Linear Algebra

PEIMS Code: N1110021
Abbreviation: LINALG
Grade Level(s): 11-12
Award of Credit: 0.5

Approved Innovative Course

- Districts must have local board approval to implement innovative courses.
- In accordance with Texas Administrative Code (TAC) §74.27, school districts must provide instruction in all essential knowledge and skills identified in this innovative course.
- Innovative courses may only satisfy elective credit toward graduation requirements.
- Please refer to TAC §74.13 for guidance on endorsements.

Course Description:

Students are introduced to linear algebra, a subject with widespread applications in other areas of mathematics, such as multivariable calculus, differential equations, and probability theory, as well as in the physical and social sciences and engineering. This is probably the student's first extensive encounter with postulational or axiomatic mathematics. However, the emphasis is on the computational and geometrical aspects of the subject, keeping the abstractions down to a minimum. Students begin with vectors and matrices and progress to systems of linear equations before gradually becoming acquainted with vector spaces and linear transformations. They learn to use the fundamental processes of Gauss-Jordan row reduction, find the inverse of a matrix, find the determinant via row reduction, find a basis using row reduction, shrink a finite spanning set to a basis, enlarge a linearly independent set to a basis, perform the Gram-Schmidt Process, diagonalize a linear operator, orthogonally diagonalize a symmetric operator, and find the dominant eigenvalue using the power method.

Essential Knowledge and Skills:

(a) Introduction. The student will to use the fundamental processes of Gauss-Jordan row reduction, find the inverse of a matrix, find the determinant via row reduction, find a basis using row reduction, shrink a finite spanning set to a basis, enlarge a linearly independent set to a basis, perform the Gram-Schmidt Process, diagonalize a linear operator, orthogonally diagonalize a symmetric operator, and find the dominant eigenvalue using the power method.

(b) Knowledge and skills.

(1) Vectors and matrices. The student understands the algebraic and geometric properties of vectors and matrices. The student is expected to:

(A) describe and perform fundamental operations with vectors;

(B) explain the dot product and use it to determine the angle between two vectors;

(C) use the concept of the projection of a vector to decompose the vector into two components;

(D) identify direct proof, proof by contrapositive, proof by contradiction, and proof by induction;

(E) explain how to negate statements with quantifiers and connectives;

(F) discuss how to disprove a statement;

(G) describe and perform fundamental operations using matrices;

(H) identify the transpose of a matrix and symmetric and skew-symmetric matrices; and

(I) discuss and perform matrix multiplication and raising square matrices to powers.

(2) Systems of linear equations. The student uses properties of a matrix to gain additional information about a system of linear equations. The student is expected to:

(A) solve systems of linear equations using Gauss-Jordan row reduction;

(B) explain the three row operations of Gauss-Jordan row reduction;

(C) identify inconsistent systems and systems with infinite solution sets;

(D) discuss the reduced echelon form of a matrix;

(E) determine the rank of a matrix;

(F) define a homogeneous system;

(G) distinguish between the trivial and nontrivial solutions of a homogeneous system;

(H) determine whether a vector is a linear combination of a given set of vectors;

(I) explain what it means for a vector to be in the row space of a matrix;

(J) find the multiplicative inverse of a square matrix (if it exists) using row reduction; and

(K) express a nonsingular matrix as a product of elementary matrices.

(3) Determinants. The student uses the determinant function to gain information about a corresponding matrix. The student is expected to:

(A) compute determinant of a 2 x 2 matrix (if possible) and a 3 x 3 matrix using the “basket weaving” technique;

(B) calculate the determinant of a 3 x 3 matrix using the method of row reduction;

(C) use cofactor expansion and the adjoint to calculate determinants;

(D) compare determining the inverse of a matrix using row reduction and using the adjoint;

(E) solve a linear system using Cramer’s Rule;

(F) discuss geometric uses of the determinant;

(G) decide whether a homogeneous system has a nontrivial solution after calculating the determinant; and

(H) determine the adjoint matrix of a given matrix.

(4) Vector spaces. The student understands the algebraic structure that defines a vector space. The student is expected to:

(A) discuss the meaning and properties of a vector space;

(B) determine whether a given set and operation meet the criteria to be called a vector space;

(C) explain the meaning of a subspace and determine whether a given subset is also a subspace;

(D) identify the meanings of linear combination and span;

(E) determine a simplified expression for all vectors in a span using the row space method;

(F) determine whether a subset of a vector space is linearly independent or dependent;

(G) discuss equivalent conditions for a subset of a vector space to be linearly independent or dependent;

(H) find a redundant vector in a given linearly dependent set;

(I) given a basis of a finite dimensional vector space, give the vector's dimension;

(J) find a basis for a subspace using the row reduction method;

(K) enlarge a linearly independent subset of a vector space to form a basis;

(L) shrink a finite spanning set to a basis;

(M) find the coordinates of a vector with respect to an ordered basis; and

(N) calculate the transition matrix between ordered bases of a subspace.

(5) Linear transformations. The student explores linear transformations of vector spaces and the relationships between transformations and matrices. The student is expected to:

(A) determine whether a mapping or function from one vector space to another is a linear transformation;

(B) explain the meaning of a linear operator and give geometric examples;

(C) find the matrix for a given linear transformation with respect to the standard basis;

(D) determine the matrix for a given linear operator with respect to the given basis;

(E) find the kernel and range for a given linear transformation;

(F) discuss the relationship between the dimension of a vector space and the dimensions of the kernel and range of the linear transformation operating on that vector space;

(G) distinguish between one-to-one and onto linear transformations;

(H) identify the term isomorphism;

(I) find an orthogonal basis for a subspace of $\boldsymbol{R}^n$ using the Gram-Schmidt process;

(J) explain how to determine whether a square nonsingular matrix is orthogonal; and

(K) identify the orthogonal complement of a subspace of $\boldsymbol{R}^n$.

(6) Eigenvalues and eigenvectors. The student explores the role of eigenvectors when analyzing certain matrices. The student is expected to:

(A) identify the terms eigenvalue and eigenvector;

(B) find the eigenvalues for a linear operator in $\boldsymbol{R}^3$ and the solution set of eigenvectors for each eigenvalue;

(C) determine the characteristic polynomial of an n x n matrix;

(D) show that a linear operator is diagonalizable;

(E) explain how diagonalization relates to find large powers of an n x n matrix; and

(F) describe the steps in the process of orthogonally diagonalizing a symmetric operator.

(7) Applications. The student explores additional applications of matrices to topics outside of the standard linear algebra curriculum. The student is expected to:

(A) find the adjacency matrix for a graph or digraph;

(B) calculate the number of paths between two points of a graph having length n using the adjacency matrix;

(C) determine the line of best fit or quadratic least squares polynomial given a set of data points;

(D) predict the future state of an interdependent system using Markov chains on a set of data;

(E) identify long-term trends in the form of a steady-state vector through the use of Markov chains;

(F) describe how matrix inverses can be used in Hill Substitution to encode and decode textual information;

(G) use coordinatization to rotate the coordinate axis and simplify the equations of conic sections; and

(H) diagonalize the matrix for any given quadratic forms so that the expression contains no mixed-product terms, only square ones.

Recommended Resources and Materials:

Andrilli, S., & Hecker, D. (2010). Elementary linear algebra (4th ed.). Amsterdam: Elsevier Academic Press.

Bretscher, O. (2004). Linear algebra with applications (3rd ed.). Upper Saddle River, N.J.: Pearson, Prentice Hall.

Recommended Course Activities:

The teacher introduces students to the vocabulary, concepts, and problem-solving processes of linear algebra. Students participate in guided practice with the teacher to master various problem-solving scenarios. Students are assigned homework problems for analysis, independent practice, and mastery. Student progress is evaluated by tests. Students do a stock market project related to linear algebra, resulting in a research paper and class presentation. Students watch videos, hear presentations by guest speakers, and take field trips to bridge the gap between the classroom and the real-world of work.

Suggested methods for evaluating student outcomes:

- Test grades given for evaluations of learning of vocabulary, notation, and problem-solving.
- Group quizzes will be used to allow students to explore harder proof- based problems in small groups.

Teacher qualifications:

Secondary Teaching Certificate in Mathematics

Recommended: Master's Degree with at least 15 graduate hours in mathematics, and continuing education in mathematics through graduate courses, summer workshops, and staff development training

Additional information: