# Guidelines for Expert Feedback on the Mathematics TEKS 

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Please review the mathematics TEKS for Kindergarten through Grade 12 and respond to the following questions. In your feedback please indicate the specific grade level/ course and student expectation number you are referring to, as appropriate.

## 1. Is a complete and logical development of mathematics concepts followed for each grade level or course?

There is a complete set of mathematics concepts at each grade level. In fact, I think there may be too many given the length of the school year especially for elementary grades. Given there are $\qquad$ weeks in a school year this would put an average of objectives taught per week per grade level. It also appears that more content was pushed down into preceding courses from algebra on. This may be a recognition that using End of Course Exams will result in greater time dedicated to content in advanced courses. In the past, the TAKS drove mathematics instruction. That is most upper level mathematics courses were not represented on the TAKS test, therefore, teachers of higher level secondary mathematics courses would deviate from their course content to address lower level mathematics content likely to be required to pass TAKS. This reduced the amount of time to cover the content in advanced courses. As a result, the content was either presented in an abbreviated form or omitted in deference to passing the TAKS. The move to End of Course Exams will preempt reviewing of generic mathematics content at the expense of real mathematics content. I applaud this effort to increase rigor and to scaffold learning that will be useful for students who are college bound. This appears to be a step in the right direction equating high school standards with college readiness.

What recommendations do you have for improvement?
a. I would like to see fewer course objectives at the elementary grades in preference for greater depth of mathematics. For example 3NO1, 3NO2, 3NO3, 3NO4, 3NO5 - these objectives treat each as discrete knowledge when in fact they are an integrated whole. In 3NO1 children are expected to "represent the value of a numeral to 10,000 using objects and pictorial models that address the notion of bundling (composing and decomposing)." This seems to be a lower level than what is required at both first and second grades with the notion of bundling as opposed to clearly base-ten notions as they were required to do at both subsequent grades. The idea of bundling is expressed through base-ten in grades 1 and 2 because students group 10 rods, sticks, or hash marks into groups of 10 , and 10 groups of 10 into a group of 100 . So they are focusing on the mathematical notation and manipulation whereas the focus of the $3^{\text {rd }}$ grade seems to be developing the skills of bundling. This does not make the most of learning theory. When I consider 3N04 "compare and order whole numbers up
to 10,000 and 3N05 "represent the comparison of two numbers to 10,000 using the symbols >, <, or =. " I think in 3NO4 the use of the term compare requires more than one number. The idea of compare is not a cleanly designed concept like the basic operations of add, subtract, multiply, and divide. So compare can be used to mean all of these plus the relational symbols of <,>, and $=$. So it seems to be that 3NO4 and 3NO5 are not really different. 3NO4 cannot be done without at least two numbers and 3NO5 specifies two numbers. So if this is to mean in 3NO4 students can compare 3, 4, 5 or more numbers this is not clear. However, if the distinction between the two is to use operator symbols in the former and relational symbols in the later this should be specified. The 3NO1-3NO5 should be rolled into one objective without regressing below the competency of prior grades.
b. In grade 4 the focus shifts to representing the numbers between whole numbers. It would be helpful to children to use the same objectives they used for building whole number base-ten ideas as they move to numbers on the other side of the decimal point. For example in first grade, "decompose the value of a numeral to 100 as a sum of so many tens and so many ones, in more than one way using objects and pictorial models. For example, 64 can be represented as 6 tens, and 4 ones, or as 5 tens, and 14 ones (representations may be bundles of an object or pictures of bundles)." A similar objective and example should be provided when exploring numbers on the other side of the decimal point. This would allow students to build on what they learned previously and for them to build on their knowledge as opposed to feeling like they are learning something completely new. There are also similar manipulatives for teaching both concepts which would also facilitate the transfer of knowledge.

## 2. Have the correct vocabulary and terminology been used?

I think the ideas of operational symbols and relational symbols should be used within the text of the objective. It is important for teachers to recognize the difference in terms and for students to know there are terms for the classes of symbols. In grade 4, the term integer is used for the first time. It is not really used to mean all integers but only a portion of the integers. I suggest that the choice of words by positive whole numbers. The idea of integers is not really introduced until $6^{\text {th }}$ grade.

## Where could changes be made for accuracy and/ or clarity?

I think it would be helpful to be cautious about using vocabulary within objectives when those vocabulary words are really not part of the instruction. Therefore, I would move the term integers to $6^{\text {th }}$ grade and I would use vocabulary as it is presented in the SEs, such as relational and operational symbols starting at least in third grade but possibly as early as 1 st grade.
3. Are there specific areas that need to be updated or reworked?

I think the changes at the secondary level shows the greatest improvements with more rigor being infused earlier. I specifically applaud the increased rigor in Algebra and Algebra II.

## 4. Are the mathematics concept/ content statements grade-level appropriate?

In all grades the content appears to be appropriate, however, as noted in the earlier question there are a few places where concepts/ content statements seem to take a step backward or fail to build on previous statements at earlier grades. As an aside, in fourth grade they begin working with rational numbers yet that word is missing. Both teachers and students need to have this vocabulary to name what they are doing and to be able communicate effectively with children who also need that vocabulary. The word later appears in $5^{\text {th }}$ grade long after the concept had been introduced almost as if it was a brand new idea.

## Are important concepts missing at any grade level?

I believe the salient concepts and content is covered at the respective grade level and secondary mathematics courses. At $6^{\text {th }}$ grade the idea of inequality in mathematical sentences is introduced. It seems as though the concept/ content statement uses or for example "6A13 write a one-variable (one-step) equation or inequality to represent constraints or conditions within a mathematical (including number lines) or real-world problem." The use of the term or does not signify if that actual performance requires students to be able to fluently express their mathematics understanding of mathematical sentences across both equations and inequalities or if crystalized understanding of just one is sufficient. My concern here is that eventually students will need to be able to represent what the answer of an inequality means and often, that representation takes the form of a graph or a number line. Representing the result of an equation is very different from representing the result of an inequality. It is important to scaffold this knowledge because it will be used throughout 7th, $8^{\text {th }}$, Algebra I and Algebra II. More time dedicated to these ideas might be more productive in subsequent grades. For example, 6A02 and 6A04 are parallel in the skills acquisition with 04 being more broad encompassing than 02. Similarly, other ideas here might be combined to provide more time for unpacking inequalities. Further, there is so much time dedicated to differentiating between expressions and equations yet it seems as though an equation is being called an expression in Algebra I. I think perhaps the expression was $49 \mathrm{x}^{\wedge} 4-\mathrm{y} \wedge 4$ but it is not clear and this might create confusion. There is enough room on that line that the clarity would not take up even more line of space. Given expressions are not part of $7^{\text {th }}$ or $8^{\text {th }}$ grade content/ concepts - less could be made of them in $6^{\text {th }}$ in favor of inequalities that are more complex and part of the content and concepts in $7^{\text {th }}, 8^{\text {th }}$, and Algebra I.

## 5. Are the Student Expectations (SEs) clear and specific?

In most cases, I was clear about what was expected. I was somewhat unclear in a few places but those were raised earlier in my review. GA02 is not as clear as it
could be. "Determine an equation with graph of a line parallel or perpendicular to a given line and that passes through a given point." Is a word missing, determine the equation from the graph of a line parallel or perpendicular to a given line that passes through a given point. I am not sure if this comment is appropriate for this section but I am unclear why the probability SEs are in geometry. They seem misplaced and do not seem to follow any of the other content or concepts presented in the course.
a. I am not clear about A2D02 "recognize that there are data sets for which it is not appropriate to model with a normal distribution." How is modeled being used here. Data either are or are not normal. By modeling if the meaning is to analyze or represent using mean as a measure of central tendency then the statement can be improved by clarifying the intent for modeling. However, if my modeling of the intent is representing the data graphically, a graphic representation is one method for visually checking to see if data are or are not normal. I think what has been confused in this portion of the SE that says "model with a normal distribution". I think this one might need a revision after a clarification of the intent of the SE.
b. I understand some of inclusion of the probability in Algebra II however, the inclusion of the following three objectives seem misplaced -A2D04 distinguish the purposes and differences among sample surveys, experiments and observation studies including explaining the role of randomization in each type of study and the scope of inference from each type of study. A2D05 use data from a sample survey to estimate population mean or population proportion including developing the margin of error through the use of simulation models for random sampling. A2D06 use data from a randomized experiment to compare two treatments and use simulation to decide if the observed differences are statistically significant. These objectives depend on probability theory that has not really been introduced; further, building an understanding of randomization does not follow from previous work in Algebra II. In combination, statistical significance has not been introduced nor have the tests that might be best suited to the task or the assumptions underlying the respective tests. The term regression has not even been introduced and only appears in Math Models However, the idea of a regression line and line of best fit might be a good choice. It might be better to examine the $\mathrm{R} \wedge 2$ value to determine the importance of a randomized experiment. The regression analysis is also easy to conduct with modern calculators and fits nicely with what students might have learned about lines of best fit (Algebra I).

## 6. Is the subject area aligned horizontally and vertically?

I am unclear about why the probability (and statistics) and assumptions underlying advanced data analytic concepts were sprinkled throughout the advanced mathematics courses. It seems to me a solid course could be developed to teach probability and statistics, and these concepts and content could be moved to a new course where students could learn, in sufficient detail, about the assumptions underlying the probability and statistics of experimental studies and the various
choices for determining the importance of the findings that could include statistical significance (and various tests for estimating statistical significance), effect sizes, and confident intervals.
a. In most instances the design team should be commended for their hard work and efforts to align the content horizontally. In general the content fits well, except where noted. However, this version seems to me to be a great improvement over the current version.
b. I have some minor concerns about the vertical alignment especially with regard to vocabulary and number of SEs within some content/ concept areas. There are a few instances where there are a number of SE's for a content/ concept area at a grade level but that content/ concept is relatively confined to that grade level. I guess there might be two arguments, 1) if it is only at one grade level it needs to be hit hard, and 2) if it is really only important at one grade level and not to subsequent mathematics success might there be something else that should be hit harder that might make a difference in subsequent mathematics courses. To make my assumptions explicit, I subscribe to the later and feel that spending more time on concepts that potentially pose more problems should receive more time and emphasis and if those item are revisited at more rigorous levels in subsequent courses it is also worth it to ensure later success. However, if this is not the prevailing sentiment perhaps a brief explanation as others are explained in the frequently asked questions section would help the reader.
c. There is a specific bright spot in the logical and sequential development of the trigonometric development beginning at geometry and progressing through precalculus. I found the integration of the concepts to be logical and a developmentally insightful presentation of the ideas that I believe may result in students making better connections to the content.
7. Should consideration be given toward adding other courses at the high school level to provide more options for students?

Given the dispersion of probability and statistics I would suggest developing a course on probability and statistics where the content can be fully developed sequentially over the period of one year instead of piece wise over several years.
8. Do you have any other suggestions for ways in which the mathematics TEKS can be improved?

I am modestly concerned about the necessity for making provisions for professional development for $8^{\text {th }}$ and $9^{\text {th }}$ grade teachers who will need substantially greater levels of content and pedagogical skills than many currently possess. In my work in schools and with teachers in Texas, albeit mostly in high-need districts, the increased expectations will be hard to fully implement or the quality of the instruction may be lacking even when these teachers have the salient content knowledge. It is important to move students in a positive direction to college and
career readiness, which, I believe, these SEs do in a big way. However, implementing one change in only one aspect of a complex and highly interpersonal system like the teaching and learning process can have catastrophic effects unless each of the components are carefully addressed. Classroom teachers, district mathematics leaders, building and district level administration, and stakeholders need to be clearly informed, with sufficient opportunities for teacher professional development prior to rolling the SEs out.

