities such as reciprocal, quotient, Pythagorean, cofunctions, even/odd, and sum and difference identities for cosine and sine to simplify trigonometric expressions; and

(N) generate and solve trigonometric equations in mathematical and real-world problems.

Source: The provisions of this §111.42 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.43. Mathematical Models with Applications, Adopted 2012 (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of this course. Prerequisite: Algebra I. This course must be taken before receiving credit for Algebra II.

(b) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and

evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) Mathematical Models with Applications is designed to build on the knowledge and skills for mathematics in Kindergarten-Grade 8 and Algebra I. This mathematics course provides a path for students to succeed in Algebra II and prepares them for various post-secondary choices. Students learn to apply mathematics through experiences in personal finance, science, engineering, fine arts, and social sciences. Students use algebraic, graphical, and geometric reasoning to recognize patterns and structure, model information, solve problems, and communicate solutions. Students will select from tools such as physical objects; manipulatives; technology, including graphing calculators, data collection devices, and computers; and paper and pencil and from methods such as algebraic techniques, geometric reasoning, patterns, and mental math to solve problems.

(4) In Mathematical Models with Applications, students will use a mathematical modeling cycle to analyze problems, understand problems better, and improve decisions. A basic mathematical modeling cycle is summarized in this paragraph. The student will:

(A) represent:

(i) identify the variables in the problem and select those that represent essential features; and

(ii) formulate a model by creating and selecting from representations such as geometric, graphical, tabular, algebraic, or statistical that describe the relationships between the variables;

(B) compute: analyze and perform operations on the relationships between the variables to draw conclusions;

(C) interpret: interpret the results of the mathematics in terms of the original problem;

(D) revise: confirm the conclusions by comparing the conclusions with the problem and revising as necessary; and

(E) report: report on the conclusions and the reasoning behind the conclusions.

(5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Mathematical modeling in personal finance. The student uses mathematical processes with graphical and numerical techniques to study patterns and analyze data related to personal finance. The student is expected to:

(A) use rates and linear functions to solve problems involving personal finance and budgeting, including compensations and deductions;

(B) solve problems involving personal taxes; and

(C) analyze data to make decisions about banking, including options for online banking, checking accounts, overdraft protection, processing fees, and debit card/ATM fees.

(3) Mathematical modeling in personal finance. The student uses mathematical processes with algebraic formulas, graphs, and amortization modeling to solve problems involving credit. The student is expected to:

(A) use formulas to generate tables to display series of payments for loan amortizations resulting from financed purchases;

(B) analyze personal credit options in retail purchasing and compare relative advantages and disadvantages of each option;

(C) use technology to create amortization models to investigate home financing and compare buying a home to renting a home; and

(D) use technology to create amortization models to investigate automobile financing and compare buying a vehicle to leasing a vehicle.

(4) Mathematical modeling in personal finance. The student uses mathematical processes with algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning. The student is expected to:

(A) analyze and compare coverage options and rates in insurance;

(B) investigate and compare investment options, including stocks, bonds, annuities, certificates of deposit, and retirement plans; and

(C) analyze types of savings options involving simple and compound interest and compare relative advantages of these options.

(5) Mathematical modeling in science and engineering. The student applies mathematical processes with algebraic techniques to study patterns and analyze data as it applies to science. The student is expected to:

(A) use proportionality and inverse variation to describe physical laws such as Hook's Law, Newton's Second Law of Motion, and Boyle's Law;

(B) use exponential models available through technology to model growth and decay in areas, including radioactive decay; and

(C) use quadratic functions to model motion.

(6) Mathematical modeling in science and engineering. The student applies mathematical processes with algebra and geometry to study patterns and analyze data as it applies to architecture and engineering. The student is expected to:

(A) use similarity, geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in architecture;

(B) use scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and non-proportional changes in surface area and volume as applied to fields;

(C) use the Pythagorean Theorem and special right-triangle relationships to calculate distances; and

(D) use trigonometric ratios to calculate distances and angle measures as applied to fields.

(7) Mathematical modeling in fine arts. The student uses mathematical processes with algebra and geometry to study patterns and analyze data as it applies to fine arts. The student is expected to:

(A) use trigonometric ratios and functions available through technology to model periodic behavior in art and music;

(B) use similarity, geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and photography;

(C) use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music; and

(D) use scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and non-proportional changes in surface area and volume as applied to fields.

(8) Mathematical modeling in social sciences. The student applies mathematical processes to determine the number of elements in a finite sample space and compute the probability of an event. The student is expected to:

(A) determine the number of ways an event may occur using combinations, permutations, and the Fundamental Counting Principle;

(B) compare theoretical to empirical probability; and

(C) use experiments to determine the reasonableness of a theoretical model such as binomial or geometric.

(9) Mathematical modeling in social sciences. The student applies mathematical processes and mathematical models to analyze data as it applies to social sciences. The student is expected to:

(A) interpret information from various graphs, including line graphs, bar graphs, circle graphs, histograms, scatterplots, dot plots, stem-and-leaf plots, and box and whisker plots, to draw conclusions from the data and determine the strengths and weaknesses of conclusions;

(B) analyze numerical data using measures of central tendency (mean, median, and mode) and variability (range, interquartile range or IQR, and standard deviation) in order to make inferences with normal distributions;

(C) distinguish the purposes and differences among types of research, including surveys, experiments, and observational studies;

(D) use data from a sample to estimate population mean or population proportion;

(E) analyze marketing claims based on graphs and statistics from electronic and print media and justify the validity of stated or implied conclusions; and

(F) use regression methods available through technology to model linear and exponential functions, interpret correlations, and make predictions.

(10) Mathematical modeling in social sciences. The student applies mathematical processes to design a study and use graphical, numerical, and analytical techniques to communicate the results of the study. The student is expected to:

(A) formulate a meaningful question, determine the data needed to answer the question, gather the appropriate data, analyze the data, and draw reasonable conclusions; and

(B) communicate methods used, analyses conducted, and conclusions drawn for a data-analysis project through the use of one or more of the following: a written report, a visual display, an oral report, or a multi-media presentation.

Source: The provisions of this §111.43 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.44. Advanced Quantitative Reasoning, Adopted 2012 (One-Half to One Credit).

(a) General requirements. Students shall be awarded one-half to one credit for successful completion of this course. Prerequisites: Geometry and Algebra II.

(b) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) In Advanced Quantitative Reasoning, students will develop and apply skills necessary for college, careers, and life. Course content consists primarily of applications of high school mathematics concepts to prepare students to become well-educated and highly informed 21st century citizens. Students will develop and apply reasoning, planning, and communication to make decisions and solve problems in applied situations involving numerical reasoning, probability, statistical analysis, finance, mathematical selection, and modeling with algebra, geometry, trigonometry, and discrete mathematics.

(4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Numeric reasoning. The student applies the process standards in mathematics to generate new understandings by extending existing knowledge. The student generates new mathematical understandings through problems involving numerical data that arise in everyday life, society, and the workplace. The student extends existing knowledge and skills to analyze real-world situations. The student is expected to:

(A) use precision and accuracy in real-life situations related to measurement and significant figures;

(B) apply and analyze published ratings, weighted averages, and indices to make informed decisions;

(C) solve problems involving quantities that are not easily measured using proportionality;

(D) solve geometric problems involving indirect measurement, including similar triangles, the Pythagorean Theorem, Law of Sines, Law of Cosines, and the use of dynamic geometry software;

(E) solve problems involving large quantities using combinatorics;

(F) use arrays to efficiently manage large collections of data and add, subtract, and multiply matrices to solve applied problems, including geometric transformations;

(G) analyze various voting and selection processes to compare results in given situations; and

(H) select and apply an algorithm of interest to solve real-life problems such as problems using recursion or iteration involving population growth or decline, fractals, and compound interest; the validity in recorded and transmitted data using checksums and hashing; sports rankings, weighted class rankings, and search engine rankings; and problems involving scheduling or routing situations using vertex-edge graphs, critical paths, Euler paths, and minimal spanning trees and communicate to peers the application of the algorithm in precise mathematical and nontechnical language.

(3) Algebraic reasoning (expressions, equations, and generalized relationships). The student applies the process standards in mathematics to create and analyze mathematical models of everyday situations to make informed decisions related to earning, investing, spending, and borrowing money by appropriate, proficient, and efficient use of tools, including technology. The student uses mathematical relationships to make connections and predictions. The student judges the validity of a prediction and uses mathematical models to represent, analyze, and solve dynamic real-world problems. The student is expected to:

(A) collect numerical bivariate data to create a scatterplot, select a function to model the data, justify the model selection, and use the model to interpret results and make predictions;

(B) describe the degree to which uncorrelated variables may or may not be related and analyze situations where correlated variables do or do not indicate a cause-and-effect relationship;

(C) determine or analyze an appropriate growth or decay model for problem situations, including linear, exponential, and logistic functions;

(D) determine or analyze an appropriate cyclical model for problem situations that can be modeled with periodic functions;

(E) determine or analyze an appropriate piecewise model for problem situations;

(F) create, represent, and analyze mathematical models for various types of income calculations to determine the best option for a given situation;

(G) create, represent, and analyze mathematical models for expenditures, including those involving credit, to determine the best option for a given situation; and

(H) create, represent, and analyze mathematical models and appropriate representations, including formulas and amortization tables, for various types of loans and investments to determine the best option for a given situation.

(4) Probabilistic and statistical reasoning. The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies. The student is expected to:

(A) use a two-way frequency table as a sample space to identify whether two events are independent and to interpret the results;

(B) use the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, in mathematical and real-world problems;

(C) calculate conditional probabilities and probabilities of compound events using tree diagrams, Venn diagrams, area models, and formulas;

(D) interpret conditional probabilities and probabilities of compound events by analyzing representations to make decisions in problem situations;

(E) use probabilities to make and justify decisions about risks in everyday life;

(F) calculate expected value to analyze mathematical fairness, payoff, and risk;

(G) determine the validity of logical arguments that include compound conditional statements by constructing truth tables;

(H) identify limitations and lack of relevant information in studies reporting statistical information, especially when studies are reported in condensed form;

(I) interpret and compare statistical results using appropriate technology given a margin of error;

(J) identify potential misuses of statistics to justify particular conclusions, including assertions of a cause-and-effect relationship rather than an association, and missteps or fallacies in logical reasoning;

(K) describe strengths and weaknesses of sampling techniques, data and graphical displays, and interpretations of summary statistics and other results appearing in a study, including reports published in the media;

(L) determine the need for and purpose of a statistical investigation and what type of statistical analysis can be used to answer a specific question or set of questions;

(M) identify the population of interest for a statistical investigation, select an appropriate sampling technique, and collect data;

(N) identify the variables to be used in a study;

(O) determine possible sources of statistical bias in a study and how bias may affect the validity of the results;

(P) create data displays for given data sets to investigate, compare, and estimate center, shape, spread, and unusual features of the data;

(Q) analyze possible sources of data variability, including those that can be controlled and those that cannot be controlled;

(R) report results of statistical studies to a particular audience, including selecting an appropriate presentation format, creating graphical data displays, and interpreting results in terms of the question studied;

(S) justify the design and the conclusion(s) of statistical studies, including the methods used; and

(T) communicate statistical results in oral and written formats using appropriate statistical and nontechnical language.

Source: The provisions of this §111.44 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.45. Independent Study in Mathematics, Adopted 2012 (One-Half to One Credit).

(a) General requirements.

(1) Students shall be awarded one-half to one credit for successful completion of this course. Prerequisites: Geometry and Algebra II.

(2) Students may repeat this course with different course content for up to three credits.

(3) The requirements for each course must be approved by the local district before the course begins.

(4) If this course is being used to satisfy requirements for the Distinguished Achievement Program, student research/products must be presented before a panel of professionals or approved by the student's mentor.

(b) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) In Independent Study in Mathematics, students will extend their mathematical understanding beyond the Algebra II level in a specific area or areas of mathematics such as theory of equations, number theory, non-Euclidean geometry, linear algebra, advanced survey of mathematics, or history of mathematics.

(4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills: mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(1) apply mathematics to problems arising in everyday life, society, and the workplace;

(2) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(3) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(4) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(5) create and use representations to organize, record, and communicate mathematical ideas;

(6) analyze mathematical relationships to connect and communicate mathematical ideas; and

(7) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Source: The provisions of this §111.45 adopted to be effective September 10, 2012, 37 TexReg 7109.

§111.46. Discrete Mathematics for Problem Solving, Adopted 2013 (One-Half to One Credit).

(a) General requirements. Students shall be awarded one-half to one credit for successful completion of this course. Prerequisite: Algebra II.

(b) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) In Discrete Mathematics for Problem Solving, students are introduced to the improved efficiency of mathematical analysis and quantitative techniques over trial-and-error approaches to management problems involving organization, scheduling, project planning, strategy, and decision making. Students will learn how mathematical topics such as graph theory, planning and scheduling, group decision making, fair division, game theory, and theory of moves can be applied to management and decision making. Students will research mathematicians of the past whose work is relevant to these topics today and read articles about current mathematicians who either teach and conduct research at major universities or work in business and industry solving real-world logistical problems. Through the study of the applications of mathematics to society's problems today, students will become better prepared for and gain an appreciation for the value of a career in mathematics.

(4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Graph theory. The student applies the concept of graphs to determine possible solutions to real-world problems. The student is expected to:

(A) explain the concept of graphs;

(B) use graph models for simple problems in management science;

(C) determine the valences of the vertices of a graph;

(D) identify Euler circuits in a graph;

(E) solve route inspection problems by Eulerizing a graph;

(F) determine solutions modeled by edge traversal in a graph;

(G) compare the results of solving the traveling salesman problem (TSP) using the nearest neighbor algorithm and using a greedy algorithm;

(H) distinguish between real-world problems modeled by Euler circuits and those modeled by Hamiltonian circuits;

(I) distinguish between algorithms that yield optimal solutions and those that give nearly optimal solutions;

(J) find minimum-cost spanning trees using Kruskal's algorithm;

(K) use the critical path method to determine the earliest possible completion time for a collection of tasks; and

(L) explain the difference between a graph and a directed graph.

(3) Planning and scheduling. The student uses heuristic algorithms to solve real-world problems. The student is expected to:

(A) use the list processing algorithm to schedule tasks on identical processors;

(B) recognize situations appropriate for modeling or scheduling problems;

(C) determine whether a schedule is optimal using the critical path method together with the list processing algorithm;

(D) identify situations appropriate for modeling by bin packing;

(E) use any of six heuristic algorithms to solve bin packing problems;

(F) solve independent task scheduling problems using the list processing algorithm; and

(G) explain the relationship between scheduling problems and bin packing problems.

(4) Group decision making. The student uses mathematical processes to apply decision-making schemes. The student analyzes the effects of multiple types of weighted voting and applies multiple voting concepts to real-world situations. The student is expected to:

(A) describe the concept of a preference schedule and how to use it;

(B) explain how particular decision-making schemes work;

(C) determine the outcome for various voting methods, given the voters' preferences;

(D) explain how different voting schemes or the order of voting can lead to different results;

(E) describe the impact of various strategies on the results of the decision-making process;

(F) explain the impact of Arrow's Impossibility Theorem;

(G) relate the meaning of approval voting;

(H) explain the need for weighted voting and how it works;

(I) identify voting concepts such as Borda count, Condorcet winner, dummy voter, and coalition; and

(J) compute the Banzhaf power index and explain its significance.

(5) Fair division. The student applies the adjusted winner procedure and Knaster inheritance procedure to real-world situations. The student is expected to:

(A) use the adjusted winner procedure to determine a fair allocation of property;

(B) use the adjusted winner procedure to resolve a dispute;

(C) explain how to reach a fair division using the Knaster inheritance procedure;

(D) solve fair division problems with three or more players using the Knaster inheritance procedure;

(E) explain the conditions under which the trimming procedure can be applied to indivisible goods;

(F) identify situations appropriate for the techniques of fair division;

(G) compare the advantages of the divider and the chooser in the divider-chooser method;

(H) discuss the rules and strategies of the divider-chooser method;

(I) resolve cake-division problems for three players using the last-diminisher method;

(J) analyze the relative importance of the three desirable properties of fair division: equitability, envy-freeness, and Pareto optimality; and

(K) identify fair division procedures that exhibit envy-freeness.

(6) Game (or competition) theory. The student uses knowledge of basic game theory concepts to calculate optimal strategies. The student analyzes situations and identifies the use of gaming strategies. The student is expected to:

(A) recognize competitive game situations;

(B) represent a game with a matrix;

(C) identify basic game theory concepts and vocabulary;

(D) determine the optimal pure strategies and value of a game with a saddle point by means of the minimax technique;

(E) explain the concept of and need for a mixed strategy;

(F) compute the optimal mixed strategy and the expected value for a player in a game who has only two pure strategies;

(G) model simple two-by-two, bimatrix games of partial conflict;

(H) identify the nature and implications of the game called "Prisoners' Dilemma";

(I) explain the game known as "chicken";

(J) identify examples that illustrate the prevalence of Prisoners' Dilemma and chicken in our society; and

(K) determine when a pair of strategies for two players is in equilibrium.

(7) Theory of moves. The student analyzes the theory of moves (TOM). The student uses the TOM and game theory to analyze conflicts. The student is expected to:

(A) compare and contrast TOM and game theory;

(B) explain the rules of TOM;

(C) describe what is meant by a cyclic game;

(D) use a game tree to analyze a two-person game;

(E) determine the effect of approaching Prisoners' Dilemma and chicken from the standpoint of TOM and contrast that to the effect of approaching them from the standpoint of game theory;

(F) describe the use of TOM in a larger, more complicated game; and

(G) model a conflict from literature or from a real-life situation as a two-by-two strict ordinal game and compare the results predicted by game theory and by TOM.

Statutory Authority: The provisions of this §111.46 issued under the Texas Education Code, §§7.102(c)(4), 28.002, and 28.025, as that section existed before amendment by House Bill 5, 83rd Texas Legislature, Regular Session, 2013.

Source: The provisions of this §111.46 adopted to be effective August 25, 2014, 38 TexReg 9027.