

Subchapter B. Middle School

§111.25. Implementation of Texas Essential Knowledge and Skills for Mathematics, Middle School, Adopted 2012.

- (a) The provisions of §§111.26-111.28 of this subchapter shall be implemented by school districts.
- (b) No later than August 31, 2013, the commissioner of education shall determine whether instructional materials ~~funding has~~ have been made available to Texas public schools ~~for materials that cover the~~ essential knowledge and skills for mathematics as adopted in §§111.26-111.28 of this subchapter.
- (c) If the commissioner makes the determination that instructional materials ~~funding has~~ have been made available under subsection (b) of this section, §§111.26-111.28 of this subchapter shall be implemented beginning with the 2014-2015 school year and apply to the 2014-2015 and subsequent school years.
- (d) If the commissioner does not make the determination that instructional materials ~~funding has~~ have been made available under subsection (b) of this section, the commissioner shall determine no later than August 31 of each subsequent school year whether instructional materials ~~funding has~~ have been made available. If the commissioner determines that instructional materials ~~funding has~~ have been made available, the commissioner shall notify the State Board of Education and school districts that §§111.26-111.28 of this subchapter shall be implemented for the following school year.
- (e) Sections 111.21-111.24 of this subchapter shall be superseded by the implementation of §§111.25-111.28 under this section.

§111.26. Grade 6, Adopted 2012.

- (a) Introduction.
 - (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency, and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
 - (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, ~~and~~ number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
 - (3) The primary focal areas in Grade 6 are number and operations; proportionality; expressions, equations, and relationships; and measurement and data. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly

complex situations. Students use concepts of proportionality to explore, develop, and communicate mathematical relationships. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other. Students connect verbal, numeric, graphic, and symbolic representations of relationships, including equations and inequalities. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, and reasoning to draw conclusions, evaluate arguments, and make recommendations. While the use of all types of technology is important, the emphasis on algebra readiness skills necessitates the implementation of graphing technology.

- (4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b) Knowledge and skills.

- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

- (A) apply mathematics to problems arising in everyday life, society, and the workplace;
- (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
- (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
- (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
- (E) create and use representations to organize, record, and communicate mathematical ideas;
- (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
- (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

- (2) Number and operations. The student applies mathematical process standards to represent and use rational numbers in a variety of forms. The student is expected to:

- (A) classify whole numbers, integers, and rational numbers using a visual representation such as a Venn diagram to describe relationships between sets of numbers;
- (B) identify a number, its opposite, and its absolute value;
- (C) locate, compare, and order integers and rational numbers using a number line;
- ~~(D) locate, compare, and order rational numbers using a number line;~~
- (D) ~~(E)~~ order a set of rational numbers arising from mathematical and real-world contexts; and
- (E) ~~(F)~~ extend representations for division to include fraction notation such as a/b represents the same number as $a \div b$ where $b \neq 0$.

- (3) Number and operations. The student applies mathematical process standards to represent addition, subtraction, multiplication, and division while solving problems and justifying solutions. The student is expected to:

- ~~(A) use an area model to represent fraction multiplication and decimal multiplication;~~

- (A) ~~[(B)]~~ recognize that dividing by a rational number and multiplying by its reciprocal result in equivalent values;
 - (B) ~~[(C)]~~ determine, with and without computation, whether a quantity is increased or decreased when multiplied by a fraction, including values greater than or less than one;
 - (C) ~~[(D)]~~ represent integer operations with concrete models and connect the actions with the models to standardized ~~[(E)]~~ algorithms;
 - ~~[(E)]~~ use prior knowledge of all four operations, including whole numbers and positive decimals, fractions, and mixed numbers not having fractions and decimals, within the same problem;
 - (D) ~~[(F)]~~ add, subtract, multiply, and divide integers fluently; and
 - (E) ~~[(G)]~~ multiply and divide positive rational numbers fluently.
- (4) Proportionality. The student applies mathematical process standards to develop an understanding of proportional relationships in problem situations. The student is expected to:
- (A) compare two rules verbally, numerically, graphically, and symbolically in the form of $y = ax$ or $y = x + a$ in order to differentiate between additive and multiplicative relationships;
 - (B) apply qualitative and quantitative reasoning to solve prediction and comparison of real-world problems involving ratios and rates;
 - (C) give examples of ratios as multiplicative comparisons of two quantities describing the same attribute;
 - (D) give examples of rates as the comparison by division of two quantities having different attributes, including rates as quotients;
 - (E) represent ratios and percents with concrete models, fractions, and decimals;
 - (F) represent benchmark fractions and percents such as 1%, 10%, 25%, 33 $\frac{1}{3}$ %, and multiples of these values using 10 by 10 grids, strip diagrams, number lines, and numbers;
 - (G) generate equivalent forms of fractions, decimals, and percents using real-world problems, including problems that involve money; and
 - (H) convert units within a measurement system, including the use of proportions and unit rates.
- (5) Proportionality. The student applies mathematical process standards to solve problems involving proportional relationships. The student is expected to:
- (A) represent mathematical and real-world problems involving ratios and rates using scale factors, tables, graphs, and proportions;
 - (B) solve real-world problems to find the whole given a part and the percent, to find the part given the whole and the percent, and to find the percent given the part and the whole, including the use of concrete and pictorial models; and
 - (C) use equivalent fractions, decimals, and percents to show equal parts of the same whole.
- (6) Expressions, equations, and relationships. The student applies mathematical process standards to use multiple representations to describe algebraic relationships. The student is expected to:
- (A) identify independent and dependent quantities from tables and graphs;
 - (B) write an equation that represents the relationship between independent and dependent quantities from a table; and

- (C) represent a given situation using verbal descriptions, tables, graphs, and equations in the form $y = kx$ or $y = x + b$.
- (7) Expressions, equations, and relationships. The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:
 - (A) generate equivalent numerical expressions using order of operations, including whole number ~~[positive]~~ exponents and prime factorization;
 - (B) distinguish between expressions and equations verbally, numerically, and algebraically;
 - (C) determine if two expressions are equivalent using concrete models, pictorial models, and algebraic representations; and
 - (D) generate equivalent expressions using the properties of operations : ~~[such as the]~~ inverse, identity, commutative, associative, and distributive properties.
- (8) Expressions, equations, and relationships. The student applies mathematical process standards to use geometry to represent relationships and solve problems. The student is expected to:
 - (A) extend previous knowledge of triangles and their properties to include the sum of angles of a triangle, the relationship between the lengths of sides and measures of angles in a triangle, and determining when three lengths form a triangle;
 - (B) model area formulas for parallelograms, trapezoids, and triangles by decomposing and rearranging parts of these shapes;
 - (C) write equations that represent problems related to the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers; and
 - (D) determine solutions for problems involving the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers.
- (9) Expressions, equations, and relationships. The student applies mathematical process standards to use equations and inequalities to represent situations. The student is expected to:
 - (A) write one-variable, one-step equations and inequalities to represent constraints or conditions within problems;
 - (B) represent solutions for one-variable, one-step equations and inequalities on number lines; and
 - (C) write corresponding real-world problems given one-variable, one-step equations or inequalities.
- (10) Expressions, equations, and relationships. The student applies mathematical process standards to use equations and inequalities to solve problems. The student is expected to:
 - (A) model and solve one-variable, one-step equations and inequalities that represent problems, including geometric concepts ~~[such as complementary and supplementary angles]~~ ; and
 - (B) determine if the given value(s) make(s) one-variable, one-step equations or inequalities true.
- (11) Measurement and data. The student applies mathematical process standards to use coordinate geometry to identify locations on a plane. The student is expected to graph points in all four quadrants using ordered pairs of rational numbers.
- (12) Measurement and data. The student applies mathematical process standards to use numerical or graphical representations to analyze problems. The student is expected to:

- (A) represent numeric data graphically, including dot plots, stem-and-leaf plots, histograms, and box plots;
 - (B) use the graphical representation of numeric data to describe the center, spread, and shape of the data distribution;
 - (C) summarize numeric data with numerical summaries, including the mean and median (measures of center) and the range and interquartile range (IQR) (measures of spread), and use these summaries to describe the center, spread, and shape of the data distribution; and
 - (D) summarize categorical data with numerical and graphical summaries, including the mode, the percent of values in each category (relative frequency table), and the percent bar graph, and use these summaries to describe the data distribution.
- (13) Measurement and data. The student applies mathematical process standards to use numerical or graphical representations to solve problems. The student is expected to:
- (A) interpret numeric data summarized in dot plots, stem-and-leaf plots, histograms, and box plots; and
 - (B) distinguish between situations that yield data with and without variability [~~such as the question "How tall am I?" which would be answered with a single height versus the question "How tall are the students in my class?" which would be answered based on heights that vary~~].
- (14) Personal financial literacy. The student applies mathematical process standards to develop an economic way of thinking and problem solving useful in one's life as a knowledgeable consumer and investor. The student is expected to:
- (A) compare the features and costs of a checking account and a debit card offered by different local financial institutions;
 - (B) distinguish between debit cards and credit cards;
 - (C) balance a check register that includes deposits, withdrawals, and transfers;
 - (D) explain why it is important to establish a positive credit history;
 - (E) describe the information in a credit report and how long it is retained; [~~and~~]
 - (F) describe the value of credit reports to borrowers and to lenders ; [~~and~~]
 - (G) explain various methods to pay for college, including through savings, grants, scholarships, student loans, and work-study; and
 - (H) compare the annual salary of several occupations requiring various levels of post-secondary education or vocational training and calculate the effects of the different annual salaries on lifetime income.

§111.27. Grade 7, Adopted 2012.

(a) Introduction.

- (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency, and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
- (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each

grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, ~~and~~ number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

- (3) The primary focal areas in Grade 7 are number and operations; proportionality; expressions, equations, and relationships; and measurement and data. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use concepts of proportionality to explore, develop, and communicate mathematical relationships, including number, geometry and measurement, and statistics and probability. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other. Students connect verbal, numeric, graphic, and symbolic representations of relationships, including equations and inequalities. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, and reasoning to draw conclusions, evaluate arguments, and make recommendations. While the use of all types of technology is important, the emphasis on algebra readiness skills necessitates the implementation of graphing technology.

- (4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b) Knowledge and skills.

- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
- (A) apply mathematics to problems arising in everyday life, society, and the workplace;
 - (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
 - (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
 - (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
 - (E) create and use representations to organize, record, and communicate mathematical ideas;
 - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and

- (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
- (2) Number and operations. The student applies mathematical process standards to represent and use rational numbers in a variety of forms. The student is expected to extend previous knowledge of sets and subsets using a visual representation [such as a Venn diagram] to describe relationships between sets of rational numbers.
- (3) Number and operations. The student applies mathematical process standards to add, subtract, multiply, and divide while solving problems and justifying solutions. The student is expected to:
- (A) add, subtract, multiply, and divide rational numbers fluently; and
- (B) apply and extend previous understandings of operations to solve problems using addition, subtraction, multiplication, and division of rational numbers.
- (4) Proportionality. The student applies mathematical process standards to represent and solve problems involving proportional relationships. The student is expected to:
- (A) represent constant rates of change in mathematical and real-world problems given pictorial, tabular, verbal, numeric, graphical, and algebraic representations, including $d = rt$;
- (B) calculate unit rates from rates in mathematical and real-world problems;
- (C) determine the constant of proportionality ($k = y/x$) within mathematical and real-world problems;
- (D) solve problems involving ratios, rates, and percents, including multi-step problems involving percent increase and percent decrease, and financial literacy problems [such as tax, tip, discount, simple interest, and commission] ; and
- (E) convert between measurement systems, including the use of proportions and the use of unit rates.
- (5) Proportionality. The student applies mathematical process standards to use geometry to describe or solve problems involving proportional relationships. The student is expected to:
- (A) generalize the critical attributes of similarity, including ratios within and between similar shapes;
- (B) describe π as the ratio of the circumference of a circle to its diameter; and
- (C) solve mathematical and real-world problems involving similar shape and scale drawings.
- (6) Proportionality. The student applies mathematical process standards to use probability and statistics to describe or solve problems involving proportional relationships [develop concepts of probability for simple and compound events] . The student is expected to:
- (A) represent sample spaces for simple and compound events using lists and tree diagrams; [and]
- (B) select and use different simulations to represent simple and compound events with and without technology ; [;]
- ~~(7) Proportionality. The student applies mathematical process standards to make predictions and determine solutions for simple and compound events. The student is expected to:~~
- ~~(C) [(A)] make predictions and determine solutions using experimental data for simple and compound events; [and]~~
- ~~(D) [(B)] make predictions and determine solutions using theoretical probability for simple and compound events ; [;]~~

- ~~(8)~~ Proportionality. The student applies mathematical process standards to find solutions in probability and statistics. The student is expected to:
- ~~(E)~~ (A) find the probabilities of a simple event and its complement and describe the relationship between the two;
 - ~~(F)~~ (B) use data from a random sample to make inferences about a population;
 - ~~(G)~~ (C) solve problems using data represented in bar graphs, dot plots, and circle graphs, including part-to-whole and part-to-part comparisons and equivalents;
 - ~~(H)~~ (D) solve problems using qualitative and quantitative predictions and comparisons from simple experiments; and
 - ~~(I)~~ (E) determine experimental and theoretical probabilities related to simple and compound events using data and sample spaces.
- ~~(7)~~ (9) Expressions, equations, and relationships. The student applies mathematical process standards to represent linear relationships using multiple representations. The student is expected to represent linear relationships using verbal descriptions, tables, graphs, and equations that simplify to the form $y = mx + b$.
- ~~(8)~~ (10) Expressions, equations, and relationships. The student applies mathematical process standards to develop geometric relationships with volume. The student is expected to:
- ~~(A)~~ (A) model the relationship between the volume of a rectangular prism and a rectangular pyramid having both congruent bases and heights and connect that relationship to the formulas;
 - ~~(B)~~ (B) explain verbally and symbolically the relationship between the volume of a triangular prism and a triangular pyramid having both congruent bases and heights and connect that relationship to the formulas; and
 - ~~(C)~~ (C) use models to determine the approximate formulas for the circumference and area of a circle and connect the models to the actual formulas.
- ~~(9)~~ (11) Expressions, equations, and relationships. The student applies mathematical process standards to solve geometric problems. The student is expected to:
- ~~(A)~~ (A) solve problems involving the volume of rectangular prisms, triangular prisms, rectangular pyramids, and triangular pyramids;
 - ~~(B)~~ (B) determine the circumference and area of circles;
 - ~~(C)~~ (C) determine the area of composite figures containing combinations ~~[any combination]~~ of rectangles, squares, parallelograms, trapezoids, triangles, semicircles, and quarter circles; and
 - ~~(D)~~ (D) solve problems involving the lateral and total surface area of a rectangular prism, rectangular pyramid, triangular prism, and triangular pyramid by determining the area of the shape's net.
- ~~(10)~~ (12) Expressions, equations, and relationships. The student applies mathematical process standards to use one-variable equations and inequalities to represent situations. The student is expected to:
- ~~(A)~~ (A) write one-variable, two-step equations and inequalities to represent constraints or conditions within problems;
 - ~~(B)~~ (B) represent solutions for one-variable, two-step equations and inequalities on number lines; and

- (C) write a corresponding real-world problem given a one-variable, two-step equation or inequality.
- (11) ~~(13)~~ Expressions, equations, and relationships. The student applies mathematical process standards to solve one-variable equations and inequalities. The student is expected to:
 - (A) model and solve one-variable, two-step equations and inequalities;
 - (B) determine if the given value(s) make(s) one-variable, two-step equations and inequalities true; and
 - (C) write and solve equations using geometry concepts, including the sum of the angles in a triangle, and angle relationships.
- (12) ~~(14)~~ Measurement and data. The student applies mathematical process standards to use statistical representations to analyze data. The student is expected to:
 - (A) compare two groups of numeric data using comparative dot plots or box plots by comparing their shapes, centers, and spreads;
 - (B) use data from a random sample to make inferences about a population; and
 - (C) compare two populations based on data in random samples from these populations, including informal comparative inferences about differences between the two populations.
- (13) ~~(15)~~ Personal financial literacy. The student applies mathematical process standards to develop an economic way of thinking and problem solving useful in one's life as a knowledgeable consumer and investor. The student is expected to:
 - (A) calculate the sales tax for a given purchase and calculate ~~(distinguish between sales tax and)~~ income tax for earned wages ;
 - (B) identify ~~(interpret)~~ the components of a personal budget, including income ; ~~(;)~~ planned savings for college, retirement, and emergencies; ~~(;)~~ taxes ; ~~(;)~~ and fixed and variable expenses , and calculate what percentage each category comprises of the total budget ;
 - (C) create and organize a financial assets and liabilities record and construct a net worth statement;
 - (D) use a family budget estimator to determine the minimum household budget and average hourly wage needed for a family to meet its basic needs in the student's city or another large city nearby;
 - ~~(D) calculate percentages for major expense categories;~~
 - (E) calculate and compare simple interest and compound interest earnings; and
 - (F) analyze and compare monetary incentives, including sales, rebates, and coupons.

§111.28. Grade 8, Adopted 2012.

(a) Introduction.

- (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency , and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
- (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills

together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, ~~and~~ number sense, and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

- (3) The primary focal areas in Grade 8 are proportionality; expressions, equations, relationships, and foundations of functions; and measurement and data. Students use concepts, algorithms, and properties of real numbers to explore mathematical relationships and to describe increasingly complex situations. Students use concepts of proportionality to explore, develop, and communicate mathematical relationships. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other. Students connect verbal, numeric, graphic, and symbolic representations of relationships, including equations and inequalities. Students begin to develop an understanding of functional relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, and reasoning to draw conclusions, evaluate arguments, and make recommendations. While the use of all types of technology is important, the emphasis on algebra readiness skills necessitates the implementation of graphing technology.

- (4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b) Knowledge and skills.

- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
- (A) apply mathematics to problems arising in everyday life, society, and the workplace;
 - (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
 - (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
 - (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
 - (E) create and use representations to organize, record, and communicate mathematical ideas;
 - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
 - (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

- (2) Number and operations. The student applies mathematical process standards to represent and use real numbers in a variety of forms. The student is expected to:
- (A) extend previous knowledge of sets and subsets using a visual representation [~~such as a Venn diagram~~] to describe relationships between sets of real numbers;
 - (B) approximate the value of an irrational number, including π and square roots of numbers less than 225, and locate that rational number approximation on a number line;
 - (C) convert between standard decimal [~~base 10~~] notation and scientific notation; and
 - (D) order a set of real numbers arising from mathematical and real-world contexts.
- (3) Proportionality. The student applies mathematical process standards to use proportional relationships to describe dilations. The student is expected to:
- (A) generalize that the ratio of corresponding sides of similar shapes are proportional, including a shape and its dilation;
 - (B) compare and contrast the attributes of a shape and its dilation(s) on a coordinate plane; and
 - (C) use an algebraic representation to explain the effect of a given positive rational scale factor applied to two-dimensional figures on a coordinate plane with the origin as the center of dilation [~~such as $(x, y) \rightarrow (0.5x, 0.5y)$~~].
- (4) Proportionality. The student applies mathematical process standards to explain proportional and non-proportional relationships involving slope. The student is expected to:
- (A) use similar right triangles to develop an understanding that slope, m , given as the rate comparing the change in y -values to the change in x -values, $(y_2 - y_1) / (x_2 - x_1)$, is the same for any two points (x_1, y_1) and (x_2, y_2) on the same line;
 - (B) graph proportional relationships, interpreting the unit rate as the slope of the line that models the relationship; and
 - (C) use data from a table or graph to determine the rate of change or slope and y -intercept in mathematical and real-world problems.
- (5) Proportionality. The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions. The student is expected to:
- (A) represent linear proportional situations with tables, graphs, and equations in the form of $y = kx$;
 - (B) represent linear non-proportional situations with tables, graphs, and equations in the form of $y = mx + b$, where $b \neq 0$;
 - (C) contrast bivariate sets of data that suggest a linear relationship with bivariate sets of data that do not suggest a linear relationship from a graphical representation;
 - (D) use a trend line that approximates the linear relationship between bivariate sets of data to make predictions;
 - (E) solve problems involving direct variation;
 - ~~(F) solve directly proportional problems;~~
 - (F) ~~(G)~~ distinguish between proportional and non-proportional situations using tables, graphs, and equations in the form $y = kx$ or $y = mx + b$, where $b \neq 0$;
 - (G) ~~(H)~~ identify functions using sets of ordered pairs, tables, mappings, and graphs;

- ~~(H)~~ ~~(H)~~ identify examples of proportional and non-proportional functions that arise from mathematical and real-world problems; and
- ~~(I)~~ ~~(I)~~ write an equation in the form $y = mx + b$ to model a linear relationship between two quantities using verbal, numerical, tabular, and graphical representations.
- (6) Expressions, equations, and relationships. The student applies mathematical process standards to develop mathematical relationships and make connections to geometric formulas. The student is expected to:
- (A) describe the volume formula $V = Bh$ of a cylinder in terms of its base area and its height;
 - (B) model the relationship between the volume of a cylinder and a cone having both congruent bases and heights and connect that relationship to the formulas; and
 - (C) use models and diagrams to explain the Pythagorean theorem.
- (7) Expressions, equations, and relationships. The student applies mathematical process standards to use geometry to solve problems. The student is expected to:
- (A) solve problems involving the volume of cylinders, cones, and spheres;
 - (B) use previous knowledge of surface area to make connections to the formulas for lateral and total surface area and determine solutions for problems involving rectangular prisms, triangular prisms, and cylinders;
 - (C) use the Pythagorean Theorem and its converse to solve problems; and
 - (D) determine the distance between two points on a coordinate plane using the Pythagorean Theorem.
- (8) Expressions, equations, and relationships. The student applies mathematical process standards to use one-variable equations or inequalities in problem situations. The student is expected to:
- (A) write one-variable equations or inequalities with variables on both sides that represent problems using rational number coefficients and constants;
 - (B) write a corresponding real-world problem when given a one-variable equation or inequality with variables on both sides of the equal sign using rational number coefficients and constants;
 - (C) model and solve one-variable equations with variables on both sides of the equal sign that represent mathematical and real-world problems using rational number coefficients and constants; and
 - (D) use informal arguments to establish facts about the angle sum and exterior angle of triangles, the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.
 - ~~[(D) write and solve equations using geometry concepts, including the angle relationships when parallel lines are cut by a transversal; and]~~
 - ~~[(E) write and solve equations using geometry concepts, including the properties of side lengths and angles in quadrilaterals.]~~
- (9) Expressions, equations, and relationships. The student applies mathematical process standards to use multiple representations to develop foundational concepts of simultaneous linear equations. The student is expected to identify and verify the values of x and y that simultaneously satisfy two linear equations in the form $y = mx + b$ from the intersections of the graphed equations.
- (10) Two-dimensional shapes. The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:

- (A) generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane;
 - (B) differentiate between transformations that preserve congruence and those that do not;
 - (C) explain the effect of translations, reflections over the x- or y-axis, and rotations limited to 90°, 180°, 270°, and 360° as applied to two-dimensional shapes on a coordinate plane using an algebraic representation [such as $(x, y) \rightarrow (x + 2, y + 2)$]; and
 - (D) model the effect on linear and area measurements of dilated two-dimensional shapes.
- (11) Measurement and data. The student applies mathematical process standards to use statistical procedures to describe data. The student is expected to:
- (A) construct a scatterplot and describe the observed data [~~trend such as positive trend, negative trend, and no trend~~] to address questions of association such as linear, non-linear, and no association between bivariate data;
 - (B) determine the mean absolute deviation and use this quantity as a measure of the average distance data are from the mean using a data set of no more than 10 data points; and
 - (C) simulate generating random samples of the same size from a population with known characteristics to develop the notion of a random sample being representative of the population from which it was selected.
- (12) Personal financial literacy. The student applies mathematical process standards to develop an economic way of thinking and problem solving useful in one's life as a knowledgeable consumer and investor. The student is expected to:
- (A) solve real-world problems comparing how interest rate and loan length affect the cost of credit;
 - (B) calculate the total cost of repaying a loan, including credit cards and easy access loans, under various rates of interest and over different periods using an online calculator;
 - (C) explain how small amounts of money invested regularly, including money saved for college and retirement, grow over time [~~grow exponentially~~];
 - (D) calculate and compare simple interest and compound interest earnings;
 - (E) [~~(D)~~] identify and explain [~~and justify~~] the advantages and disadvantages of different payment methods; [~~such as stored value cards, debit cards, and online payment systems; and~~]
 - (F) [~~(E)~~] analyze [~~financial~~] situations to determine if they represent [~~the situation is a~~] financially responsible decisions [~~decision~~] and identify the benefits of financial responsibility and the costs of financial irresponsibility; and [~~7~~]
 - (G) estimate the cost of a two-year and four-year college education, including family contribution, and devise a periodic savings plan for accumulating the money needed to contribute to the total cost of attendance for at least the first year of college.