

Advanced Quantitative Reasoning (IMRA25)

Subject: Mathematics

Grade: 11

Expectations: 43

Breakouts: 177

(a) Introduction.

1. The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
2. The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
3. In Advanced Quantitative Reasoning, students will develop and apply skills necessary for college, careers, and life. Course content consists primarily of applications of high school mathematics concepts to prepare students to become well-educated and highly informed 21st century citizens. Students will develop and apply reasoning, planning, and communication to make decisions and solve problems in applied situations involving numerical reasoning, probability, statistical analysis, finance, mathematical selection, and modeling with algebra, geometry, trigonometry, and discrete mathematics.
4. Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b) Knowledge and Skills Statements

- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
 - (A) apply mathematics to problems arising in everyday life, society, and the workplace;
 - (i) apply mathematics to problems arising in everyday life
 - (ii) apply mathematics to problems arising in society

- (iii) apply mathematics to problems arising in the workplace
- (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
- (i) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process
 - (ii) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the reasonableness of the solution
- (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
- (i) select tools, including real objects as appropriate, to solve problems
 - (ii) select tools, including manipulatives as appropriate, to solve problems
 - (iii) select tools, including paper and pencil as appropriate, to solve problems
 - (iv) select tools, including technology as appropriate, to solve problems
 - (v) select techniques, including mental math as appropriate, to solve problems
 - (vi) select techniques including estimation as appropriate, to solve problems
 - (vii) select techniques, including number sense as appropriate, to solve problems
- (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
- (i) communicate mathematical ideas using multiple representations, including symbols as appropriate
 - (ii) communicate mathematical ideas using multiple representations, including diagrams as appropriate
 - (iii) communicate mathematical ideas using multiple representations, including graphs as appropriate
 - (iv) communicate mathematical ideas using multiple representations, including language as appropriate
 - (v) communicate mathematical reasoning using multiple representations, including symbols as appropriate
 - (vi) communicate mathematical reasoning using multiple representations, including diagrams as appropriate
 - (vii) communicate mathematical reasoning using multiple representations, including graphs as appropriate
 - (viii) communicate mathematical reasoning using multiple representations, including language as appropriate
 - (ix) communicate [mathematical ideas'] implications using multiple representations, including symbols as appropriate
 - (x) communicate [mathematical ideas'] implications using multiple representations, including diagrams as appropriate
 - (xi) communicate [mathematical ideas'] implications using multiple representations, including graphs as appropriate
 - (xii) communicate [mathematical ideas'] implications using multiple representations, including language as appropriate

- (xiii) communicate [mathematical reasoning's] implications using multiple representations, including symbols as appropriate
- (xiv) communicate [mathematical reasoning's] implications using multiple representations, including diagrams as appropriate
- (xv) communicate [mathematical reasoning's] implications using multiple representations, including graphs as appropriate
- (xvi) communicate [mathematical reasoning's] implications using multiple representations, including language as appropriate

(E) create and use representations to organize, record, and communicate mathematical ideas;

- (i) create representations to organize mathematical ideas
- (ii) create representations to record mathematical ideas
- (iii) create representations to communicate mathematical ideas
- (iv) use representations to organize mathematical ideas
- (v) use representations to record mathematical ideas
- (vi) use representations to communicate mathematical ideas

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

- (i) analyze mathematical relationships to connect mathematical ideas
- (ii) analyze mathematical relationships to communicate mathematical ideas

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

- (i) display mathematical ideas using precise mathematical language in written or oral communication
- (ii) display mathematical arguments using precise mathematical language in written or oral communication
- (iii) explain mathematical ideas using precise mathematical language in written or oral communication
- (iv) explain mathematical arguments using precise mathematical language in written or oral communication
- (v) justify mathematical ideas using precise mathematical language in written or oral communication
- (vi) justify mathematical arguments using precise mathematical language in written or oral communication

(2) Numeric reasoning. The student applies the process standards in mathematics to generate new understandings by extending existing knowledge. The student generates new mathematical understandings through problems involving numerical data that arise in everyday life, society, and the workplace. The student extends existing knowledge and skills to analyze real-world situations. The student is expected to:

(A) use precision and accuracy in real-life situations related to measurement and significant figures;

- (i) use precision in real-life situations related to measurement
- (ii) use precision in real-life situations related to significant figures
- (iii) use accuracy in real-life situations related to measurement

- (iv) use accuracy in real-life situations related to significant figures
- (B) apply and analyze published ratings, weighted averages, and indices to make informed decisions;
 - (i) apply published ratings to make informed decisions
 - (ii) apply weighted averages to make informed decisions
 - (iii) apply indices to make informed decisions
 - (iv) analyze published ratings to make informed decisions
 - (v) analyze weighted averages to make informed decisions
 - (vi) analyze indices to make informed decisions
- (C) solve problems involving quantities that are not easily measured using proportionality;
 - (i) solve problems involving quantities that are not easily measured using proportionality
- (D) solve geometric problems involving indirect measurement, including similar triangles, the Pythagorean Theorem, Law of Sines, Law of Cosines, and the use of dynamic geometry software;
 - (i) solve geometric problems involving indirect measurement, including similar triangles
 - (ii) solve geometric problems involving indirect measurement, including the Pythagorean Theorem
 - (iii) solve geometric problems involving indirect measurement, including Law of Sines
 - (iv) solve geometric problems involving indirect measurement, including Law of Cosines
 - (v) solve geometric problems involving indirect measurement, including the use of dynamic geometry software
- (E) solve problems involving large quantities using combinatorics;
 - (i) solve problems involving large quantities using combinatorics
- (F) use arrays to efficiently manage large collections of data and add, subtract, and multiply matrices to solve applied problems, including geometric transformations;
 - (i) use arrays to efficiently manage large collections of data
 - (ii) add matrices to solve applied problems, including geometric transformations
 - (iii) subtract matrices to solve applied problems, including geometric transformations
 - (iv) multiply matrices to solve applied problems, including geometric transformations
- (G) analyze various voting and selection processes to compare results in given situations; and
 - (i) analyze various voting processes to compare results in given situations
 - (ii) analyze various selection processes to compare results in given situations
- (H) select and apply an algorithm of interest to solve real-life problems such as problems using recursion or iteration involving population growth or decline, fractals, and compound interest; the validity in recorded and transmitted data using checksums and hashing; sports rankings, weighted class rankings, and search engine rankings; and problems involving scheduling or routing situations using vertex-edge graphs, critical paths, Euler paths, and

minimal spanning trees and communicate to peers the application of the algorithm in precise mathematical and nontechnical language.

- (i) select an algorithm of interest to solve real-life problems
- (ii) apply an algorithm of interest to solve real-life problems

(3) Algebraic reasoning (expressions, equations, and generalized relationships). The student applies the process standards in mathematics to create and analyze mathematical models of everyday situations to make informed decisions related to earning, investing, spending, and borrowing money by appropriate, proficient, and efficient use of tools, including technology. The student uses mathematical relationships to make connections and predictions. The student judges the validity of a prediction and uses mathematical models to represent, analyze, and solve dynamic real-world problems. The student is expected to:

- (A) collect numerical bivariate data to create a scatterplot, select a function to model the data, justify the model selection, and use the model to interpret results and make predictions;
 - (i) collect numerical bivariate data to create a scatterplot
 - (ii) select a function to model the data
 - (iii) justify the model selection
 - (iv) use the model to interpret results
 - (v) use the model to make predictions
- (B) describe the degree to which uncorrelated variables may or may not be related and analyze situations where correlated variables do or do not indicate a cause-and-effect relationship;
 - (i) describe the degree to which uncorrelated variables may or may not be related
 - (ii) analyze situations where correlated variables do or do not indicate a cause-and-effect relationship
- (C) determine or analyze an appropriate growth or decay model for problem situations, including linear, exponential, and logistic functions;
 - (i) determine or analyze an appropriate growth or decay model for problem situations, including linear functions
 - (ii) determine or analyze an appropriate growth or decay model for problem situations, including exponential functions
 - (iii) determine or analyze an appropriate growth or decay model for problem situations, including logistic functions
- (D) determine or analyze an appropriate cyclical model for problem situations that can be modeled with periodic functions;
 - (i) determine or analyze an appropriate cyclical model for problem situations that can be modeled with periodic functions
- (E) determine or analyze an appropriate piecewise model for problem situations;
 - (i) determine or analyze an appropriate piecewise model for problem situations
- (F) create, represent, and analyze mathematical models for various types of income calculations to determine the best option for a given situation;

- (i) create mathematical models for various types of income calculations to determine the best option for a given situation
 - (ii) represent mathematical models for various types of income calculations to determine the best option for a given situation
 - (iii) analyze mathematical models for various types of income calculations to determine the best option for a given situation
- (G) create, represent, and analyze mathematical models for expenditures, including those involving credit, to determine the best option for a given situation; and
- (i) create mathematical models for expenditures, including those involving credit, to determine the best option for a given situation
 - (ii) represent mathematical models for expenditures, including those involving credit, to determine the best option for a given situation
 - (iii) analyze mathematical models for expenditures, including those involving credit, to determine the best option for a given situation
- (H) create, represent, and analyze mathematical models and appropriate representations, including formulas and amortization tables, for various types of loans and investments to determine the best option for a given situation.
- (i) create mathematical models, including formulas, for various types of loans to determine the best option for a given situation
 - (ii) create mathematical models, including formulas, for various types of investments to determine the best option for a given situation
 - (iii) create mathematical models, including amortization tables, for various types of loans to determine the best option for a given situation
 - (iv) create mathematical models, including amortization tables, for various types of investments to determine the best option for a given situation
 - (v) create appropriate representations, including formulas, for various types of loans to determine the best option for a given situation
 - (vi) create appropriate representations, including formulas, for various types of investments to determine the best option for a given situation
 - (vii) create appropriate representations, including amortization tables, for various types of loans to determine the best option for a given situation
 - (viii) create appropriate representations, including amortization tables, for various types of investments to determine the best option for a given situation
 - (ix) represent mathematical models, including formulas, for various types of loans to determine the best option for a given situation
 - (x) represent mathematical models, including formulas, for various types of investments to determine the best option for a given situation
 - (xi) represent mathematical models, including amortization tables, for various types of loans to determine the best option for a given situation

- (xii) represent mathematical models, including amortization tables, for various types of investments to determine the best option for a given situation
- (xiii) represent appropriate representations, including formulas, for various types of loans to determine the best option for a given situation
- (xiv) represent appropriate representations, including formulas, for various types of investments to determine the best option for a given situation
- (xv) represent appropriate representations, including amortization tables, for various types of loans to determine the best option for a given situation
- (xvi) represent appropriate representations, including amortization tables, for various types of investments to determine the best option for a given situation
- (xvii) analyze mathematical models, including formulas, for various types of loans to determine the best option for a given situation
- (xviii) analyze mathematical models, including formulas, for various types of investments to determine the best option for a given situation
- (xix) analyze mathematical models, including amortization tables, for various types of loans to determine the best option for a given situation
- (xx) analyze mathematical models, including amortization tables, for various types of investments to determine the best option for a given situation
- (xxi) analyze appropriate representations, including formulas, for various types of loans to determine the best option for a given situation
- (xxii) analyze appropriate representations, including formulas, for various types of investments to determine the best option for a given situation
- (xxiii) analyze appropriate representations, including amortization tables, for various types of loans to determine the best option for a given situation
- (xxiv) analyze appropriate representations, including amortization tables, for various types of investments to determine the best option for a given situation

(4) Probabilistic and statistical reasoning. The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies. The student is expected to:

- (A) use a two-way frequency table as a sample space to identify whether two events are independent and to interpret the results;
 - (i) use a two-way frequency table as a sample space to identify whether two events are independent
 - (ii) use a two-way frequency table as a sample space to interpret the results
- (B) use the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, in mathematical and real-world problems;
 - (i) use the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, in mathematical problems

- (ii) use the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, in real-world problems
- (C) calculate conditional probabilities and probabilities of compound events using tree diagrams, Venn diagrams, area models, and formulas;
- (i) calculate conditional probabilities using tree diagrams
 - (ii) calculate conditional probabilities using Venn diagrams
 - (iii) calculate conditional probabilities using area models
 - (iv) calculate conditional probabilities using formulas
 - (v) calculate probabilities of compound events using tree diagrams
 - (vi) calculate probabilities of compound events using Venn diagrams
 - (vii) calculate probabilities of compound events using area models
 - (viii) calculate probabilities of compound events using formulas
- (D) interpret conditional probabilities and probabilities of compound events by analyzing representations to make decisions in problem situations;
- (i) interpret conditional probabilities by analyzing representations to make decisions in problem situations
 - (ii) interpret probabilities of compound events by analyzing representations to make decisions in problem situations
- (E) use probabilities to make and justify decisions about risks in everyday life;
- (i) use probabilities to make decisions about risks in everyday life
 - (ii) use probabilities to justify decisions about risks in everyday life
- (F) calculate expected value to analyze mathematical fairness, payoff, and risk;
- (i) calculate expected value to analyze mathematical fairness
 - (ii) calculate expected value to analyze payoff
 - (iii) calculate expected value to analyze risk
- (G) determine the validity of logical arguments that include compound conditional statements by constructing truth tables;
- (i) determine the validity of logical arguments that include compound conditional statements by constructing truth tables
- (H) identify limitations and lack of relevant information in studies reporting statistical information, especially when studies are reported in condensed form;
- (i) identify limitations in studies reporting statistical information, especially when studies are reported in condensed form
 - (ii) identify lack of relevant information in studies reporting statistical information, especially when studies are reported in condensed form
- (I) interpret and compare statistical results using appropriate technology given a margin of error;

- (i) interpret statistical results using appropriate technology given a margin of error
 - (ii) compare statistical results using appropriate technology given a margin of error
- (J) identify potential misuses of statistics to justify particular conclusions, including assertions of a cause-and-effect relationship rather than an association, and missteps or fallacies in logical reasoning;
- (i) identify potential misuses of statistics to justify particular conclusions, including assertions of a cause-and-effect relationship rather than an association
 - (ii) identify missteps or fallacies in logical reasoning
- (K) describe strengths and weaknesses of sampling techniques, data and graphical displays, and interpretations of summary statistics and other results appearing in a study, including reports published in the media;
- (i) describe strengths of sampling techniques
 - (ii) describe strengths of data
 - (iii) describe strengths of graphical displays
 - (iv) describe weaknesses of sampling techniques
 - (v) describe weaknesses of data
 - (vi) describe weaknesses graphical displays
 - (vii) describe strengths of interpretations of summary statistics including reports published in the media
 - (viii) describe weaknesses of interpretations of summary statistics including reports published in the media
 - (ix) describe strengths of interpretations of other results appearing in a study, including reports published in the media
 - (x) describe weaknesses of interpretations of other results appearing in a study, including reports published in the media
- (L) determine the need for and purpose of a statistical investigation and what type of statistical analysis can be used to answer a specific question or set of questions;
- (i) determine the need for a statistical investigation
 - (ii) determine the purpose of a statistical investigation
 - (iii) determine what type of statistical analysis can be used to answer a specific question or set of questions
- (M) identify the population of interest for a statistical investigation, select an appropriate sampling technique, and collect data;
- (i) identify the population of interest for a statistical investigation
 - (ii) select an appropriate sampling technique
 - (iii) collect data
- (N) identify the variables to be used in a study;
- (i) identify the variables to be used in a study
- (O) determine possible sources of statistical bias in a study and how bias may affect the validity of the results;

- (i) determine possible sources of statistical bias in a study
 - (ii) determine how bias may affect the validity of the results
- (P) create data displays for given data sets to investigate, compare, and estimate center, shape, spread, and unusual features of the data;
- (i) create data displays for given data sets to investigate center of the data
 - (ii) create data displays for given data sets to investigate shape of the data
 - (iii) create data displays for given data sets to investigate spread of the data
 - (iv) create data displays for given data sets to investigate unusual features of the data
 - (v) create data displays for given data sets to compare center of the data
 - (vi) create data displays for given data sets to compare shape of the data
 - (vii) create data displays for given data sets to compare spread of the data
 - (viii) create data displays for given data sets to compare unusual features of the data
 - (ix) create data displays for given data sets to estimate center of the data
 - (x) create data displays for given data sets to estimate shape of the data
 - (xi) create data displays for given data sets to estimate spread of the data
 - (xii) create data displays for given data sets to estimate unusual features of the data
- (Q) analyze possible sources of data variability, including those that can be controlled and those that cannot be controlled;
- (i) analyze possible sources of data variability, including those that can be controlled
 - (ii) analyze possible sources of data variability, including those that cannot be controlled
- (R) report results of statistical studies to a particular audience, including selecting an appropriate presentation format, creating graphical data displays, and interpreting results in terms of the question studied;
- (i) report results of statistical studies to a particular audience, including selecting an appropriate presentation format
 - (ii) report results of statistical studies to a particular audience, including creating graphical data displays
 - (iii) report results of statistical studies to a particular audience, including interpreting results in terms of the question studied
- (S) justify the design and the conclusion(s) of statistical studies, including the methods used; and
- (i) justify the design of statistical studies, including the methods used
 - (ii) justify the conclusion(s) of statistical studies, including the methods used
- (T) communicate statistical results in oral and written formats using appropriate statistical and nontechnical language.
- (i) communicate statistical results in oral formats using appropriate statistical language
 - (ii) communicate statistical results in oral formats using nontechnical language

- (iii) communicate statistical results in written formats using appropriate statistical language
- (iv) communicate statistical results in written formats using appropriate nontechnical language