The State Board of Education (SBOE) adopts new §§111.29-111.31, concerning middle school Texas Essential Knowledge and Skills (TEKS) for mathematics. Section 111.29 is adopted without changes to the proposed text as published in the February 28, 2025 issue of the *Texas Register* (50 TexReg 1080) and will not be republished. Section 111.30 and §111.31 are adopted with changes to the proposed text as published in the February 28, 2025 issue of the *Texas Register* (50 TexReg 1080) and will be republished. The adopted new sections add TEKS to support middle school advanced mathematics programs designed to enable students to enroll in Algebra I in Grade 8.

REASONED JUSTIFICATION: The board received training from a standards writing advisor at the July 2014 meeting. The standards writing advisor provided additional training to Texas Education Agency (TEA) staff in October 2014 to support future facilitation of the TEKS review committees.

In 2017, the SBOE significantly revised the process for the review and revision of the TEKS. At the November 2018 meeting, the SBOE approved updates to the 2017 TEKS review and revision process to better clarify the process. The updated process was used for the review of the physical education, health education, and science TEKS.

At the January 2021 meeting, the board held a work session to discuss the timeline for the TEKS review and revision process and associated activities, including updates to State Board for Educator Certification teacher assignment rules and certification exams, adoption of instructional materials, and the completion of the Texas Resource Review. The board discussed potential adjustments to the TEKS and Instructional Materials Review and Adoption Schedule. At the April 2021 meeting, the SBOE approved revisions to the TEKS and Instructional Materials Review and Adoption Schedule.

At the April 2023 SBOE meeting, the board discussed and approved changes to the TEKS review process, including approving a process for selecting work group members.

At the April 2024 meeting, TEA staff shared an overview of upcoming interrelated needs for TEKS review and revision and instructional materials review and approval (IMRA) and identified two needs related to mathematics, including options for instructional materials for accelerated learning and establishing TEKS to support middle school advanced mathematics pathways. At the June 2024 meeting, the board approved moving forward with the establishment of TEKS for middle school advanced mathematics and inclusion of advanced mathematics in a future IMRA process.

Applications to serve on the middle school advanced mathematics TEKS work group were collected by TEA in July and August 2024. TEA provided SBOE members with the applications for approval to serve on the work group in late August.

At the September 2024 SBOE meeting, the board directed the work group to present two models for middle school advanced mathematics TEKS. One model was to be based on the importance of keeping the Grade 6 TEKS similar to the current TEKS and to combine the Grade 7 and Grade 8 TEKS into Grade 7. The SBOE gave the work group leeway to analyze models from Barbers Hill Independent School District (ISD), Tomball ISD, and other school districts to develop recommendations for the second model. Additionally, the SBOE directed the work group to recommend one of the two models for the SBOE's further consideration. Work groups convened for two face-to-face meetings to develop recommendations for the proposed TEKS for middle school advanced mathematics in October 2024.

A public hearing was conducted and a discussion item regarding TEKS for middle school advanced mathematics was presented to the Committee of the Full Board at the November 2024 SBOE meeting. At that time, the SBOE selected the second model as the plan for the middle school advanced mathematics programs. The work group met in December 2024 to finalize its recommendations for the second model.

The following changes were made to the rules since published as proposed.

Section 111.30(c)(8)(C) was amended to read, "identify examples of proportional and non-proportional relationships that arise from mathematical and real-world problems."

The section title for §111.31 was amended by adding the roman numeral "I" after "Algebra."

Section 111.31(b) was amended to read, "This course is recommended for students in Grade 8. Prerequisite: Middle School Advanced Mathematics, Grade 7 or Mathematics, Grade 8. Students shall be awarded one credit that satisfies the Algebra I requirement for high school graduation. This course satisfies the requirement for any course which identifies Algebra I as a prerequisite."

The first sentence in §111.31(c)(4) was amended to read, "In Grade 8, Middle School Advanced Mathematics, Algebra I, students will build on the knowledge and skills for mathematics in Middle School Advanced Mathematics, Grades 6 and 7, which provide a foundation in linear relationships, number and operations, and proportionality. Students will study linear, quadratic, and exponential functions and their related transformations, equations, and associated solutions."

The last sentence in §111.31(c)(4) was amended to read, "The use of technology, including graphing tools, is essential in Grade 8, Middle School Advanced Mathematics, Algebra I, to bridge conceptual understanding and procedural fluency."

The SBOE approved the new rules for first reading and filing authorization at its January 31, 2025 meeting and for second reading and final adoption at its April 11, 2025 meeting.

In accordance with TEC, §7.102(f), the SBOE approved the new rules for adoption by a vote of two-thirds of its members to specify an effective date earlier than the beginning of the 2025-2026 school year. The earlier effective date will enable districts to begin preparing for implementation of TEKS that support a middle school advanced mathematics program. The effective date is 20 days after filing as adopted with the Texas Register.

SUMMARY OF COMMENTS AND RESPONSES: The public comment period on the proposal began February 28, 2025, and ended at 5:00 p.m. on March 31, 2025. The SBOE also provided an opportunity for registered oral and written comments at its April 2025 meeting in accordance with the SBOE board operating policies and procedures. Following is a summary of the public comments received and corresponding responses.

Comment. A teacher, two administrators, and a community member asked which State of Texas Assessments of Academic Readiness (STAAR®) would be administered to students in the middle school advanced mathematics program at the end of the year.

Response. This comment is outside the scope of the proposed rulemaking.

Comment. A teacher expressed concern about the inclusion of student expectations from the Grade 8 TEKS in the proposed TEKS for the Grade 8, Middle School Advanced Mathematics, Algebra course, which would limit advanced mathematics students to taking the Grade 7 STAAR® test in Grade 7. The commenter stated that all the Grade 8 TEKS should be compacted into the Grade 7, Middle School Advanced Mathematics course.

Response. The SBOE disagrees and has determined that incorporating certain student expectations from the Grade 8 mathematics TEKS is appropriate and scaffolds middle school students' transition into Algebra I content in Grade 8. The SBOE also provides the following clarification. The assessment a student takes is a local school system decision.

Comment. A teacher stated that under the proposed new TEKS, students would be skipping the difficult and important Grade 7 STAAR® assessment.

Response. This comment is outside the scope of the proposed rulemaking. The assessment a student takes is determined by the local school system.

Comment. Two administrators and a community member asked if new STAAR® assessments would be developed for the proposed middle school advanced mathematics courses.

Response. This comment is outside the scope of the proposed rulemaking.

Comment. A teacher expressed concern that the arrangement of the student expectations in the middle school advanced mathematics TEKS may negatively impact student success on the STAAR® assessments.

Response. The SBOE disagrees and has determined that the arrangement of the student expectations in the middle school advanced mathematics TEKS are appropriate as proposed and should not negatively impact performance on the STAAR® assessment.

Comment. Two administrators expressed concern regarding the inclusion of circle graphs in the proposed student expectation in 19 TAC §111.29(c)(14)(D).

Response. The SBOE disagrees and has determined that circle graphs are appropriately included in the student expectation in 19 TAC §111.29(c)(14)(D).

Comment. Three administrators asked whether proposed new 19 TAC §111.31, Grade 8, Middle School Advanced Mathematics, Algebra, could be retitled to ensure that students would take the Algebra I end-of-course exam and receive credit for Algebra I.

Response. The SBOE agrees that additional clarification to the title for the Grade 8 middle school advanced mathematics standards is warranted and took action to amend the title for 19 TAC §111.31 to read Grade 8, Middle School Advanced Mathematics, Algebra I.

Comment. A teacher expressed concern with the policy for automatic enrollment of Grade 5 students in a middle school advanced mathematics program. The teacher stated that parents do not understand that they must opt their child out of the program and there are already students who struggled in advanced mathematics programs and have since opted out.

Response. This comment is outside the scope of the proposed rulemaking. The requirement for automatic enrollment of certain students in middle school advanced mathematics is a statutory requirement.

Comment. An administrator stated that the re-wording of the TEKS for the advanced mathematics courses may cause confusion for teachers who teach both on-level and advanced courses.

Response. The SBOE disagrees and has determined that the wording in the TEKS for the proposed middle school advanced mathematics courses is sufficiently clear.

Comment. An administrator asked why districts were not given the decision on how to split the Grade 7 TEKS between Grade 6 and Grade 7.

Response. The SBOE provides the following clarification. The proposed middle school advanced mathematics TEKS for Grades 6, 7, and 8 are optional. School districts maintain the ability to create and offer locally developed courses and make decisions to arrange the TEKS in their middle school advanced mathematics programs differently.

Comment. A teacher asked why there is a push to get the middle school advanced mathematics TEKS adopted if they are optional.

Response. The SBOE provides the following clarification. Beginning with the 2024-2025 school year, all school systems are required to offer a middle school advanced mathematics program that prepares students to enroll in Algebra I in Grade 8 in accordance with TEC, §28.029. The middle school advanced mathematics TEKS provide an option that districts can use to meet the requirement in state law that is already in effect. The adoption of these TEKS will enable the SBOE to call for instructional materials to support implementation of middle school advanced mathematics.

Comment. A teacher asked how an optional set of standards helps transient students.

Response. The SBOE provides the following clarification. The middle school advanced mathematics TEKS provide an option that districts can use to meet the requirement in state law that all school systems offer a middle school advanced mathematics program that prepares students to enroll in Algebra I in Grade 8. If a transient student moves from one school system that implements these standards to another school system that implements these standards, the student is less likely to experience disruption in learning.

Comment. An administrator stated there is a lack of clarity in the proposed rules. The administrator suggested revising the phrase "may be implemented" in the general requirements in §111.29(a) to read, "The provisions of this section are one option districts may use to implement Texas Education Code, §28.029."

Response. The SBOE disagrees that the suggested revision is necessary and has determined that the general requirements in §111.29(a) are sufficiently clear as proposed.

Comment. An administrator asked whether the proposed new TEKS for middle school advanced mathematics would replace Accelerated Math 6/7 and Accelerated Math 7/8 and Algebra I in their district.

Response. The SBOE provides the following clarification. The proposed middle school advanced mathematics TEKS for Grades 6, 7, and 8 may be implemented by school districts beginning with the 2025-2026 school year. Districts have the option to implement the new middle school advanced mathematics TEKS or to create or continue to offer their own locally developed middle school advanced mathematics courses.

Comment. An administrator stated it is important for students to have the opportunity to enroll in Algebra I as eighth graders.

Response. The SBOE agrees and took action to adopt proposed TEKS for middle school advanced mathematics that prepare students for Algebra I in Grade 8 that districts may offer beginning with the 2025-2026 school year as part of their middle school advanced mathematics programs.

Comment. A teacher stated that students should be taught all the TEKS as they are necessary for student success.

Response. The SBOE agrees that all the knowledge and skills in the existing mathematics TEKS for Grades 6-8 are necessary for student success. However, the SBOE has determined that there are certain student expectations that can be combined with or subsumed into more advanced student expectations to compact instruction for an accelerated mathematics program that prepares students to take Algebra I in Grade 8.

Comment. A teacher provided a copy of the TEKS breakdown the teacher uses with honors middle school students on an accelerated mathematics pathway.

Response. The SBOE provides the following clarification. The proposed middle school advanced mathematics TEKS for Grades 6-8 are optional standards that school systems may use as part of their middle school advanced mathematics programs. School systems have the option to implement the new middle school advanced mathematics TEKS or to create or continue to offer their own locally developed middle school advanced mathematics courses.

Comment. A teacher expressed concern that there are too many student expectations in the proposed TEKS for Grade 7.

Response. The SBOE disagrees and has determined that the number of student expectations in the proposed TEKS for Grade 7 is appropriate as proposed.

Comment. Two administrators stated that Algebra I should remain the same course as is currently listed in 19 TAC §111.39 whether it is taught in Grade 8 or 9.

Response. The SBOE agrees that the existing high school course, §111.39, Algebra I, is an appropriate and rigorous option for both high school and certain middle school students. However, the SBOE disagrees that §111.39, Algebra I, should be the only option for Grade 8 students in middle school advanced mathematics programs. The SBOE took action to approve for adoption new TEKS for Grade 8, Middle School Advanced Mathematics, Algebra, that

includes and scaffolds Algebra I content with certain Grade 8 student expectations. In response to other comments, the SBOE took action to retitle Grade 8, Middle School Advanced Mathematics, Algebra, as Grade 8, Middle School Advanced Mathematics, Algebra I, to clarify that the course fulfills the graduation requirement for Algebra I.

Comment. A community member stated that high-quality instructional materials are needed for the proposed TEKS and will support these efforts.

Response. This comment is outside the scope of the proposed rulemaking.

Comment. An administrator recommended that if language is changed to add clarity for the advanced mathematics courses, the language should be revised in the on-level TEKS, too.

Response. This comment is outside the scope of the proposed rulemaking. The existing on-level TEKS for Grades 6-8 are not included in the proposed rulemaking.

Comment. Two administrators and a community member stated that some standards were missing from the proposed middle school advanced mathematics courses.

Response. The SBOE disagrees and has determined that all the middle school mathematics standards for Grades 6, 7, and 8 are addressed in the middle school advanced mathematics TEKS. The SBOE provides the following clarification. The student expectations in the middle school advanced mathematics TEKS are either identical to the student expectations in the existing on-level TEKS, have minor revisions, or have been subsumed into similar standards that meet or exceed the knowledge and skills of the original student expectation.

Comment. A teacher and an administrator stated that renumbering student expectations in the middle school advanced mathematics courses causes confusion for teachers who teach both on-level and advanced courses and would have a negative impact on reporting, analyzing, and acting upon data on campus, district, and STAAR® assessments.

Response. The SBOE disagrees and has determined that renumbering student expectations to reflect the order in which they appear in the proposed middle school advanced mathematics courses would not cause confusion and is necessary and appropriate as proposed.

Comment. A teacher stated that there is not sufficient time to teach probability before STAAR®, but it is heavily tested on the Texas Success Initiative assessment for college admission.

Response. This comment is outside the scope of the proposed rulemaking.

Comment. An administrator asked whether there will be new student expectations within the proposed new courses.

Response. The SBOE provided the following clarification. There are no new knowledge and skills statements in the proposed middle school advanced mathematics TEKS. Some student expectations from the student expectations in the existing on-level mathematics TEKS have been subsumed into other student expectations or rearranged, but no new student expectations or knowledge and skills have been added.

Comment. A teacher stated that some Grade 7 student expectations can completely replace Grade 6 student expectations.

Response. The SBOE agrees that some Grade 7 student expectations can replace Grade 6 student expectations if the knowledge and skills are fully addressed. The SBOE took action to subsume certain student expectations from the on-level Grade 6 TEKS in mathematics into similar student expectations in Grade 7 to compact instruction and avoid duplication.

Comment. A teacher stated that instead of teaching the student expectation in §111.26(b)(2)(A) in the current Grade 6 TEKS, natural numbers could be added and taught within the student expectation in §111.27(b)(2)(A) in Grade 7.

Response. The SBOE disagrees and has determined that the student expectation in §111.26(b)(2)(A) related to natural numbers is appropriately included in the proposed new TEKS for §111.29, Grade 6, Middle School Advanced Mathematics.

Comment. A teacher stated that two-step equations and inequalities can be taught instead of one-step equations and inequalities in Grade 6.

Response. The SBOE agrees and has determined that both two-step equations and inequalities and one-step equations and inequalities should be taught in Grade 6.

Comment. A teacher recommended including "comparing data representations" with the different types of graphs already covered in Grade 6.

Response. The SBOE disagrees and has determined the addition of "comparing data representations" with the different types of graphs already covered in Grade 6, Middle School Advanced Mathematics TEKS, is unnecessary.

Comment. A teacher stated that in addition to using frequency percent bars in Grade 6, circle graphs can be taught as another representation of the same thing.

Response. The SBOE disagrees and has determined that introducing circle graphs and frequency percent bars separately provides more opportunity to practice percents and is appropriate as proposed.

Comment. An administrator and community member expressed support for the addition of the middle school advanced mathematics TEKS.

Response. The SBOE agrees and took action to approve proposed new §§111.29-111.31. In response to other comments, the SBOE took action to amend the course title and clarify language in the general requirements in the Grade 8, Middle School Advanced Mathematics, Algebra, TEKS and to clarify language in one student expectation in the Grade 7, Middle School Advanced Mathematics TEKS.

Comment. An administrator and community member asked if students who complete the Grade 6, Middle School Advanced Mathematics course should automatically advance into the Grade 7, Middle School Advanced Mathematics course.

Response. The SBOE provides the following clarification. Grade placement is a local decision and should be based on demonstrated proficiency; however, advancing a student from Grade 6, Middle School Advanced Mathematics course to the Grade 7, Middle School Advanced Mathematics course would be appropriate.

STATUTORY AUTHORITY. The new sections are adopted under Texas Education Code (TEC), §7.102(c)(4), which requires the State Board of Education (SBOE) to establish curriculum and graduation requirements; TEC, §28.002(a), which identifies the subjects of the required curriculum; TEC, §28.002(c), which requires the SBOE to identify by rule the essential knowledge and skills of each subject in the required curriculum that all students should be able to demonstrate and that will be used in evaluating instructional materials and addressed on the state assessment instruments; and TEC, §28.029, which requires school districts and open-enrollment charter schools to develop an advanced mathematics program for middle school students that is designed to enable those students to enroll in Algebra I in Grade 8.

CROSS REFERENCE TO STATUTE. The new sections implement Texas Education Code, §§7.102(c)(4), 28.002(a) and (c), and 28.029.

<rule>

§111.29. Grade 6, Middle School Advanced Mathematics, Adopted 2025.

(a) Implementation. The provisions of this section may be implemented by school districts beginning with the 2025-2026 school year.

(b) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency, and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) To increase the number of students who complete advanced mathematics courses in high school, the middle school advanced mathematics courses are designed to enable students to complete Algebra I by the end of Grade 8.

(4) The primary focal areas in Grade 6, Middle School Advanced Mathematics are numeracy; proportionality; expressions, equations, and relationships; and data science. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use concepts of proportionality to explore, develop, and communicate mathematical relationships, including number, geometry and measurement, and statistics. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other. Students connect verbal, numeric, graphic, and symbolic representations of relationships, including equations and inequalities. Students begin to develop a foundational understanding of functions. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, and reasoning to draw conclusions, evaluate arguments, and make recommendations. The use of technology, including graphing tools, is essential in middle school advanced mathematics courses to master algebra readiness skills by bridging conceptual understanding and procedural fluency.

(5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Numeracy--foundations of rational numbers. The student applies mathematical process standards to represent and use rational numbers in a variety of forms. The student is expected to:

(A) classify sets and subsets using a visual representation such as a Venn diagram or a hierarchy to describe relationships between sets of rational numbers;

(B) identify a number, its opposite, and its absolute value;

(C) represent benchmark fractions and percents such as 1%, 10%, 25%, 33 1/3%, and multiples of these values using 10 by 10 grids, strip diagrams, number lines, and numbers as proportional relationships;

(D) generate equivalent forms of fractions, decimals, and percents using real-world problems as proportional relationships, including problems that involve money;

(E) use equivalent fractions, decimals, and percents to show equal parts of the same whole as proportional relationships;

(F) locate, compare, and order integers and rational numbers using a number line;

(G) order a set of rational numbers arising from mathematical and real-world contexts; and

(H) use coordinate geometry to identify locations on a plane, including graphing points in all four quadrants using ordered pairs of rational numbers.

(3) Numeracy--operations with rational numbers. The student applies mathematical process standards to represent addition, subtraction, multiplication, and division while solving problems and justifying solutions. The student is expected to:

(A) recognize that dividing by a rational number and multiplying by its reciprocal result in equivalent values;

(B) determine, with and without computation, whether a quantity is increased or decreased when multiplied by a fraction, including values greater than or less than one;

(C) extend representations for division to include fraction notation such as a/b represents the same number as $a \div b$ where $b \neq 0$;

(D) represent integer operations with concrete models and connect the actions with the models to standardized algorithms;

(E) add, subtract, multiply, and divide integers fluently;

(F) add, subtract, multiply, and divide rational numbers;

(G) generate equivalent numerical expressions using order of operations, including whole number exponents and prime factorization;

(H) balance a check register that includes deposits, withdrawals, and transfers; and

(I) create and organize a financial assets and liabilities record and construct a net worth statement.

(4) Numeracy--applications of percents. The student applies mathematical process standards to solve problems involving percents as proportional relationships. The student is expected to:

(A) solve real-world problems to find the whole given a part and the percent, to find the part given the whole and the percent, and to find the percent given the part and the whole, including the use of concrete and pictorial models; and

(B) calculate the sales tax for a given purchase and calculate income tax for earned wages.

(5) Proportionality--foundations of ratios and rates. The student applies mathematical process standards to develop an understanding of proportional relationships in problem situations. The student is expected to:

(A) give examples of ratios as multiplicative comparisons of two quantities describing the same attribute;

(B) give examples of rates as the comparison by division of two quantities having different attributes, including rates as quotients;

(C) represent ratios and percents with concrete models, fractions, and decimals; and

(D) represent mathematical and real-world problems involving ratios and rates using scale factors, tables, graphs, and proportions.

(6) Proportionality--applications of ratios and rates. The student applies mathematical process standards to solve problems involving proportional relationships. The student is expected to:

(A) apply qualitative and quantitative reasoning to solve prediction and comparison of real-world problems involving ratios and rates;

(B) calculate unit rates from rates in mathematical and real-world problems; and

(C) convert within and between measurement systems, including the use of proportions and the use of unit rates.

(7) One-variable expressions, equations, and relationships--foundations of one-variable relationships. The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:

(A) distinguish between expressions and equations verbally, numerically, and algebraically;

(B) determine if two expressions are equivalent using concrete models, pictorial models, and algebraic representations; and

(C) generate equivalent expressions using the properties of operations: inverse, identity, commutative, associative, and distributive properties.

(8) One-variable expressions, equations, and relationships--applications of one-variable relationships. The student applies mathematical process standards to use equations and inequalities to represent situations and solve problems. The student is expected to:

(A) write one-variable, one- and two-step equations and inequalities to represent constraints or conditions within problems;

(B) write corresponding real-world problems given one-variable, one- and two-step equations or inequalities;

(C) represent solutions for one-variable, one- and two-step equations and inequalities on number lines;

(D) model and solve one-variable, one-step equations and inequalities that represent problems, including geometric concepts;

(E) model and solve one-variable, two-step equations and inequalities; and

(F) determine if the given value(s) make(s) one-variable, one- and two-step equations and inequalities true.

(9) Two-variable equations and relationships--foundations of linear relationships. The student applies mathematical process standards to use multiple representations to describe algebraic relationships. The student is expected to:

(A) identify independent and dependent quantities from tables and graphs;

(B) write an equation that represents the relationship between independent and dependent quantities from a table;

(C) represent a given situation using verbal descriptions, tables, graphs, and equations in the form $y = kx$ or $y = x + b$; and

(D) compare two rules verbally, numerically, graphically, and symbolically in the form of $y = ax$ or $y = x + a$ in order to differentiate between additive and multiplicative relationships.

(10) Two-variable equations and relationships--applications of proportional relationships. The student applies mathematical process standards to represent and solve problems involving proportional relationships. The student is expected to represent constant rates of change in mathematical and real-world problems given pictorial, tabular, verbal, numeric, graphical, and algebraic representations, including $d = rt$.

(11) Geometric expressions, equations, and relationships--foundations of geometric concepts equations. The student applies mathematical process standards to use geometry to represent relationships. The student is expected to:

(A) model area formulas for parallelograms, trapezoids, and triangles by decomposing and rearranging parts of these shapes; and

(B) write equations that represent problems related to the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers.

(12) Geometric expressions, equations, and relationships--applications of geometric concepts. The student applies mathematical process standards to use geometry to represent relationships and solve problems. The student is expected to:

(A) extend previous knowledge of triangles and their properties to include the sum of angles of a triangle, the relationship between the lengths of sides and measures of angles in a triangle, and determining when three lengths form a triangle;

(B) determine solutions for problems involving the area of rectangles, parallelograms, trapezoids, and triangles where dimensions are positive rational numbers;

(C) solve problems involving the volume of right rectangular prisms and triangular prisms; and

(D) write and solve equations using geometry concepts, including the sum of the angles in a triangle, and angle relationships.

(13) Data science--foundations of measurement and data. The student applies mathematical process standards to represent and analyze data. The student is expected to:

(A) distinguish between situations that yield data with and without variability; and

(B) represent numeric data graphically, including dot plots, stem-and-leaf plots, histograms, and box plots.

(14) Data science--applications of measurement and data. The student applies mathematical process standards to use numerical or graphical representations to analyze and solve problems. The student is expected to:

(A) use the graphical representation of numeric data to describe the center, spread, and shape of the data distribution;

(B) summarize numeric data with numerical summaries, including the mean and median (measures of center) and the range and interquartile range (IQR) (measures of spread), and use these summaries to describe the center, spread, and shape of the data distribution;

(C) interpret numeric data summarized in dot plots, stem-and-leaf plots, histograms, and box plots;

(D) solve problems using data represented in bar graphs, dot plots, and circle graphs, including part-to-whole and part-to-part comparisons and equivalents;

(E) compare two groups of numeric data using comparative dot plots or box plots by comparing their shapes, centers, and spreads; and

(F) summarize categorical data with numerical and graphical summaries, including the mode, the percent of values in each category (relative frequency table), and the percent bar graph, and use these summaries to describe the data distribution.

(15) Personal financial literacy--money management. The student applies mathematical process standards to develop an economic way of thinking and problem solving useful in one's life as a knowledgeable consumer and investor. The student is expected to:

(A) compare the features and costs of a checking account and a debit card offered by different local financial institutions;

(B) identify and explain the advantages and disadvantages of different payment methods, including distinguishing between debit cards and credit cards;

(C) explain why it is important to establish a positive credit history;

(D) describe the information in a credit report and how long it is retained;

(E) describe the value of credit reports to borrowers and to lenders;

(F) explain various methods to pay for college, including through savings, grants, scholarships, student loans, and work-study; and

(G) compare the annual salary of several occupations requiring various levels of post-secondary education or vocational training and calculate the effects of the different annual salaries on lifetime income.

§111.30. Grade 7, Middle School Advanced Mathematics, Adopted 2025.

(a) Implementation. The provisions of this section may be implemented by school districts beginning with the 2025-2026 school year.

(b) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency, and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and

evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) To increase the number of students who complete advanced mathematics courses in high school, the middle school advanced mathematics courses are designed to enable students to complete Algebra I by the end of Grade 8.

(4) The primary focal areas in Grade 7, Middle School Advanced Mathematics are numeracy; proportionality; expressions, equations, and relationships; and data science. Students use concepts, algorithms, and properties of real numbers to explore mathematical relationships and to describe increasingly complex situations. Students use concepts of proportionality to explore, develop, and communicate mathematical relationships, including number, geometry and measurement, and statistics and probability. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other. Students connect verbal, numeric, graphic, and symbolic representations of relationships, including equations and inequalities. Students continue to develop a foundational understanding of functions. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, and reasoning to draw conclusions, evaluate arguments, and make recommendations. The use of technology, including graphing tools, is essential in middle school advanced mathematics courses to master algebra readiness skills by bridging conceptual understanding and procedural fluency.

(5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Numeracy--foundations of real numbers. The student applies mathematical process standards to represent and use real numbers in a variety of forms. The student is expected to:

(A) extend previous knowledge of sets and subsets using a visual representation to describe relationships between sets of real numbers;

(B) approximate the value of an irrational number, including π and square roots of numbers less than 225, and locate that rational number approximation on a number line;

(C) convert between standard decimal notation and scientific notation; and

(D) order a set of real numbers arising from mathematical and real-world contexts.

(3) Numeracy--operations with rational numbers. The student applies mathematical process standards to add, subtract, multiply, and divide while solving problems and justifying solutions. The student is expected to:

(A) add, subtract, multiply, and divide rational numbers fluently; and

(B) apply and extend previous understandings of operations to solve problems using addition, subtraction, multiplication, and division of rational numbers.

(4) Numeracy--applications of percents. The student applies mathematical process standards to represent and solve problems involving percents as proportional relationships. The student is expected to:

(A) solve problems involving ratios, rates, and percents, including multi-step problems involving percent increase and percent decrease, and financial literacy problems;

(B) calculate and compare simple interest and compound interest earnings;

(C) analyze and compare monetary incentives, including sales, rebates, and coupons;

(D) solve real-world problems comparing how interest rate and loan length affect the cost of credit;

(E) calculate the total cost of repaying a loan, including credit cards and easy access loans, under various rates of interest and over different periods using an online calculator;

(F) explain how small amounts of money invested regularly, including money saved for college and retirement, grow over time; and

(G) estimate the cost of a two-year and four-year college education, including family contribution, and devise a periodic savings plan for accumulating the money needed to contribute to the total cost of attendance for at least the first year of college.

(5) Proportionality--geometric ratios. The student applies mathematical process standards to use geometry to describe or solve problems involving proportional relationships such as dilations. The student is expected to:

(A) describe π as the ratio of the circumference of a circle to its diameter;

(B) generalize the critical attributes of similarity, including ratios within and between similar shapes;

(C) solve mathematical and real-world problems involving similar shape and scale drawings;

(D) compare and contrast the attributes of a shape and its dilation(s) on a coordinate plane; and

(E) use an algebraic representation to explain the effect of a given positive rational scale factor applied to two-dimensional figures on a coordinate plane with the origin as the center of dilation.

(6) Proportionality--probability. The student applies mathematical process standards to use probability and statistics to describe or solve problems involving proportional relationships. The student is expected to:

(A) represent sample spaces for simple and compound events using lists and tree diagrams;

(B) select and use different simulations to represent simple and compound events with and without technology;

(C) make predictions and determine solutions using experimental data for simple and compound events;

(D) make predictions and determine solutions using theoretical probability for simple and compound events;

(E) find the probabilities of a simple event and its complement and describe the relationship between the two;

(F) solve problems using qualitative and quantitative predictions and comparisons from simple experiments; and

(G) determine experimental and theoretical probabilities related to simple and compound events using data and sample spaces.

(7) One-variable expressions, equations, and relationships--applications of one-variable relationships. The student applies mathematical process standards to use one-variable equations or inequalities in problem situations. The student is expected to:

(A) represent solutions for one-variable, two-step inequalities on number lines;

(B) model and solve one-variable, two-step inequalities;

(C) write one-variable equations or inequalities with variables on both sides that represent problems using rational number coefficients and constants;

(D) write a corresponding real-world problem when given a one-variable equation or inequality with variables on both sides of the equal sign using rational number coefficients and constants; and

(E) model and solve one-variable equations with variables on both sides of the equal sign that represent mathematical and real-world problems using rational number coefficients and constants.

(8) Two-variable equations and relationships--foundations of linear relationships. The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions. The student is expected to:

(A) determine the constant of proportionality ($k = y/x$) within mathematical and real-world problems;

(B) distinguish between proportional and non-proportional situations using tables, graphs, and equations in the form $y = kx$ or $y = mx + b$, where $b \neq 0$; and

(C) identify examples of proportional and non-proportional relationships that arise from mathematical and real-world problems.

(9) Two-variable equations and relationships--applications of linear relationships. The student applies mathematical process standards to represent linear relationships using multiple representations. The student is expected to represent linear proportional and non-proportional relationships using verbal descriptions, tables, graphs, and equations that simplify to the form $y = mx + b$.

(10) Geometric expressions, equations, and relationships--foundations of geometric concepts. The student applies mathematical process standards to develop geometric relationships and solve problems. The student is expected to:

(A) use models to determine the approximate formulas for the circumference and area of a circle and connect the models to the actual formulas;

(B) solve problems involving the lateral and total surface area of a rectangular prism, rectangular pyramid, triangular prism, and triangular pyramid by determining the area of the shape's net;

(C) describe the volume formula $V = Bh$ of a cylinder in terms of its base area and its height;

(D) model the relationship between the volume of a rectangular prism and a rectangular pyramid having both congruent bases and heights and connect that relationship to the formulas;

(E) explain verbally and symbolically the relationship between the volume of a triangular prism and a triangular pyramid having both congruent bases and heights and connect that relationship to the formulas;

(F) model the relationship between the volume of a cylinder and a cone having both congruent bases and heights and connect that relationship to the formulas;

(G) use models and diagrams to explain the Pythagorean theorem; and

(H) use informal arguments to establish facts about the angle sum and exterior angle of triangles, the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

(11) Geometric expressions, equations, and relationships--applications of geometric concepts. The student applies mathematical process standards to solve geometric problems. The student is expected to:

(A) determine the circumference and area of circles;

(B) determine the area of composite figures containing combinations of rectangles, squares, parallelograms, trapezoids, triangles, semicircles, and quarter circles;

(C) use previous knowledge of surface area to make connections to the formulas for lateral and total surface area and determine solutions for problems involving rectangular prisms, triangular prisms, and cylinders;

(D) solve problems involving the volume of rectangular pyramids and triangular pyramids;

(E) solve problems involving the volume of cylinders, cones, and spheres;

(F) use the Pythagorean theorem and its converse to solve problems; and

(G) determine the distance between two points on a coordinate plane using the Pythagorean theorem.

(12) Geometric expressions, equations, and relationships--transformations. The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:

(A) generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane;

(B) differentiate between transformations that preserve congruence and those that do not;

(C) explain the effect of translations, reflections over the x- or y-axis, and rotations limited to 90°, 180°, 270°, and 360° as applied to two-dimensional shapes on a coordinate plane using an algebraic representation; and

(D) model the effect on linear and area measurements of dilated two-dimensional shapes.

(13) Data science--applications of measurement and data. The student applies mathematical process standards to use statistical representations and procedures to analyze and describe data. The student is expected to:

(A) use data from a random sample to make inferences about a population;

(B) compare two populations based on data in random samples from these populations, including informal comparative inferences about differences between the two populations;

(C) simulate generating random samples of the same size from a population with known characteristics to develop the notion of a random sample being representative of the population from which it was selected; and

(D) determine the mean absolute deviation and use this quantity as a measure of the average distance data are from the mean using a data set of no more than 10 data points.

(14) Personal financial literacy--money management. The student applies mathematical process standards to develop an economic way of thinking and problem solving useful in one's life as a knowledgeable consumer and investor. The student is expected to:

(A) identify the components of a personal budget, including income; planned savings for college, retirement, and emergencies; taxes; and fixed and variable expenses, and calculate what percentage each category comprises of the total budget;

(B) use a family budget estimator to determine the minimum household budget and average hourly wage needed for a family to meet its basic needs in the student's city or another large city nearby; and

(C) analyze situations to determine if they represent financially responsible decisions and identify the benefits of financial responsibility and the costs of financial irresponsibility.

§111.31. Grade 8, Middle School Advanced Mathematics, Algebra I (One Credit), Adopted 2025.

(a) Implementation. The provisions of this section may be implemented by school districts beginning with the 2025-2026 school year.

(b) General requirements. This course is recommended for students in Grade 8. Prerequisite: Middle School Advanced Mathematics, Grade 7 or Mathematics, Grade 8. Students shall be awarded one credit that satisfies the Algebra I requirement for high school graduation. This course satisfies the requirement for any course which identifies Algebra I as a prerequisite.

(c) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) To increase the number of students who complete advanced mathematics courses in high school, the middle school advanced mathematics courses are designed to enable students to complete Algebra I by the end of Grade 8.

(4) In Grade 8, Middle School Advanced Mathematics, Algebra I, students will build on the knowledge and skills for mathematics in Middle School Advanced Mathematics, Grades 6 and 7, which provide a foundation in linear relationships, number and operations, and proportionality. Students will study linear, quadratic, and exponential functions and their related transformations, equations, and associated solutions. Students will connect functions and their associated solutions in both mathematical and real-world situations. Students will use technology to collect and explore data and analyze statistical relationships. In addition, students will study polynomials of degree one and two, radical expressions, sequences, and laws of exponents. Students will generate and solve linear systems with two equations and two variables and will create new functions through transformations. The use of technology, including graphing tools, is essential in Grade 8, Middle School Advanced Mathematics, Algebra I, to bridge conceptual understanding and procedural fluency.

(5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(d) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

(A) determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities;

(B) write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points;

(C) write linear equations in two variables given a table of values, a graph, and a verbal description;

(D) write and solve equations involving direct variation;

(E) write the equation of a line that contains a given point and is parallel to a given line;

(F) write the equation of a line that contains a given point and is perpendicular to a given line;

(G) write an equation of a line that is parallel or perpendicular to the x- or y- axis and determine whether the slope of the line is zero or undefined;

(H) write linear inequalities in two variables given a table of values, a graph, and a verbal description; and

(I) write systems of two linear equations given a table of values, a graph, and a verbal description.

(3) Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

(A) use similar right triangles to develop an understanding that slope, m, given as the rate comparing the change in y-values to the change in x-values, $(y2 - y1)/ (x2 - x1)$, is the same for any two points $(x1, y1)$ and $(x2, y2)$ on the same line;

(B) graph proportional relationships, interpreting the unit rate as the slope of the line that models the relationship;

(C) determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$;

(D) calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical and real-world problems;

(E) use data from a table or graph to determine the rate of change or slope and y-intercept in mathematical and real-world problems;

(F) graph linear functions on the coordinate plane and identify key features, including x-intercept, y-intercept, zeros, and slope, in mathematical and real-world problems;

(G) graph the solution set of linear inequalities in two variables on the coordinate plane;

(H) determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, and $f(bx)$ for specific values of a, b, c, and d;

(I) graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist;

(J) estimate graphically the solutions to systems of two linear equations with two variables in real-world problems; and

(K) graph the solution set of systems of two linear inequalities in two variables on the coordinate plane.

(4) Linear functions, equations, and inequalities. The student applies the mathematical process standards to formulate statistical relationships and evaluate their reasonableness based on real-world data. The student is expected to:

(A) construct a scatterplot and describe the observed data to address questions of association such as linear, non-linear, and no association between bivariate data;

(B) contrast bivariate sets of data that suggest a linear relationship with bivariate sets of data that do not suggest a linear relationship from a graphical representation;

(C) use a trend line that approximates the linear relationship between bivariate sets of data to make predictions;

(D) calculate, using technology, the correlation coefficient between two quantitative variables and interpret this quantity as a measure of the strength of the linear association;

(E) compare and contrast association and causation in real-world problems; and

(F) write, with and without technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.

(5) Linear functions, equations, and inequalities. The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:

(A) solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides;

(B) solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides; and

(C) solve systems of two linear equations with two variables for mathematical and real-world problems.

(6) Quadratic functions and equations. The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to:

(A) determine the domain and range of quadratic functions and represent the domain and range using inequalities;

(B) write equations of quadratic functions given the vertex and another point on the graph, write the equation in vertex form ($f(x) = a(x - h)^2 + k$), and rewrite the equation from vertex form to standard form ($f(x) = ax^2 + bx + c$); and

(C) write quadratic functions when given real solutions and graphs of their related equations.

(7) Quadratic functions and equations. The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:

(A) graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry;

(B) describe the relationship between the linear factors of quadratic expressions and the zeros of their associated quadratic functions; and

(C) determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, and $f(bx)$ for specific values of a, b, c, and d.

(8) Quadratic functions and equations. The student applies the mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:

(A) solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula; and

(B) write, using technology, quadratic functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.

(9) Exponential functions and equations. The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:

(A) determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities;

(B) interpret the meaning of the values of a and b in exponential functions of the form $f(x) = ab^x$ in real-world problems;

(C) write exponential functions in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay;

(D) graph exponential functions that model growth and decay and identify key features, including y-intercept and asymptote, in mathematical and real-world problems; and

(E) write, using technology, exponential functions that provide a reasonable fit to data and make predictions for real-world problems.

(10) Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. The student is expected to:

(A) add and subtract polynomials of degree one and degree two;

(B) multiply polynomials of degree one and degree two;

(C) determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend;

(D) rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property;

(E) factor, if possible, trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two; and

(F) decide if a binomial can be written as the difference of two squares and, if possible, use the structure of a difference of two squares to rewrite the binomial.

(11) Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to rewrite algebraic expressions into equivalent forms. The student is expected to:

(A) simplify numerical radical expressions involving square roots; and

(B) simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.

(12) Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:

(A) identify functions using sets of ordered pairs and mappings;

(B) decide whether relations represented verbally, tabularly, graphically, and symbolically define a function;

(C) evaluate functions, expressed in function notation, given one or more elements in their domains;

(D) identify terms of arithmetic and geometric sequences when the sequences are given in function form using recursive processes;

(E) write a formula for the n^{th} term of arithmetic and geometric sequences, given the value of several of their terms; and

(F) solve mathematic and scientific formulas, and other literal equations, for a specified variable.