



Welcome to the Math Research- Based Topics!

Strong Foundations Framework Grant Learning Opportunity
July 2022

Recap: Strong Foundations Framework Grant Learning Opportunities

We have many ways to learn more about this grant!

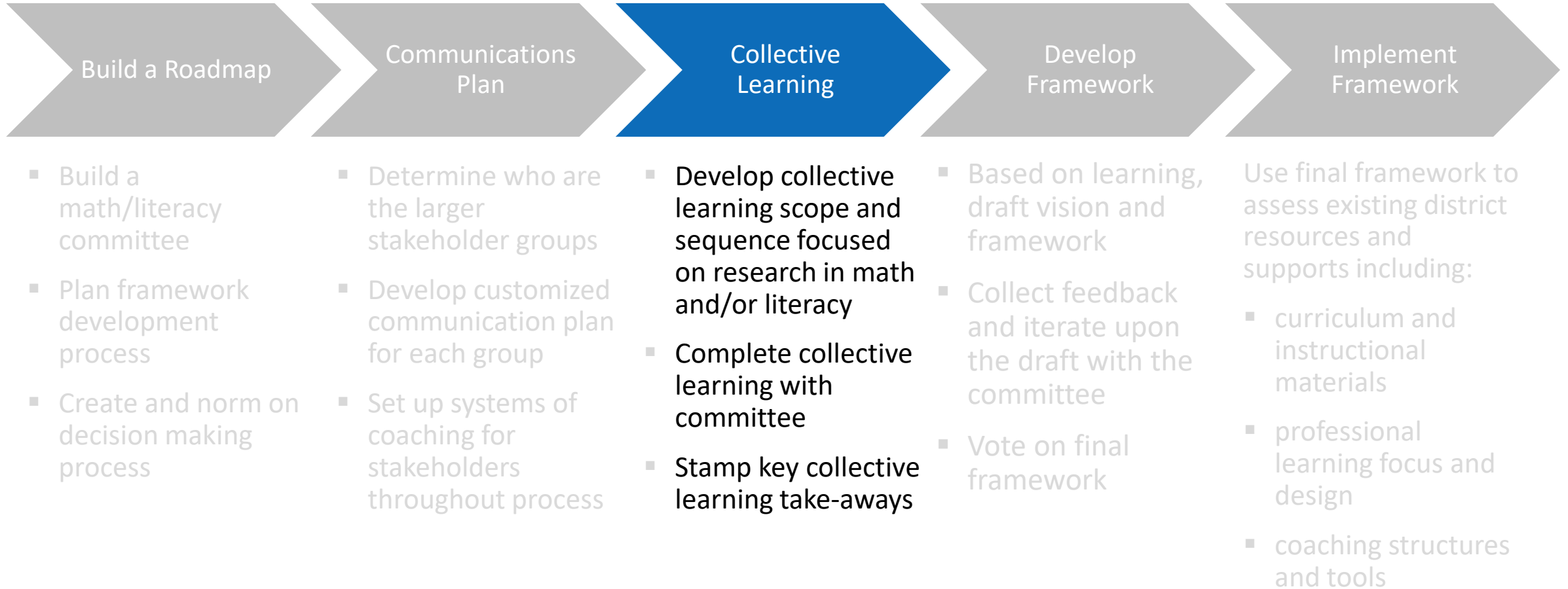
- 1. District Panel:** Sign up for our last District Panel to hear from districts already engaging in this work.
 - Date: 8/18; Registration: [Here](#)
- 2. RLA Research-Based Topics:** Sign up for the August research topic session for RLA ([register here](#)).
- 3. TEA Consultation [Optional]:** District leaders may sign up for one 30-minute session with TEA [here](#) to help determine what application decision may be best based on local context.

Purpose of Session

- Allow LEAs interested in the Strong Foundations Framework Grant to learn more about research topics in math
- Allow LEAs to get a ***short sample*** of the collective learning series and “step back” their district would take to dig deeper into the research

Recap: Collective Learning Series in Strong Foundations Framework Grant

LEAs will contract with an **approved provider** to go through the following steps, with the flexibility to customize for their local context



What are the essential best practices in mathematics instruction?

Math Research-based Instructional Strategies (RBIS)

1

Balance Conceptual & Procedural

Pursue **rigor by balancing conceptual understanding, procedural skill and fluency**. Apply this balanced understanding to mathematical **applications** as required by the standards in the TEKS.

2

Depth of key concepts

Focus on math content that **aligns to and meets the rigor of the TEKS** for each grade level, **while concentrating time and effort** on going deep on the **most important topics** for the grade level.

3

Coherence of Key Concepts

Connect concepts within and across grades along a strategic progression of learning so that new understandings are built on previous foundations. Mathematics tells a **continuous, connected story**.

4

Productive Struggle

Students engage in productive problem solving, engaging in **multiple opportunities for practice, discussion, representations, and writing** that requires them to explain and revise their thinking.

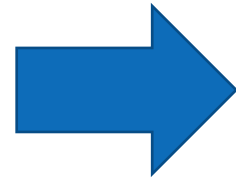
5

Assessment Practices

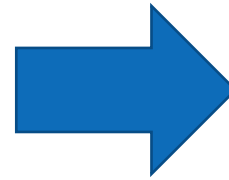
Leverage HQIM **embedded assessments** to drive instruction.

RBIS Background Information

TEA developed a set of
Research-based
Instructional Strategies



RBIS



Session Norms & Parking Lot

- Be fully present
- Use technology appropriately
- Disagree with ideas, not people
- Have fun!

**Got questions?
Please drop them in the
chat.**

What are the RBIS?

RBIS are...

- A set of research-based practices that highlights misconceptions common in the field
- Topics that require conceptual or philosophical shifts in approach to instruction
- A set of practices that are supported by research and should be present in classrooms, regardless of instructional materials
- A set of practices that relate directly to the design of instructional materials AND/OR the approach required to implement them well

RBIS are NOT...

- Topics that are commonly agreed upon (e.g., materials should be aligned to the standards)
- Topics not related to curriculum and instructional materials (e.g., classroom management best practices)

RBIS in Context



Why is it important to look towards the research?

Research tells us there are clear **best practices in instruction** by content and associated topics such as assessment and supporting special populations. Because these practices directly connect to **improving students' academic achievement and experience**, they should inform school, district, and state-wide **visions for instruction** and increase use of **high-quality instructional materials (HQIM)**. The RBIS also demonstrate **why HQIM is important** and what is required to **implement HQIM** well.

RBIS 1: Balance Conceptual & Procedural

What are the essential best practices in mathematics instruction?

Math Research-Based Instructional Strategies (RBIS)

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5

Assessment
Practices

Leverage HQIM **embedded assessments** to drive instruction.



Do the Math

Work independently to complete the following...

1. Simplify the expression.
2. Script out how you would explain how to solve this problem to a student.
3. Begin to reflect on how you have seen this skill taught in TX classrooms.

$$1\frac{3}{4} \div \frac{1}{2}$$

PISA Results & Trends

U.S. 15-year-old students' average score **below** OECD average

Average Score Higher than U.S. Average			
<i>Singapore</i>	564	Austria	497
<i>Hong Kong (China)</i>	548	New Zealand	495
<i>Macau (China)</i>	544	<i>Vietnam</i>	495
<i>Chinese Taipei</i>	542	<i>Russian Federation</i>	494
Japan		Sweden	494
<i>B-S-J-G (China)</i>	531	Australia	494
Korea, Republic of	524	France	493
Switzerland	521	United Kingdom	492
Estonia	520	Czech Republic	492
Canada	516	Portugal	492
Netherlands	512	OECD Average	490
Denmark	511	Italy	490
Finland	511	Iceland	488
Slovenia	510	Spain	486
Belgium	507	Luxembourg	486
Germany	506	Latvia	482
Poland	504	<i>Malta</i>	479
Ireland	504	<i>Lithuania</i>	478
Norway	502	United States	470

Note: *Italics* indicate non-OECD countries and education systems.

Key Points

- U.S. student performance on the **curriculum-based** assessment was notably stronger than it was on the assessment that required students to **apply their understandings** to novel, real-world problems.
- When students had to **apply their learning to new contexts**, the relative ranking of U.S. students, in comparison to other countries, declined.
- **Reflect:** What connections are you making between the results of these two studies and how students perform in math around the state?



Back to Division of Fraction

Compare the following responses. What do you notice?

$$1\frac{3}{4} \div \frac{1}{2}$$

Response 1: “I would convert $1\frac{3}{4}$ to fourths, which would give me $\frac{7}{4}$. Then to divide by $\frac{1}{2}$, I would invert $\frac{1}{2}$ and multiply. So, I would multiply $\frac{7}{4}$ by 2 and I would get $\frac{14}{4}$, and then I would divide 14 by 4 to get it back to my mixed number, $3\frac{2}{4}$ and then I would reduce that into $3\frac{1}{2}$.

Response 2: “This question is asking us how many halves there are in $1\frac{3}{4}$. So, to divide $1\frac{3}{4}$ by $\frac{1}{2}$ we multiply $1\frac{3}{4}$ by the reciprocal of $\frac{1}{2}$ (which is $\frac{2}{1}$) and we get $3\frac{1}{2}$.

Division of Fraction

CONCEPTUAL

Response 2: "This question is asking us how many halves there are in $1\frac{3}{4}$.

So, to divide $1\frac{3}{4}$ by $\frac{1}{2}$ we multiply $1\frac{3}{4}$ by the reciprocal of $\frac{1}{2}$ (which is $\frac{2}{1}$) and we get $3\frac{1}{2}$."

PROCEDURAL

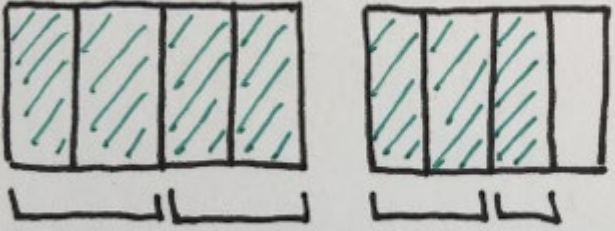
$$1\frac{3}{4} \div \frac{1}{2}$$

Division of Fractions – A Balanced Approach

$$1\frac{3}{4} \div \frac{1}{2}$$

Conceptual

Determine how many "halves" there are in $1\frac{3}{4}$.



1 1 1 $\frac{1}{2}$ of a half.

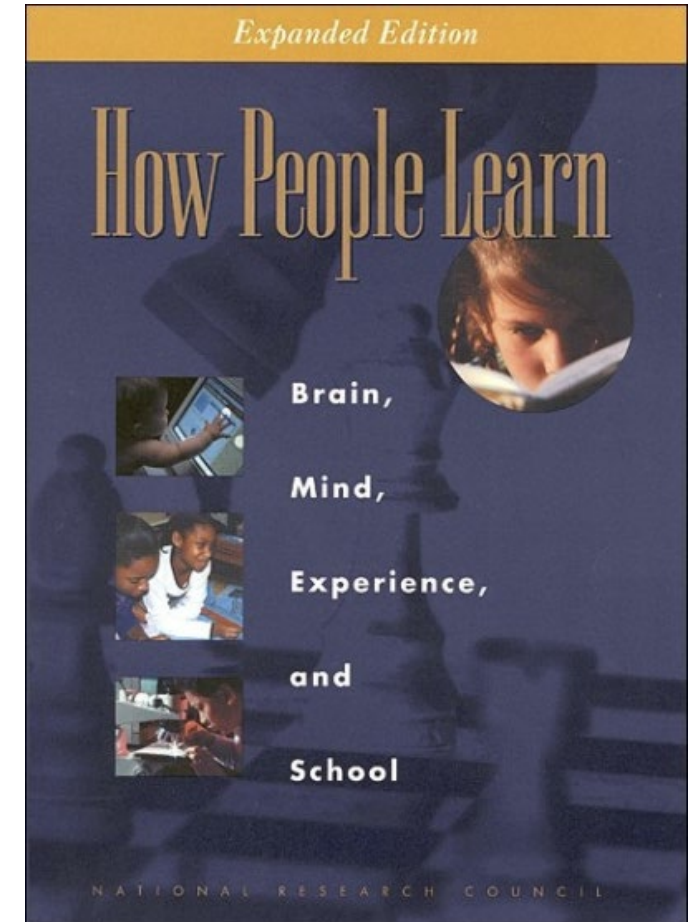
Answer: $3\frac{1}{2}$

Procedural

$$1\frac{3}{4} \div \frac{1}{2}$$
$$\frac{7}{4} \cdot \frac{2}{1}$$
$$\frac{14}{4} = 3\frac{3}{4} = \boxed{3\frac{1}{2}}$$

The Science of Learning: Three Key Findings

- Engage Preconceptions
- Understanding Requires Factual Knowledge and Conceptual Frameworks
- A Metacognitive Approach Enables Self-Monitoring



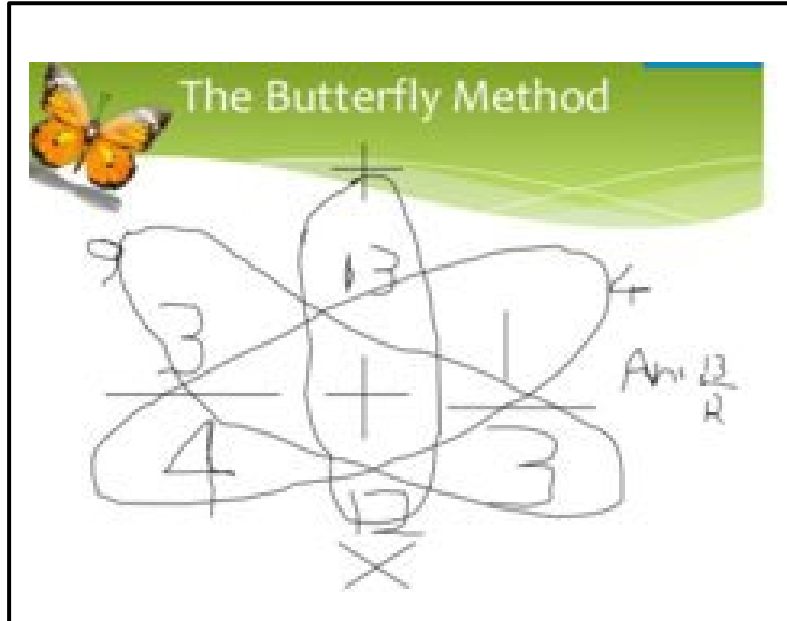
National Research Council, *How People Learn: Brain, Mind, Experience and School*. National Academy Press, 2000..

Procedural Skill & Fluency allows for Automaticity

- Students can carry out procedures **flexibly, accurately, efficiently,** and **appropriately.**
- Student mastery of key fluency allows for the **automaticity** that helps to **access** and manipulate more **advanced concepts.**

$$\begin{array}{r} 9x - 28 = -1 \\ +28 \quad +28 \\ \hline 9x = 27 \\ 9 \quad 9 \\ \hline x = 3 \end{array}$$

Tricks versus Procedural Understanding



$$\frac{3}{4} + \frac{3}{5} + \frac{1}{3} =$$

Key Idea: Tricks are **context dependent**. When students have a balanced procedural and conceptual understanding, they can apply a consistent framework to all contexts; **the rules of math do not change**.

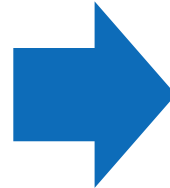
Key Points: Trick versus Procedural Understanding

- The ability to apply a "trick" is not the same as true procedural fluency.
- When students are introduced to tricks, it removes all elements of balance of procedural and conceptual understanding.
- When students do not have conceptual understanding, they are not set up to apply their learning and they have fluency with a trick, not a procedure that can be applied in different contexts.

Balance Conceptual and Procedural Instruction

Misconception:

Balancing conceptual, procedural, and application means there should be equal time spent on each component of rigor.



RBIS Approach:

Conceptual understanding is often the primary focus and comes before procedural fluency and application, but each of these components of rigor are intertwined in quality tasks.

Procedural fluency is fact practice.



The TEKS define procedural fluency as “skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.” Fact practice is important, but there is more to procedural fluency than memorizing math facts.

Students need to master math facts before they can engage in conceptual problem-solving.



Students can engage in rich, conceptually-based tasks while continuing to develop their procedural fluency.

RBIS 2 & 3 : Depth and Coherence of Key Concepts



Do The Math

- Order the following elementary level math problems to illustrate the correct grade level progression. Justify your reasoning.

A

1. For the following problems, draw a picture using the rectangular fraction model and write the answer. Simplify your answer, if possible.

a. $\frac{1}{3} - \frac{1}{4} =$

b. $\frac{2}{3} - \frac{1}{2} =$

B

1. Each fraction strip is 1 whole. All the fraction strips are equal in length. Color 1 fractional unit in each strip. Then, answer the questions below.

$\frac{1}{2}$

$\frac{1}{3}$

$\frac{1}{5}$

$\frac{1}{4}$

$\frac{1}{9}$

2. Circle *less than* or *greater than*. Whisper the complete sentence.

a. $\frac{1}{2}$ is less than $\frac{1}{3}$ b. $\frac{1}{9}$ is less than $\frac{1}{2}$
 greater than

c. $\frac{1}{4}$ is less than $\frac{1}{2}$ d. $\frac{1}{4}$ is less than $\frac{1}{9}$
 greater than

e. $\frac{1}{5}$ is less than $\frac{1}{3}$ f. $\frac{1}{5}$ is less than $\frac{1}{4}$
 greater than

g. $\frac{1}{2}$ is less than $\frac{1}{5}$ h. 6 fifths is less than 3 thirds
 greater than greater than

C

1. Draw an area model for each pair of fractions, and use it to compare the two fractions by writing >, <, or = on the line. The first two have been partially done for you. Each rectangle represents 1.

a. $\frac{1}{2} < \frac{2}{3}$ $\frac{2}{3}$

b. $\frac{4}{5} > \frac{3}{4}$

c. $\frac{3}{5} > \frac{4}{7}$

d. $\frac{3}{7} < \frac{2}{6}$

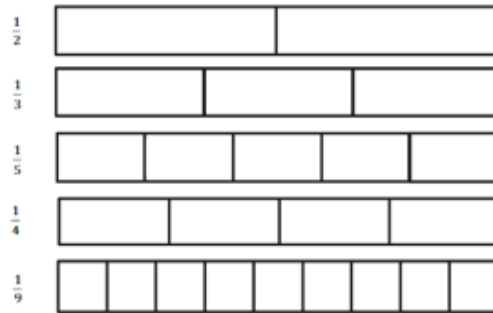


Depth and Coherence of Key Concepts Examples

How do these examples tell a connected, coherent story about the progression of the skills students must build around fractions?

B 3rd Grade

1. Each fraction strip is 1 whole. All the fraction strips are equal in length. Color 1 fractional unit in each strip. Then, answer the questions below.

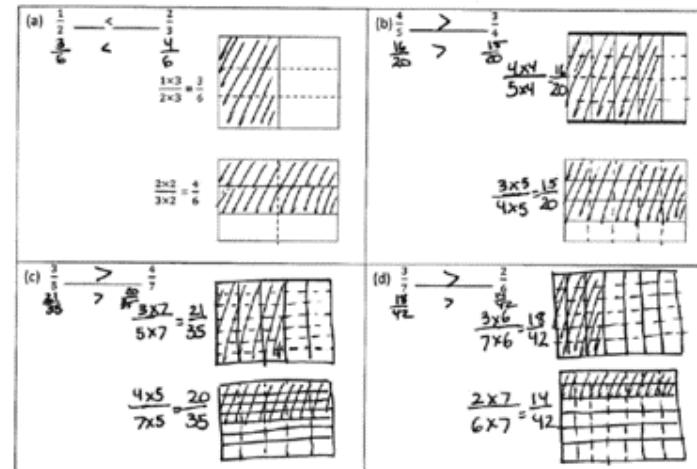


2. Circle less than or greater than. Whisper the complete sentence.

- | | | | | | |
|---------------------|--------------|---------------|---------------------|--------------|---------------|
| a. $\frac{1}{2}$ is | less than | $\frac{1}{3}$ | b. $\frac{1}{9}$ is | less than | $\frac{1}{2}$ |
| | greater than | | | greater than | |
| c. $\frac{1}{4}$ is | less than | $\frac{1}{2}$ | d. $\frac{1}{4}$ is | less than | $\frac{1}{9}$ |
| | greater than | | | greater than | |
| e. $\frac{1}{5}$ is | less than | $\frac{1}{3}$ | f. $\frac{1}{5}$ is | less than | $\frac{1}{4}$ |
| | greater than | | | greater than | |
| g. $\frac{1}{2}$ is | less than | $\frac{1}{5}$ | h. 6 fifths is | less than | 3 thirds |
| | greater than | | | greater than | |

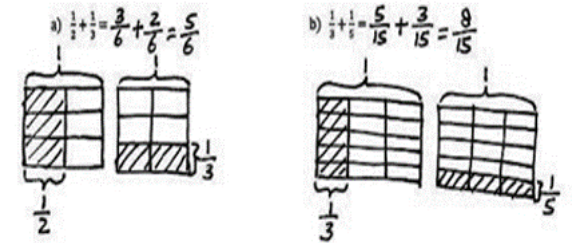
C 4th Grade

1. Draw an area model for each pair of fractions, and use it to compare the two fractions by writing $>$, $<$, or $=$ on the line. The first two have been partially done for you. Each rectangle represents 1.



A 5th Grade

1. For the following problems, draw a picture using the rectangular fraction model and write the answer. Simplify your answer.

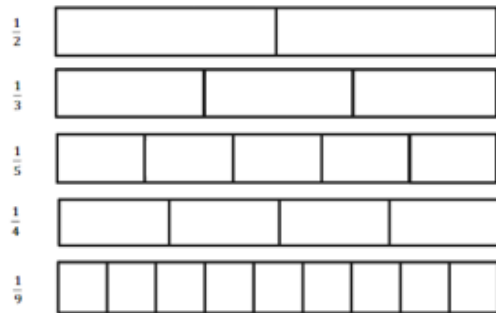


Eureka Math TEKS Edition. Grade 3, Module 5, Lesson 11
Eureka Math TEKS Edition. Grade 4, Module 5, Lesson 14
Eureka Math TEKS Edition. Grade 5, Module 3, Lesson 3

Depth & Coherence of Key Concepts Examples

B 3rd Grade

1. Each fraction strip is 1 whole. All the fraction strips are equal in length. Color 1 fractional unit in each strip. Then, answer the questions below.



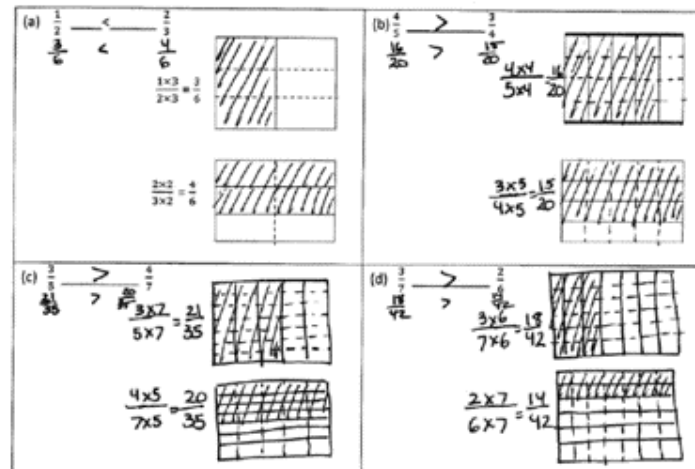
2. Circle less than or greater than. Whisper the complete sentence.

- a. $\frac{1}{2}$ is less than $\frac{1}{3}$ or greater than $\frac{1}{3}$
 b. $\frac{1}{9}$ is less than $\frac{1}{2}$ or greater than $\frac{1}{2}$
 c. $\frac{1}{4}$ is less than $\frac{1}{2}$ or greater than $\frac{1}{2}$
 d. $\frac{1}{4}$ is less than $\frac{1}{9}$ or greater than $\frac{1}{9}$

Students compare unit fractions by looking at their size. 3.3H

C 4th Grade

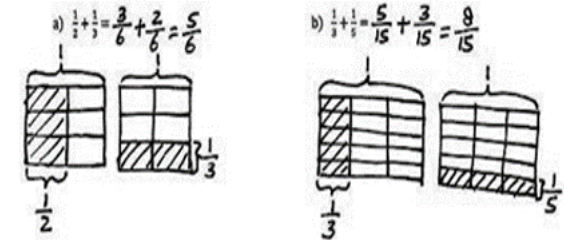
1. Draw an area model for each pair of fractions, and use it to compare the two fractions by writing $>$, $<$, or $=$ on the line. The first two have been partially done for you. Each rectangle represents 1.



Students begin to compare fractions with different numerators and denominators. 4.3D

A 5th Grade

1. For the following problems, draw a picture using the rectangular fraction model and write the answer. Simplify your answer.



Students represent and solve addition and subtraction of fractions with unequal denominators. 5.3H

What are the essential best practices in mathematics instruction?

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Students engage in productive problem solving, engaging in **multiple opportunities for practice, discussion, representations, and writing** that requires them to explain and revise their thinking.

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Assessment Practices

Leverage HQIM **embedded assessments** to drive instruction.

Depth of Key Concepts

2

Depth of Key Concepts

Focus on math content that **aligns to and meets the rigor of the TEKS** for each grade level, **while concentrating time and effort** on going deep on the **most important topics** for the grade level.

Meet rigor of the TEKS

Prepare students to identify appropriate concepts to tackle real-world, relevant tasks through an alignment to the TEKS and a balance of conceptual and procedural fluency.

Concentrate time and effort

Utilize high-quality instructional materials to ensure that the majority of class time is spent going deep on the most important topics for the grade level or course.

Most important topics

Identify the focal points that build coherence across grade levels and provide a foundation for strong mathematics understanding of algebra and beyond.

Depth of Key Concepts - Task 1



Solve. What skills are students demonstrating when completing this task?

Equivalent Fractions (only numerators missing)

Grade 3 Fractions Worksheet

Complete the equivalent fractions.

1. $\frac{2}{5} = \frac{\quad}{30}$

2. $\frac{\quad}{2} = \frac{6}{12}$

3. $\frac{2}{3} = \frac{\quad}{27}$

4. $\frac{1}{2} = \frac{\quad}{18}$

5. $\frac{\quad}{3} = \frac{12}{18}$

6. $\frac{4}{5} = \frac{\quad}{25}$

Depth of Key Concepts - Task 2



Solve. What skills are students demonstrating when completing this task?

1. Jerry put 7 equally spaced hooks on a straight wire so students could hang up their coats. The whole length is from the first hook to the last hook.

a. On the picture below, label the fraction of the wire's length where each hook is located.



b. At what fraction is Betsy's coat if she hangs it at the halfway point?

c. Write a fraction that is equivalent to your answer for Part (b).

2. Jerry used the picture below to show his son how to find a fraction equal to $\frac{2}{3}$. Explain what Jerry might have said and done using words, pictures, and numbers.



Depth of Key Concepts Comparison

Equivalent Fractions (only numerators missing)

Grade 3 Fractions Worksheet

Complete the equivalent fractions.

1. $\frac{2}{5} = \frac{\quad}{30}$

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3. $\frac{2}{3} = \frac{\quad}{27}$

4. $\frac{1}{2} = \frac{\quad}{18}$

5. $\frac{3}{3} = \frac{12}{18}$

6. $\frac{4}{5} = \frac{\quad}{25}$

3(3)(F) The student is expected to represent equivalent fractions with denominators of 2, 3, 4, 6, and 8 using a variety of objects and pictorial models, including number lines.

1. Jerry put 7 equally spaced hooks on a straight wire so students could hang up their coats. The whole length is from the first hook to the last hook.

- a. On the picture below, label the fraction of the wire's length where each hook is located.



- b. At what fraction is Betsy's coat if she hangs it at the halfway point?

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Depth of Key Concepts Comparison

Equivalent Fractions (only numerators missing)

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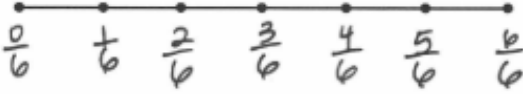
1. $\frac{2}{5} = \frac{\quad}{30}$ 2. $\frac{2}{2} = \frac{6}{12}$ 3. $\frac{2}{3} = \frac{\quad}{27}$

4. $\frac{1}{2} = \frac{\quad}{18}$ 5. $\frac{3}{3} = \frac{12}{18}$ 6. $\frac{4}{5} = \frac{\quad}{25}$

- Grade 3 students should be using a **variety of objects, models, or the number line** to develop their understanding of fraction equivalence.
- Students should be demonstrating that they can recognize and generate simple equivalent fractions on a number line or area model and **explaining** why they are equivalent.

1. Jerry put 7 equally spaced hooks on a straight wire so students could hang up their coats. The whole length is from the first hook to the last hook.

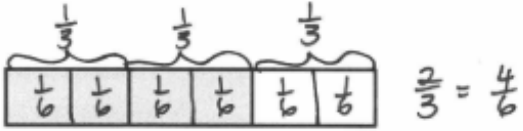
a. On the picture below, label the fraction of the wire's length where each hook is located.



b. At what fraction is Betsy's coat if she hangs it at the halfway point? $\frac{3}{6}$

c. Write a fraction that is equivalent to your answer for Part (b). $\frac{1}{2}$

2. Jerry used the picture below to show his son how to find a fraction equal to $\frac{2}{3}$. Explain what Jerry might have said and done using words, pictures, and numbers.



I made each $\frac{1}{3}$ into 2 smaller, equal parts. So then it wasn't just thirds anymore, it was sixths too! I can see from the shading that $\frac{2}{3}$ is the same as $\frac{4}{6}$.

RBIS 3: Coherence of Key Concepts

Coherence of Key Concepts

3

Coherence of Key Concepts

Connect concepts within and across grades along a strategic progression of learning so that new understandings are built on previous foundations. Mathematics tells a **continuous, connected story**.

Within grade levels

Build new ideas on the foundation of what students have learned during the current school year in previous and future lessons and units.

Across grade levels

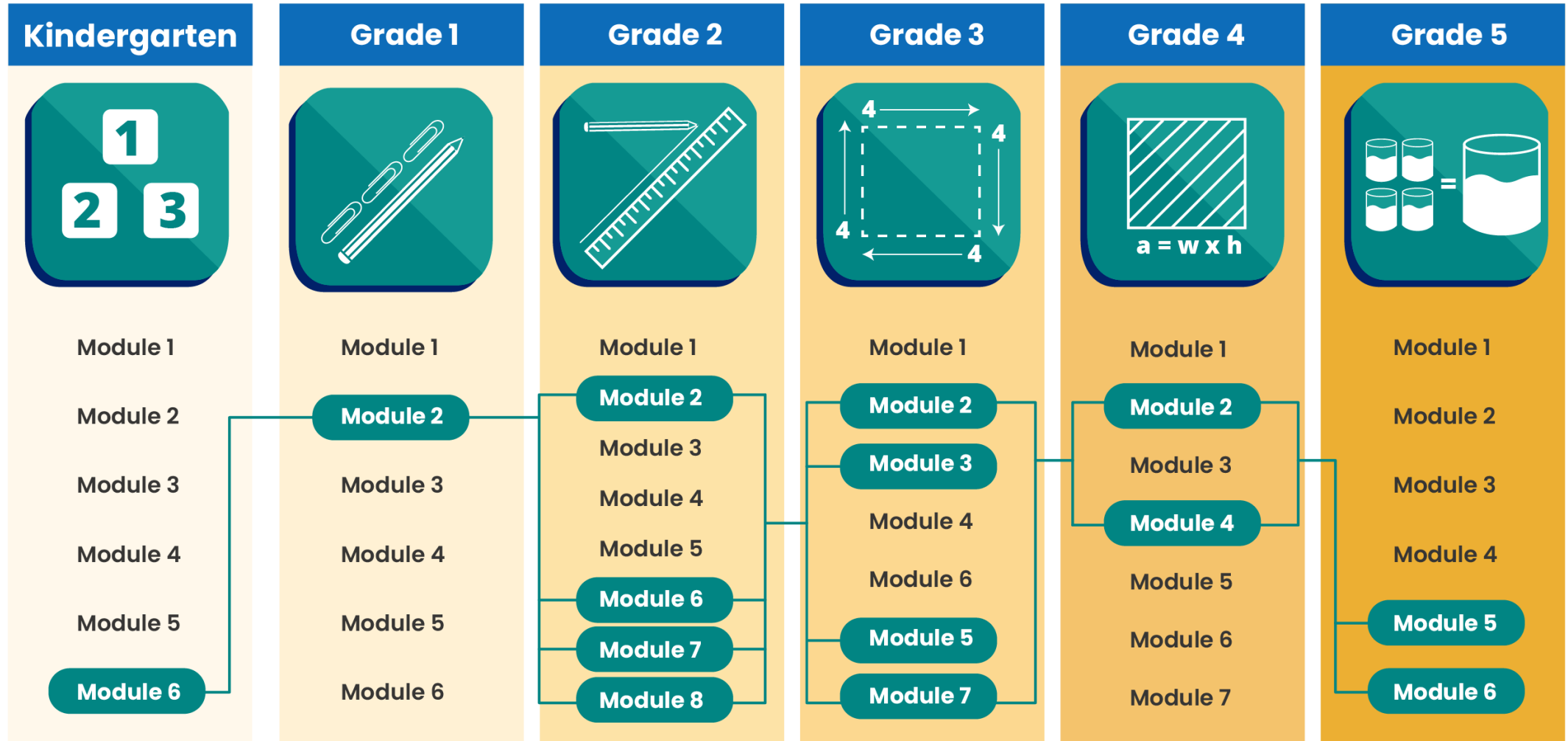
Build upon key concepts in previous and current grade levels as foundational knowledge that could serve as gatekeepers for new ideas in the next grade level and future math courses.

Continuous, connected story

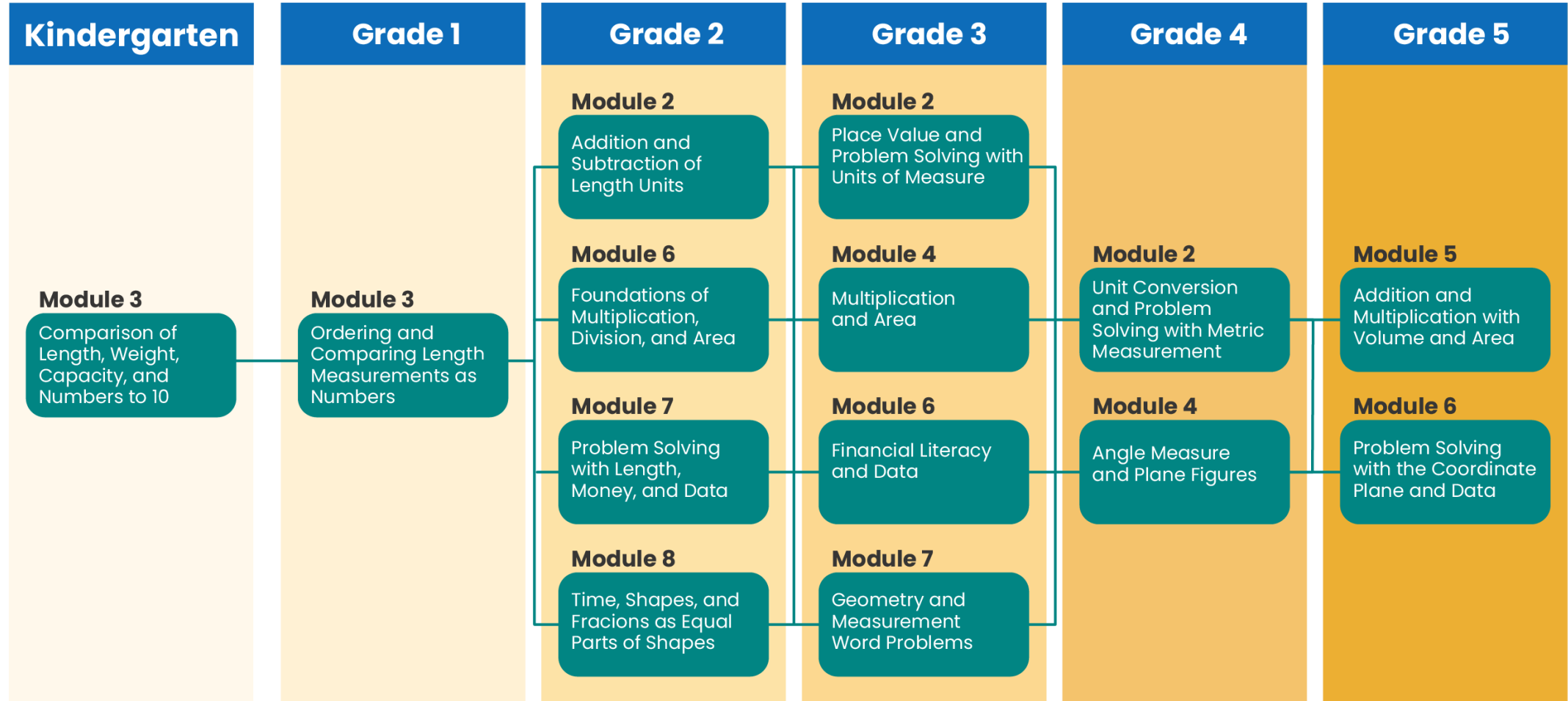
Mathematics concepts and skills create an ongoing, coherent learning experience throughout a students' educational journey.

Coherence of Key Concepts

Over time, students develop knowledge of key mathematical concepts. Concepts connect within and across grades along a strategic learning progression.



Coherence of Key Concepts



Example: Coherence in Geometry and Measurement

Beginning with “Length, Weight, Capacity, and Numbers to 10” in kindergarten and progressing all the way up to “Problem Solving with the Coordinate Plane and Data” in grade 5, Eureka Math TEKS Edition builds coherence in the foundations of geometry and measurement.

Coherence of Key Concepts: Grades 6-8

Grade 6	Grade 7	Grade 8
(6) Expressions, equations, and relationships. The student applies mathematical process standards to use multiple representations to describe algebraic relationships. The student is expected to:	Representing Linear Relationships	
	(7) Expressions, equations, and relationships. The student applies mathematical process standards to represent linear relationships using multiple representations. The student is expected to:	(5) Proportionality. The student applies mathematical
	(A) represent linear relationships using verbal descriptions, tables, graphs, and equations that simplify to $y = mx + b$.	
(B) write an equation that represents the relationship between independent and dependent quantities from a table.		(I) write an equation in the form $y = mx + b$ to model a linear relationship between two quantities using verbal, numerical, tabular, and graphical representations.
(C) represent situations described by tables and equations in the form $y = kx$ or $y = mx + b$.		(F) distinguish between proportional and non-proportional situations using tables, graphs, and equations in the form $y = kx$ or $y = mx + b$, where $b \neq 0$.
		(H) identify examples of proportional and non-proportional functions that arise from mathematical and real-world problems.

(I) Write an equation in the form $y = mx + b$ to model a linear relationship between two quantities using verbal, numerical, tabular, and graphical representations

(B) write an equation that represents the relationship between independent and dependent quantities from a table

Grade 8

Module 2: Linear Relationships
Topic 1: From Proportions to Linear Relationships
Topic 2: Linear Relationships

Grade 7

Module 1: Thinking Proportionally
Topic 3: Proportionality

Module 3: Reasoning Algebraically
Topic 4: Multiple Representations of Equations

Grade 6

Module 4: Determining Unknown Quantities
Topic 3: Graphing Quantitative Relationships

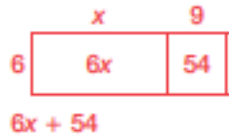
Examples of Building Coherence Across Grade Levels with HQIM



Order the following math problems to illustrate the correct grade level progression. Justify your reasoning.

4. Draw a model for each expression, and then simplify.

a. $6(x + 9)$

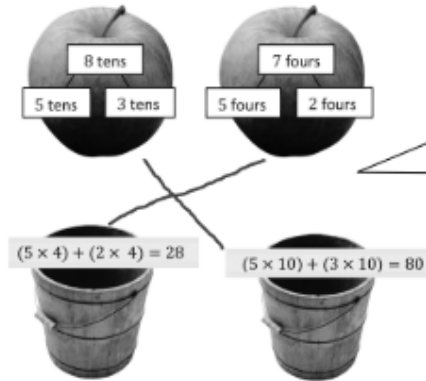


b. $7(2b - 5)$



A

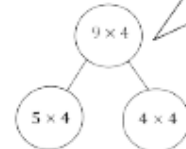
1. Match the number bond on an apple with the equation on a bucket that shows the same total.



The number bonds in the apples help me see how I can find the total by adding the two smaller parts together. I can match the apples with the equations below that show the same two parts and total.

2. Solve.

$9 \times 4 = \underline{\quad 36 \quad}$



I can think of this total as 9 fours. There are many ways to break apart 9 fours, but I'm going to break it apart as 5 fours and 4 fours because 5 is a friendly number.

$(\underline{5} \times 4) + (\underline{4} \times 4) = 9 \times 4$

$\underline{20} + \underline{16} = \underline{36}$

$9 \times 4 = \underline{36}$

B

a. 425×4

	400	20	5
4	$1,600$	80	20
$4(400 + 20 + 5)$ $(4 \times 400) + (4 \times 20) + (4 \times 5)$			

b. 534×7

	500	30	4
7	$3,500$	210	28
$7(500 + 30 + 4)$ $(7 \times 500) + (7 \times 30) + (7 \times 4)$			

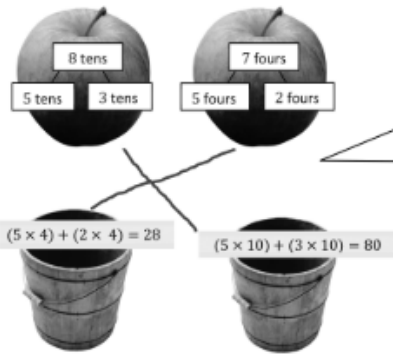
C

Examples of Building Coherence Across Grade Levels with HQIM

Grade 3

B

1. Match the number bond on an apple with the equation on a bucket that shows the same total.

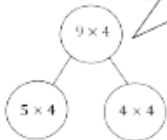


The number bonds in the apples help me see how I can find the total by adding the two smaller parts together. I can match the apples with the equations below that show the same two parts and total.

2. Solve.

$$9 \times 4 = \underline{\quad 36 \quad}$$

I can think of this total as 9 fours. There are many ways to break apart 9 fours, but I'm going to break it apart as 5 fours and 4 fours because 5 is a friendly number.



$$(\underline{5} \times 4) + (\underline{4} \times 4) = 9 \times 4$$

$$\underline{20} + \underline{16} = \underline{36}$$

$$9 \times 4 = \underline{36}$$

Eureka Math TEKS Edition Module 1, Lesson 18

Students apply the distributive property to decompose units. 3.4G

Grade 4

C

Eureka Math TEKS Edition Module 3, Lesson 11

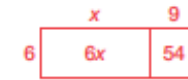
Students represent the product of 2 two-digit numbers using area models. 4.4C/4.4D

Grade 6

A

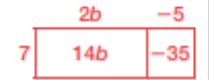
4. Draw a model for each expression, and then simplify.

a. $6(x + 9)$



$$6x + 54$$

b. $7(2b - 5)$



$$14b - 35$$

Carnegie Learning Texas Math Solution 6-12Module 1

Students generate equivalent expressions using the distributive property. 6.7D

Types of Coherence

Within HQIM

Teachers and selected materials utilize consistent vocabulary terms and tools year-to-year from elementary to secondary levels to support all learners, including emergent bilingual students.

Year-to-Year

Grade level content builds year-to-year. Teachers regularly connect or ask students to connect what they have learned from previous years.

"In third grade you learned how to add and subtract fractions with the same denominator, this year we will learn how to add and subtract fractions with different denominators. Let's start with what we know ..."

Unit-to-Unit

Units are sequenced to build on each other over the course of the school year. Students and teacher regularly connect and build on what they know from previous units.

"Last unit we studied linear expressions between two quantities, this unit we will begin to discuss what happens when there is not a constant rate of change."

Day-to-Day

Teachers and students make connections and build on what they know from previous lessons.

"Over the past few days we have been studying complex fractions, their meaning, and comparing numbers, yesterday we used modeling to ... today we will continue to model ..."

RBIS 4: Productive Struggle

What are the essential best practices in mathematics instruction?

Math Research-Based Instructional Strategies (RBIS)

1

Balance
Conceptual &
Procedural

Pursue **rigor** by **balancing conceptual understanding, procedural skill and fluency,** and **application** as required by the standards in the TEKS.

2

Depth of Key
Concepts

Focus on math content that **aligns to and meets the rigor of the TEKS** for each grade level, **while concentrating time and effort** on going deep on the **most important topics** for the grade level.

3

Coherence
of Key Concepts

Connect concepts within and across grades along a strategic progression of learning so that new understandings are built on previous foundations. Mathematics tells a **continuous, connected story.**

4

Productive
Struggle

Students engage in productive problem solving, engaging in **multiple opportunities for practice, discussion, representations, and writing** that requires them to explain and revise their thinking.

5

Assessment
Practices

Leverage HQIM **embedded assessments** to drive instruction.

Defining productive struggle

4

Productive Struggle

Students engage in productive problem solving, including **multiple opportunities for practice, discussion, representations, and writing** that requires them to explain and revise their thinking.

“...students **expend effort to make sense of mathematics**, to figure something out that is not immediately apparent...The struggle we have in mind comes from **solving problems that are within reach and grappling with key mathematical ideas** that are comprehensible but not yet well formed (Hiebert et al., 1996).”

“...productive struggle comprises **the work that students do to make sense of a situation and determine a course of action when a solution strategy is not stated**, implied, or immediately obvious. From an equity perspective, this implies that each and **every student must have the opportunity to struggle** with challenging mathematics and to receive support that encourages their persistence without removing the challenge.” (NCTM publications 2007, 2017)

Defining productive struggle

4

Productive Struggle

Student engage in productive problem solving, including **multiple opportunities for practice, discussion, representations, and writing** that requires them to explain and revise their thinking.

Maintains Rigor

Provides students time to collaboratively problem solve using different representations and then asking them to explain their thinking

Sets up all students to Engage

Tasks should have multiple entry points so that students can use different solution paths to solve and make connections

Develops Independent Problem Solvers

Acknowledging when students' effort supports their thinking and mathematical understanding, thus developing their capacity to persevere in the face of challenging content

Observing productive struggle...

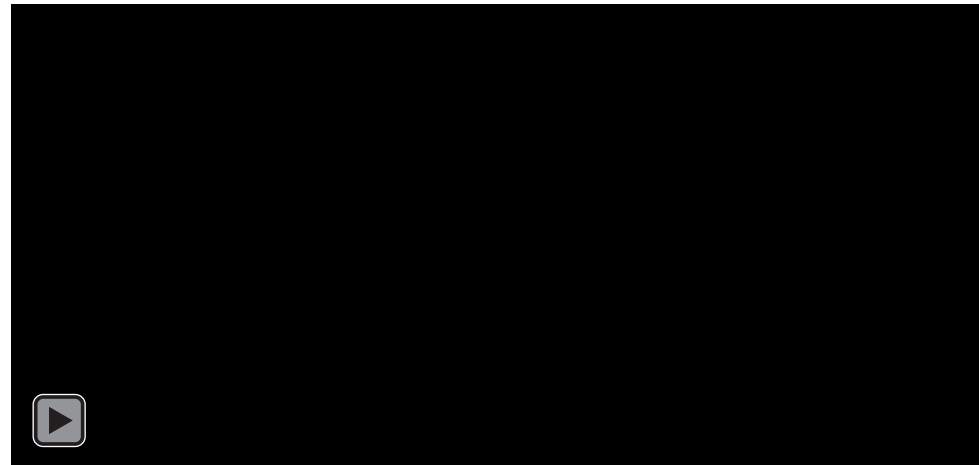


4

Productive Struggle

Students engage in productive problem solving, including **multiple opportunities for practice, discussion, representations, and writing** that requires them to explain and revise their thinking.

- **Observe and Reflect:** Watch the following instructional video. Is this an example of productive struggle? Using language from the RBIS, explain why or why not.









“Natalie has 30 jellybeans. Her mom gives her 23 more jellybeans. How many jellybeans does Natalie have?”

Productive Struggle is NOT

- students just showing their work on paper
- something to give only when students are at a certain age or grade level
- asking all students how they got an answer
- just for students who have the wrong answer.
- giving students challenging work or work above grade level.
- something that could harm a student's development



5 Types of Common Teacher Responses to Struggle

	Example	Impact on Student Learning
1. Telling 2. Directed Guidance	 <i>"Not quite. Instead, draw 23 cubes..."</i>	 More likely to lower the demands of the task and remove struggle.
3. Unfocused or Vague Direction	 <i>"Read the problem again and check your work..."</i>	 More likely to provide general suggestions that are not helpful to the particular task.
4. Probing Guidance 5. Affordance	 <i>"Can you tell me how your picture shows how you solved the problem..."</i>	 More likely to maintain the demands of the task and support productive struggle

Warshauer, Hiroko Kawaguchi. "Productive Struggle in Middle School Mathematics Classrooms." *Journal of Mathematics Teacher Education* 17, no.4 (2015): 375-399. [p. 387]

Strategies to Support Productive Struggle

Strategy	What it looks like
Question	"Teachers ask questions that help students focus on their thinking and identify the source of their struggle, then encourage students to build on their thinking or look at other ways to approach the problem."
Encourage	"Teachers encourage students to reflect on their work and support student struggle in their effort and not just in getting the correct answers."
Give Time	"Teachers give time and support for students to manage their struggles through adversity and failure by not stepping in too soon or too much, thereby taking the intellectual work away from the students."
Acknowledge	"Teachers acknowledge that struggle is an important part of learning and doing mathematics."

Probing Guidance

Affordance

What examples of these have you observed today?

Dos and Don'ts of Productive Struggle

DO...	Rationale
Give your students time to engage in productive struggle.	<i>You know your students, and you <u>prepared</u> your lesson to meet their needs. <u>Trust</u> that they can accomplish what you prepared for them.</i>
Ask questions when students are stuck.	<i>What have you done <u>before</u> that might be <u>useful</u> now? What seems <u>important</u> in the problem? How is this the same or different as what you've seen before?</i>
Encourage students to solve problems in different ways.	<i>Students need to feel <u>comfortable</u> trying different strategies. Celebrate <u>creativity</u> by encouraging students to <u>share</u> their thinking with the class.</i>
Praise students' effort on both successful and unsuccessful attempts .	<i>These actions send the message to students that you value <u>risk-taking</u> and trying out ideas. Have students <u>reflect</u> on what they learned from their unsuccessful efforts and how those efforts helped them decide what method(s) to try next. <u>Math is not just about getting the right answer.</u></i>

The Struggle is Real (and Productive) Mike Linskey, Great Minds: Eureka Math Blog <https://gm.greatminds.org/math/blog/eureka/the-struggle-is-real-and-productive>

STAAR redesign reflects RBIS best practices

Changes are coming to help improve alignment

- Classroom practices that over-use multiple choice questions, rely on only short reading passages, and limit student writing can get small, short-term gains on STAAR, but evidence has shown they don't lead to high performance or long-term student mastery
- Strong instructional practices lead to increased student understanding and stronger performance on STAAR
- **It is possible for the state summative assessment to be designed so that it better aligns with strong instructional practices, while still accurately measuring student mastery**

For math specifically, many of the changes will be in new item types to allow for students to respond in new ways


These new question types allow students to respond to questions in a way that they'll see in their classrooms and high-quality instructional materials

The following new question types may be included in the specified Mathematics tests starting in Spring 2023

*Question Type	Question Type Description	STAAR Math Test Titles
Equation editor	Student can write responses in the form of fractions, expressions, equations, or inequalities.	Grades 3-8 EOC
Text Entry	Student responds by typing a brief string of text such as a number, word, or phrase.	Grades 3-8 EOC
Graphing	Student selects, points, draws lines, drags bar graphs, and perform other functions to independently create different types of graphs.	Grades 3-8 EOC
Number line	Student selects a point, an open or closed circle, and a direction arrow to demonstrate a solution set on a number line.	Grades 6-8 EOC
Inline choice	Student selects the correct answer(s) from one or more drop-down menu(s).	Grades 3-8 EOC
Hot spot	Student responds by selecting one or more specific areas of a graphic.	Grades 3-8 EOC
Fraction model	Student represents a fraction by dividing an object into the correct number of sections to indicate the denominator and clicking to shade the appropriate number of sections to indicate the numerator.	Grades 3-5
Drag and drop	Student evaluates a given number of options (words, numbers, symbols, etc.) and chooses which response(s) to drag to a given area (a diagram, map, chart, etc.).	Grades 3-8 EOC
Match table grid	Student matches statements or objects to different categories presented in a table grid.	Grades 6-8 EOC
Multiselect	Student can select more than one correct answer from a set of possible answers.	Grades 3-8 EOC

Max possible points per question

 2 points

 1 or 2 points dependent upon question

*Not all new question types will appear on every test every year



TEXAS ASSESSMENT

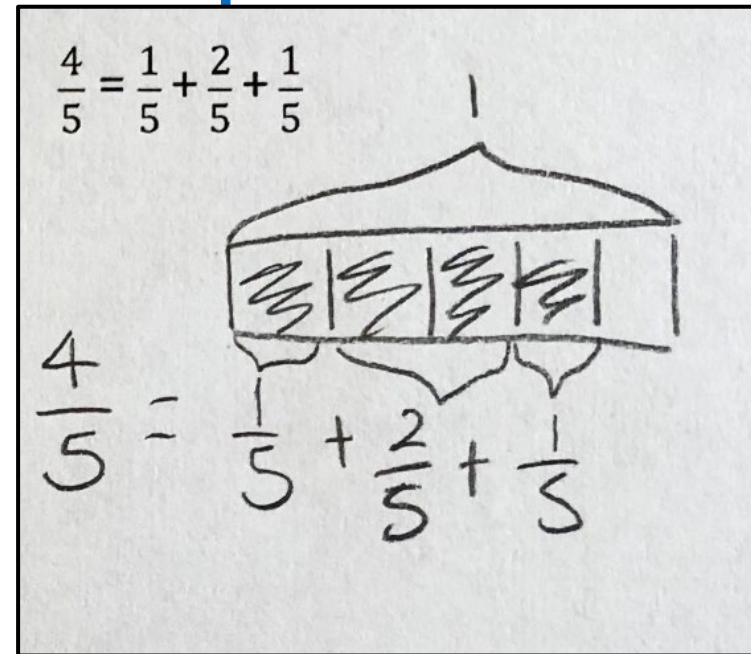


In the classroom, students are asked to engage with content in multiple ways to gain and express understanding (I)

Grade 4 Math TEKS

- 4.3A: represent a fraction a/b as a sum of fractions $1/b$, where a and b are whole numbers and $b > 0$, including when $a > b$
- 4.3B: decompose a fraction in more than one way into a sum of fractions with the same denominator using concrete and pictorial models and recording results with symbolic representations.

Example: “Draw and label a strip diagram to model the decomposition”

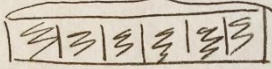


New STAAR question types are more like the kind teachers ask in class (I)

Math, Grade 4 Lesson

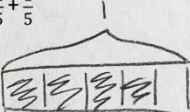
2. Draw and label strip diagrams to model each decomposition.

a. $1 = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$



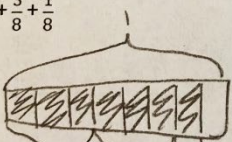
$1 = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$

b. $\frac{4}{5} = \frac{1}{5} + \frac{2}{5} + \frac{1}{5}$



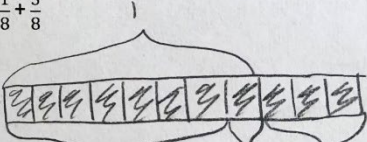
$\frac{4}{5} = \frac{1}{5} + \frac{2}{5} + \frac{1}{5}$

c. $\frac{7}{8} = \frac{3}{8} + \frac{3}{8} + \frac{1}{8}$



$\frac{7}{8} = \frac{3}{8} + \frac{3}{8} + \frac{1}{8}$

d. $\frac{11}{8} = \frac{7}{8} + \frac{1}{8} + \frac{3}{8}$



$\frac{11}{8} = \frac{7}{8} + \frac{1}{8} + \frac{3}{8}$


In this lesson, students are using shaded fraction models to show their understanding of adding fractions

Potential new STAAR question

In a bag of balloons, $\frac{2}{8}$ of the balloons are red and $\frac{5}{8}$ of the balloons are blue. What fraction of the balloons in the bag are either red or blue?

Complete the model so that it is shaded to represent the fraction of the balloons that are either red or blue.

Select the parts you want to shade.



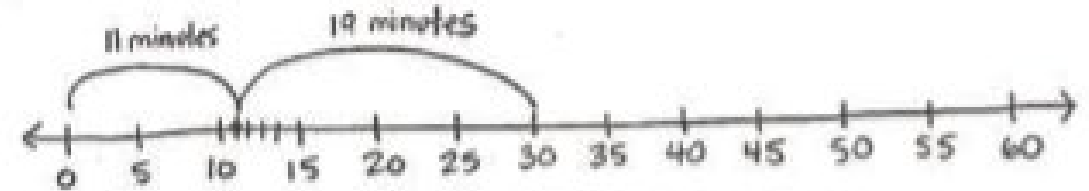
This potential new STAAR question asks students to shade in a fraction model to represent the addition of two fractions

In the 3rd grade classroom, students are asked to engage with time

Grade 3 Math TEKS

- 3.7A determine the solutions to problems involving addition and subtraction of time intervals in minutes using pictorial models or tools such as a 15-minute event plus a 30-minute event equals 45 minutes

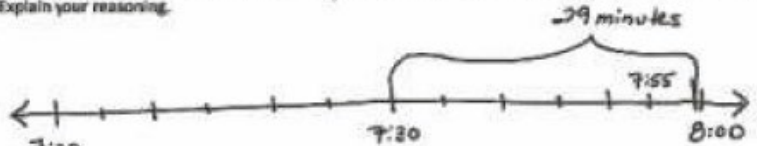
Example: “Draw and use a number line to solve problems”



New STAAR question types are more like the kind teachers ask in class

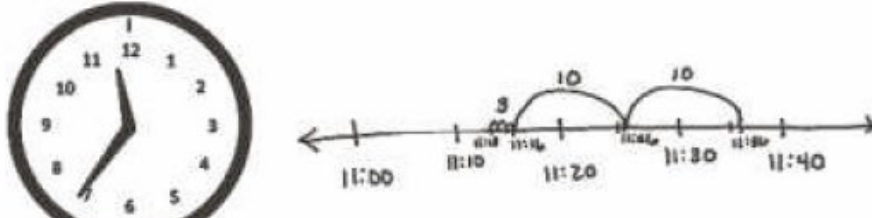
Eureka Math TEKS, Grade 3 Module 2

b. Austin's bus arrives at 7:55am. If he starts his chores at 7:30am, will he be done in time to meet his bus?
Explain your reasoning.



No, Austin will be late. It takes four minutes more than he has time for.

5. Gilberto's cat sleeps in the sun for 23 minutes. It wakes up at the time shown on the clock below. What time did the cat go to sleep?



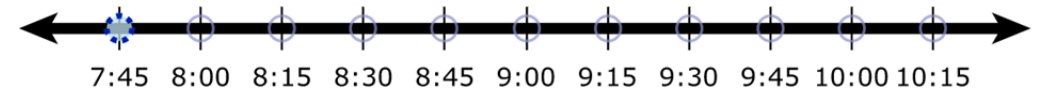
Potential new STAAR question

Miguel decided to watch a show on television:

- It took him 15 minutes to choose a show to watch.
- After choosing a show, he watched the show for 1 hour.

If Miguel started choosing a show at 7:45, at what time did he stop watching the show?

Select **ONE** location on the number line to plot the point.



In this lesson, students are using number lines to show their understanding of elapsed time

This potential new STAAR question asks students to use a number line and solve for the end time



Reflection

How does the research on how student learn in math support student success on STAAR?

Breakout Rooms – Reflect and Debrief



Directions:

- 1** From what we went over today, does your LEA have a vision or framework aligned to research?

- 2** How does this math research support the needs of all learners in your LEA?

- 3** Where do you see strengths or gaps in your instructional practices for math?

Next Steps

- **District Panels [Optional]:** Sign up for District Panels to hear from districts already engaging in this work
 - Date: 8/18; Registration: [Here](#)
- **Research Overview Series [Optional]:** Sign up for overview of research topics series aligned with STAAR Redesign to see if your district may want to explore further
 - Date: 8/19 ([Math](#)) and 8/25 ([RLA](#))
- **TEA Consultation [Optional]:** District leaders may sign up for one 30-minute session with TEA [here](#) to help determine what application decision may be best based on local context
- **Apply to grant [Required]**
 - Open: June 22nd, 2022; Closing: July 29th *and* August 26th