ATTACHMENT II
Text of Proposed Revisions to 19 TAC

Chapter 111. Texas Essential Knowledge and Skills for Mathematics

Subchapter A. Elementary

§111.1. Implementation of Texas Essential Knowledge and Skills for Mathematics, Elementary, Adopted 2012.

(a)   The provisions of §§111.2-111.7 of this subchapter shall be implemented by school districts.

(b)   No later than August 31, 2013, the commissioner of education shall determine whether instructional materials funding has [have] been made available to Texas public schools for materials that cover the essential knowledge and skills for mathematics as adopted in §§111.2-111.7 of this subchapter.

(c)   If the commissioner makes the determination that instructional materials funding has [have] been made available under subsection (b) of this section, §§111.2-111.7 of this subchapter shall be implemented beginning with the 2014-2015 school year and apply to the 2014-2015 and subsequent school years.

(d)   If the commissioner does not make the determination that instructional materials funding has [have] been made available under subsection (b) of this section, the commissioner shall determine no later than August 31 of each subsequent school year whether instructional materials funding has [have] been made available. If the commissioner determines that instructional materials funding has [have] been made available, the commissioner shall notify the State Board of Education and school districts that §§111.2-111.7 of this subchapter shall be implemented for the following school year.

(e)   Sections 111.11-111.17 of this subchapter shall be superseded by the implementation of §§111.1-111.7 under this section.

§111.2. Kindergarten, Adopted 2012.

(a)   Introduction.

(1)   The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency , and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2)   The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, [and] number sense , and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or

        justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3)   For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Kindergarten are expected to perform their work without the use of calculators.

(4)   The primary focal areas in Kindergarten are understanding counting and cardinality, understanding addition as joining and subtraction as separating, and comparing objects by measurable [~~measureable~~] attributes.

    (A)   Students develop number and operations through several fundamental concepts. Students know number names and the counting sequence. Counting and cardinality lay a solid foundation for number. Students apply the principles of counting to make the connection between numbers and quantities.

    (B)   Students use meanings of numbers to create strategies for solving problems and responding to practical situations involving addition and subtraction.

    (C)   Students identify characteristics of objects that can be measured and directly compare objects according to these measurable [~~measureable~~] attributes.

(5)   Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b)   Knowledge and skills.

(1)   Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

    (A)   apply mathematics to problems arising in everyday life, society, and the workplace;

    (B)   use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

    (C)   select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

    (D)   communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

    (E)   create and use representations to organize, record, and communicate mathematical ideas;

    (F)   analyze mathematical relationships to connect and communicate mathematical ideas; and

    (G)   display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2)   Number and operations. The student applies mathematical process standards to understand how to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system. The student is expected to:

    (A)   count forward and backward to at least 20 with and without objects;

    (B)   read, write, and represent whole numbers from 0 to at least 20 with and without objects or pictures;

(C) count a set of objects up to at least 20 and demonstrate that the last number said tells the number of objects in the set regardless of their arrangement or order ;

(D) recognize instantly the quantity of a small group of objects in organized and random arrangements;

(E) generate a set using concrete and pictorial models that represents a number that is more than, less than, and equal to a given number up to 20;

(F) generate a number that is one more than or one less than another number up to at least 20;

(G) compare sets of objects up to at least 20 in each set using comparative language;

(H) use comparative language to describe two numbers up to 20 presented as written numerals; and

(I) compose and decompose numbers up to 10 with objects and pictures.

(3) Number and operations. The student applies mathematical process standards to develop an understanding of addition and subtraction situations in order to solve problems. The student is expected to:

(A) model the action of joining to represent addition and the action of separating to represent subtraction;

(B) solve word problems using objects and drawings to find sums up to 10 and differences within 10; and

(C) explain the strategies used to solve problems involving adding and subtracting within 10 using spoken words, concrete and pictorial models, and number sentences.

(4) Number and operations. The student applies mathematical process standards to identify coins in order to recognize the need for monetary transactions. The student is expected to identify U.S. coins by name, including pennies, nickels, dimes, and quarters.

(5) Algebraic reasoning. The student applies mathematical process standards to identify the pattern in the number word list. The student is expected to recite numbers up to at least 100 by ones and tens beginning with any given number. [:]

[(A) recite numbers up to at least 100 by ones and tens beginning with any given number; and]

[(B) represent addition and subtraction with objects, drawings, situations, verbal explanations, or number sentences.]

(6) Geometry and measurement. The student applies mathematical process standards to analyze attributes of two-dimensional shapes and three-dimensional solids to develop generalizations about their properties. The student is expected to:

(A) identify two-dimensional shapes, including circles, triangles, rectangles, and squares as special rectangles;

(B) identify three-dimensional solids, including cylinders, cones, spheres, and cubes, in the real world;

(C) identify two-dimensional components of three-dimensional objects [such as the face of a tissue box is a rectangle] ;

(D) identify attributes of two-dimensional shapes using informal and formal geometric language interchangeably [such as number of corners or vertices and number of sides] ;

(E) classify and sort a variety of regular and irregular two- and three-dimensional figures regardless of orientation or size; and

(F) create two-dimensional shapes using a variety of materials and drawings.

(7)   Geometry and measurement. The student applies mathematical process standards to directly compare measurable [~~measureable~~] attributes. The student is expected to:

   (A)   give an example of a measurable attribute of a given object, including length, capacity, and weight; and

   (B)   compare two objects with a common measurable [~~measureable~~] attribute to see which object has more of/less of the attribute and describe the difference.

(8)   Data analysis. The student applies mathematical process standards to collect and organize data to make it useful for interpreting information. The student is expected to:

   (A)   collect, sort, and organize data into two or three categories;

   (B)   use data to create real-object and picture graphs; and

   (C)   draw conclusions from real-object and picture graphs.

(9)   Personal financial literacy. The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:

   (A)   identify ways to earn income;

   (B)   differentiate between money received as income and money received as gifts;

   (C)   list simple skills required for jobs [~~such as bus driver, librarian, cashier, or cook~~] ; and

   (D)   distinguish between wants and needs and identify income as a source to meet one's wants and needs.

§111.3. Grade 1, Adopted 2012.

(a)   Introduction.

   (1)   The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency , and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

   (2)   The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, [~~and~~] number sense , and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

   (3)   For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in

carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 1 are expected to perform their work without the use of calculators.

(4)   The primary focal areas in Grade 1 are understanding and applying place value, solving problems involving addition and subtraction, and composing and decomposing two-dimensional shapes and three-dimensional solids.

(A)   Students use relationships within the numeration system to understand the sequential order of the counting numbers and their relative magnitude.

(B)   Students extend their use of addition and subtraction beyond the actions of joining and separating to include comparing and combining. Students use properties of operations and the relationship between addition and subtraction to solve problems. By comparing a variety of solution strategies, students use efficient, accurate, and generalizable methods to perform operations.

(C)   Students use basic shapes and spatial reasoning to model objects in their environment and construct more complex shapes. Students are able to identify, name, and describe basic two-dimensional shapes and three-dimensional solids.

(5)   Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b)   Knowledge and skills.

(1)   Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A)   apply mathematics to problems arising in everyday life, society, and the workplace;

(B)   use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C)   select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D)   communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E)   create and use representations to organize, record, and communicate mathematical ideas;

(F)   analyze mathematical relationships to connect and communicate mathematical ideas; and

(G)   display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2)   Number and operations. The student applies mathematical process standards to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system related to place value. The student is expected to:

(A)   recognize instantly the quantity of structured arrangements [such as seen on a die or a ten frame] ;

(B)   use concrete and pictorial models to compose and decompose numbers up to 120 in more than one way as so many hundreds, so many tens, and so many ones;

(C)   use objects, pictures, and expanded and standard forms to represent numbers up to 120;

(D)   generate a number that is greater than or less than a given whole number up to 120;

(E)   use place value to compare whole numbers up to 120 using comparative language;

(F)   order whole numbers up to 120 using place value and open number lines; and

(G)   represent the comparison of two numbers to 100 using the symbols >, <, or =.

(3)   Number and operations. The student applies mathematical process standards to develop and use strategies for whole number addition and subtraction computations in order to solve problems. The student is expected to:

(A)   use concrete and pictorial models to determine the sum of a multiple of 10 and a one-digit number in problems up to 99;

(B)   use objects and pictorial models to solve word problems involving joining, separating, and comparing sets within 20 and unknowns as any one of the terms in the problem such as 2 + 4 = [ ]; 3 + [ ] = 7; and 5 = [ ] - 3;

(C)   compose 10 with two or more addends with and without concrete objects;

(D)   apply basic fact strategies to add and subtract within 20, including making 10 and decomposing a number leading to a 10;

(E)   explain strategies used to solve addition and subtraction problems up to 20 using spoken words, objects, pictorial models, and number sentences; and

(F)   generate and solve problem situations when given a number sentence involving addition or [and] subtraction of numbers within 20.

(4)   Number and operations. The student applies mathematical process standards to identify coins, their values, and the relationships among them in order to recognize the need for monetary transactions. The student is expected to:

(A)   identify U.S. coins, including pennies, nickels, dimes, and quarters, by value and describe the relationships among them;

(B)   write a number with the cent symbol to describe the value of a coin; and

(C)   use relationships to count by twos, fives, and tens to determine the value of a collection of pennies, nickels, and/or [and] dimes.

(5)   Algebraic reasoning. The student applies mathematical process standards to identify and apply number patterns within properties of numbers and operations in order to describe relationships. The student is expected to:

(A)   recite numbers forward and backward from any given number between 1 and 120;

[(B)   skip count by twos, fives, and tens to 100;]

(B) [(C)]   skip count by twos, fives, and tens to determine the total number of objects up to 120 in a set;

(C) [(D)]   use relationships to determine the number that is 10 more and 10 less than a given number up to 120;

(D) [(E)]represent word problems involving addition and subtraction of whole numbers up to 20 using concrete and pictorial models and number sentences;

(E) [(F)] understand that the equal sign represents a relationship where expressions [statements] on each side of the equal sign represent the same value(s) [are true] ;

(F) [(G)]determine the unknown whole number in an addition or subtraction equation when the unknown may be any one of the three or four terms in the equation; and

[(H) identify relationships between addition facts and related subtraction sentences such as 3 + 2 = 5 and 5 − 2 = 3; and]

(G) [(I)] apply properties of operations [as strategies] to add and subtract two or three numbers [such as if 2 + 3 = 5 is known, then 3 + 2 = 5] .

(6) Geometry and measurement. The student applies mathematical process standards to analyze attributes of two-dimensional shapes and three-dimensional solids to develop generalizations about their properties. The student is expected to:

(A) classify and sort regular and irregular two-dimensional shapes based on attributes using informal geometric language;

(B) distinguish between attributes that define a two-dimensional or three-dimensional figure [such as a closed figure with three sides is a triangle or a solid with exactly six rectangular faces is a rectangular prism] and attributes that do not define the shape [such as orientation or color] ;

(C) create two-dimensional figures, including circles, triangles, rectangles, and squares, as special rectangles, rhombuses, and hexagons;

(D) identify two-dimensional shapes, including circles, triangles, rectangles, and squares, as special rectangles, rhombuses, and hexagons and describe their attributes using formal geometric language [such as vertex and side] ;

(E) identify three-dimensional solids, including spheres, cones, cylinders, rectangular prisms (including cubes), and triangular prisms, and describe their attributes using formal geometric language [such as vertex, edge, and face] ;

(F) compose two-dimensional shapes by joining two, three, or four figures to produce a target shape in more than one way if possible;

(G) partition two-dimensional figures [such as circles and rectangles] into two and four fair shares or equal parts and describe the parts using words [such as "halves," "half of," "fourths," or "quarters"] ; and

(H) identify examples and non-examples of halves and fourths.

(7) Geometry and measurement. The student applies mathematical process standards to select and use units to describe length and time. The student is expected to:

(A) use measuring tools [such as adding machine tape, ribbon, or string] to measure the length of objects to reinforce the continuous nature of linear measurement;

(B) illustrate [demonstrate] that the length of an object is the number of same-size units of length that, when laid end-to-end with no gaps or overlaps, reach from one end of the object to the other;

(C) measure the same object/distance with units of two different lengths and describe how and why the measurements differ;

(D) describe a length to the nearest whole unit using a number and a unit [such as five craft sticks] ; and

(E) tell time to the hour and half hour using analog and digital clocks.

(8) Data analysis. The student applies mathematical process standards to organize data to make it useful for interpreting information and solving problems. The student is expected to:

(A) collect, sort, and organize data in up to three categories using models/representations such as tally marks or T-charts;

(B) use data to create picture and bar-type graphs; and

(C)   draw conclusions and generate and answer questions using information from picture and bar-type graphs.

(9)   Personal financial literacy. The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:

(A)   define money earned as income;

[(B)   explain how human capital is related to work;]

(B) [(C)]identify income as a means of obtaining goods and services, oftentimes making choices between wants and needs;

(C) [(D)]   distinguish between spending and saving; and

(D) [(E)]consider charitable giving.

§111.4. Grade 2, Adopted 2012.

(a)   Introduction.

(1)   The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency , and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2)   The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, [and] number sense , and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3)   For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 2 are expected to perform their work without the use of calculators.

(4)   The primary focal areas in Grade 2 are making comparisons within the base-10 place value system, solving problems with addition and subtraction within 1,000 [100] , and building foundations for multiplication.

(A)   Students develop an understanding of the base-10 place value system and place value concepts. The students' understanding of base-10 place value includes ideas of counting

in units and multiples of thousands, hundreds, tens, and ones and a grasp of number relationships, which students demonstrate in a variety of ways.

(B) Students identify situations in which addition and subtraction are useful to solve problems. Students develop a variety of strategies to use efficient, accurate, and generalizable methods to add and subtract multi-digit whole numbers.

(C) Students use the relationship between skip counting and equal groups of objects to represent the addition or subtraction of equivalent sets, which builds a strong foundation for multiplication and division.

(5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Number and operations. The student applies mathematical process standards to understand how to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system related to place value. The student is expected to:

(A) use concrete and pictorial models to compose and decompose numbers up to 1,200 in more than one way as a sum of so many thousands, hundreds, tens, and ones;

(B) use standard, word, and expanded forms to represent numbers up to 1,200;

(C) generate a number that is greater than or less than a given whole number up to 1,200;

(D) use place value to compare and order whole numbers up to 1,200 using comparative language, numbers, and symbols (>, <, or =);

(E) locate the position of a given whole number on an open number line; and

(F) name the whole number that corresponds to a specific point on a number line . [; and]

[(G)   order whole numbers up to 1,200 using place value and open number lines.]

(3) Number and operations. The student applies mathematical process standards to recognize and represent fractional units and communicates how they are used to name parts of a whole. The student is expected to:

(A)   partition objects [such as strips, lines, regular polygons, and circles] into equal parts and name the parts, including halves, fourths, and eighths, using words [such as "one-half" or "three-fourths"] ;

(B)   explain that the more fractional parts used to make a whole, the smaller the part; and the fewer the fractional parts, the larger the part;

(C)   use concrete models to count fractional parts beyond one whole using words [such as "one-fourth," "two-fourths," "three-fourths," "four-fourths," "five-fourths," or "one and one-fourth,"] and recognize how many parts it takes to equal one whole [such as four-fourths equals one whole] ; and

(D)   identify examples and non-examples of halves, fourths, and eighths.

(4)   Number and operations. The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve addition and subtraction problems with efficiency and accuracy. The student is expected to:

(A)   recall basic facts to add and subtract within 20 with automaticity;

(B)   [use mental strategies, flexible methods, and algorithms based on knowledge of place value and equality to] add up to four two-digit numbers and subtract two-digit numbers using mental strategies and algorithms based on knowledge of place value and properties of operations ;

(C)   solve one-step and multi-step word problems involving addition and subtraction within 1,000 [of two-digit numbers] using a variety of strategies based on place value, including algorithms; and

(D)   generate and solve problem situations for a given mathematical number sentence involving addition and subtraction of whole numbers within 1,000 [100] .

(5)   Number and operations. The student applies mathematical process standards to determine the value of coins in order to solve monetary transactions. The student is expected to:

(A)   determine the value of a collection of coins up to one dollar; and

(B)   use the cent symbol, dollar sign, and the decimal point to name the value of a collection of coins.

(6)   Number and operations. The student applies mathematical process standards to connect repeated addition and subtraction to multiplication and division situations that involve equal groupings and shares. The student is expected to:

(A)   model, create, and describe contextual multiplication situations in which equivalent sets of concrete objects are joined; and

(B)   model, create, and describe contextual division situations in which a set of concrete objects is separated into equivalent sets.

(7)   Algebraic reasoning. The student applies mathematical process standards to identify and apply number patterns within properties of numbers and operations in order to describe relationships. The student is expected to:

(A)   [use relationships and objects to] determine whether a number up to 40 is even or odd using pairings of objects to represent the number ;

(B)   use an understanding of place value [relationships] to determine the number that is 10 or 100 more or less than a given number up to 1,200; and

(C)   represent and solve addition and subtraction word problems where unknowns may be any one of the terms in the problem.

(8)   Geometry and measurement. The student applies mathematical process standards to analyze attributes of two-dimensional shapes and three-dimensional solids [two- and three-dimensional geometric figures] to develop generalizations about their properties. The student is expected to:

(A)   create two-dimensional shapes based on given attributes, including number of sides and vertices;

[(B)   identify attributes of a quadrilateral, a pentagon, and an octagon;]

(B) [(C)]classify and sort three-dimensional solids, including spheres, cones, cylinders, [spheres, triangular and] rectangular prisms (including [, and] cubes as special rectangular prisms), and triangular prisms, based on attributes using formal geometric language [such as vertex, edge, and face] ;

(C) [(D)]   classify and sort polygons with 12 or fewer sides according to attributes, including identifying the number of sides and number of vertices;

(D) [(E)]compose two-dimensional shapes and three-dimensional solids with given properties or attributes [such as build a rectangle out of unit squares or build a rectangular prism out of unit cubes] ; and

(E) [(F)] decompose two-dimensional shapes such as cutting out a square from a rectangle, dividing a shape in half, or partitioning a rectangle into identical triangles and identify the resulting geometric parts.

(9)   Geometry and measurement. The student applies mathematical process standards to select and use units to describe length, area, and time. The student is expected to:

(A)   find the length of objects using concrete models for standard units of length [such as the edges of inch tiles or centimeter cubes] ;

(B)   describe the inverse relationship between the size of the unit and the number of units needed to equal the length of an object [such as the longer the unit, the fewer needed and the shorter the unit, the more needed] ;

(C)   represent whole numbers as distances from any given location on a number line;

(D)   determine the length of an object to the nearest marked [half] unit using rulers, yardsticks, meter sticks, or measuring tapes;

(E)   determine a solution to a problem involving length, including estimating lengths;

(F)   use concrete models of square units to find the area of a rectangle by covering it with no gaps or overlaps, counting to find the total number of square units, and describing the measurement using a number and the unit [such as 24 square units] ; and

(G)   read and write time to the nearest [five- and] one-minute increment [increments] using analog and digital clocks and distinguish between a.m. and p.m.

(10)   Data analysis. The student applies mathematical process standards to organize data to make it useful for interpreting information and solving problems. The student is expected to:

(A)   explain that the length of a bar in a bar graph or the number of pictures in a pictograph represents the number of data points for a given category;

(B)   organize a collection of data with up to four categories using pictographs and bar graphs with intervals of one or more;

(C)   write and solve one-step word problems involving addition or subtraction using data represented within pictographs and bar graphs with intervals of one; and

(D)   draw conclusions and make predictions from information in a graph.

(11) Personal financial literacy. The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:

(A) calculate how money saved can accumulate into a larger amount over time;

(B) explain that saving is an alternative to spending;

(C) distinguish between a deposit and a withdrawal;

(D) identify examples of borrowing and distinguish between responsible and irresponsible borrowing;

(E) identify examples of lending and use concepts of benefits and costs to evaluate lending decisions; and

(F) differentiate between producers and consumers and calculate the cost to produce a simple item [such as a shirt, a pitcher of lemonade, or a class art project] .

§111.5. Grade 3, Adopted 2012.

(a) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency , and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, [and] number sense , and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 3 are expected to perform their work without the use of calculators.

(4) The primary focal areas in Grade 3 are place value, operations of whole numbers, and understanding fractional units. These focal areas are supported throughout the mathematical strands of number and operations, algebraic reasoning, geometry and measurement, and data analysis. In Grades 3-5, the number set is limited to positive rational numbers. In number and operations, students will focus on applying place value, comparing and ordering whole numbers,

connecting multiplication and division, and understanding and representing fractions as numbers and equivalent fractions. In algebraic reasoning, students will use multiple representations of problem situations, determine missing values in number sentences, and represent real-world relationships using number pairs in a table and verbal descriptions. In geometry and measurement, students will identify and classify two-dimensional figures according to common attributes, decompose composite figures formed by rectangles to determine area, determine the perimeter of polygons, solve problems involving time, and measure liquid volume (capacity) or weight. In data analysis, students will represent and interpret data.

    (5)   Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b)   Knowledge and skills.

    (1)   Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

        (A)   apply mathematics to problems arising in everyday life, society, and the workplace;

        (B)   use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

        (C)   select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

        (D)   communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

        (E)   create and use representations to organize, record, and communicate mathematical ideas;

        (F)   analyze mathematical relationships to connect and communicate mathematical ideas; and

        (G)   display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

    (2)   Number and operations. The student applies mathematical process standards to represent and compare whole numbers and understand relationships related to place value. The student is expected to:

        (A)   compose and decompose numbers up to 100,000 [in more than one way] as a sum of so many ten thousands, so many thousands, so many hundreds, so many tens, and so many ones using objects, pictorial models, and numbers, including expanded notation as appropriate;

        (B)   describe the mathematical relationships found in the base-10 place value system through the hundred thousands [100,000s] place;

        (C)   represent a number on a number line as being between two consecutive multiples of 10; 100; 1,000; or 10,000 and use words [such as "closer to," "is about," or "is nearly"] to describe relative size of numbers in order to round whole numbers; and

        (D)   compare and order whole numbers up to 100,000 and represent comparisons using the symbols >, <, or =.

    (3)   Number and operations. The student applies mathematical process standards to represent and explain fractional units. The student is expected to:

        (A)   represent fractions greater than zero and less than or equal to one with denominators of 2, 3, 4, 6, and 8 using concrete objects and pictorial models, including strip diagrams and number lines;

(B) determine the corresponding fraction greater than zero and less than or equal to one with denominators of 2, 3, 4, 6, and 8 given [of] a specified point on a number line;

(C) explain that the unit fraction 1/*b* represents the quantity formed by one part of a whole that has been partitioned into *b* equal parts where *b* is a non-zero whole number;

(D) compose and decompose a fraction *a*/*b* with a numerator greater than zero and less than or equal to *b* as a sum of parts 1/*b*;

(E) solve problems involving partitioning an object or a set of objects among two or more recipients using pictorial representations of fractions with denominators of 2, 3, 4, 6, and 8 [such as two children share five cookies] ;

(F) represent equivalent fractions with denominators of 2, 3, 4, 6, and 8 using a variety of objects and pictorial models, including number lines;

(G) explain that two fractions are equivalent if and only if they are both represented by the same point on the number line or represent the same portion of a same size whole for an area model; and

(H) compare two fractions having the same numerator or denominator in problems by reasoning about their sizes and justifying the conclusion using symbols, words, objects, and pictorial models [such as comparing the size of pieces when sharing a candy bar equally among four people or equally among three people] .

(4) Number and operations. The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve problems with efficiency and accuracy. The student is expected to:

(A) solve with fluency one-step and two-step [multi-step] problems involving addition and subtraction within 1,000 using strategies based on place value, properties of operations, and the relationship between addition and subtraction;

(B) round [use strategies, including rounding] to the nearest 10 or 100 or use [and] compatible numbers [,] to estimate solutions to addition and subtraction problems;

(C) determine the value of a collection of coins and bills;

(D) determine the total number of objects when equally-sized groups of objects are combined or arranged in arrays up to 10 by 10;

(E) represent multiplication facts by using a variety of approaches such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line, and skip counting;

(F) recall facts to multiply up to 10 by 10 with automaticity and recall the corresponding division facts;

(G) use strategies and algorithms, including the standard algorithm, to multiply a two-digit number by a one-digit number. Strategies may include mental math, partial products, and the commutative, associative, and distributive properties;

(H) determine the number of objects in each group when a set of objects is partitioned into equal shares or a set of objects is shared equally;

(I) [use divisibility rules to] determine if a number is even or odd using divisibility rules ;

(J) determine a quotient using the relationship between multiplication and division [such as the quotient of 40 ÷ 8 can be found by determining what factor makes 40 when multiplied by 8] ; and

      (K)   solve one-step and <u>two-step</u> [~~multi-step~~] problems involving multiplication and division within 100 using strategies based on objects; pictorial models, including arrays, area models, and equal groups; properties of operations; or recall of facts.

(5)   Algebraic reasoning. The student applies mathematical process standards to analyze and create patterns and relationships. The student is expected to:

      (A)   represent [~~and solve~~] one- and two-step problems involving addition and subtraction of whole numbers to 1,000 using pictorial models <u>.</u> [~~such as strip diagrams and~~] number lines <u>,</u> and equations;

      (B)   represent and solve one- and two-step multiplication and division problems within 100 using arrays, strip diagrams, and equations;

      (C)   describe a multiplication expression as a comparison such as 3 x 24 represents 3 times as much as 24;

      (D)   determine the unknown whole number in a multiplication or division equation relating three whole numbers when the unknown is either a missing factor or product [~~such as the value 4 makes 3 x [ ] = 12 a true equation~~] <u>; and</u>

      (E)   represent real-world relationships using number pairs in a table and verbal descriptions [~~such as 1 insect has 6 legs, 2 insects have 12 legs, and so forth~~] <u>.</u>

(6)   Geometry and measurement. The student applies mathematical process standards to analyze attributes of two-dimensional geometric figures to develop generalizations about their properties. The student is expected to:

      (A)   classify and sort two- and three-dimensional solids, including cones, cylinders, spheres, triangular and rectangular prisms, and cubes, based on attributes using formal geometric language [~~such as vertex, edge, and face~~] <u>;</u>

      (B)   use attributes to recognize rhombuses, parallelograms, trapezoids, rectangles, and squares as examples of quadrilaterals and draw examples of quadrilaterals that do not belong to any of these subcategories;

      (C)   determine the area of rectangles with whole number side lengths in problems using multiplication related to the number of rows times the number of unit squares in each row;

      (D)   decompose composite figures formed by rectangles into non-overlapping rectangles to determine the area of the original figure using the additive property of area; and

      (E)   decompose two congruent two-dimensional figures into parts with equal areas and express the area of each part as a unit fraction of the whole and recognize that equal shares of identical wholes need not have the same shape.

(7)   Geometry and measurement. The student applies mathematical process standards to select appropriate units, strategies, and tools to solve problems involving customary <u>and metric</u> measurement. The student is expected to:

      (A)   represent fractions of halves, fourths, and eighths as distances from zero on a number line;

      (B)   determine the perimeter of a polygon or a missing length when given perimeter and remaining side lengths in problems;

      (C)   determine the solutions to problems involving addition and subtraction of time intervals in minutes using pictorial models or tools such as a 15-minute event plus a 30-minute event equals 45 minutes;

      (D)   determine when it is appropriate to use measurements of liquid volume (capacity) or weight; and

      (E)   determine liquid volume (capacity) or weight using appropriate units and tools.

(8)   Data analysis. The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:

      (A)   summarize a data set with multiple categories using a frequency table, dot plot, pictograph, or bar graph with scaled intervals; and

      (B)   solve one- and two-step problems using categorical data represented with a frequency table, dot plot, pictograph, or bar graph with scaled intervals.

(9)   Personal financial literacy. The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:

      (A)   explain the connection between human capital/labor [capital] and income;

      (B)   describe the relationship between the availability or scarcity of resources and how that impacts cost;

      (C)   identify the costs and benefits of planned and unplanned spending decisions;

      (D)   explain that credit is used when wants or needs exceed the ability to pay and that it is the borrower's responsibility to pay it back to the lender, usually with interest;

      (E)   list reasons to save and explain the benefit of a savings plan , including for college ; and

      (F)   identify decisions involving income, spending, saving, credit, and charitable giving.

§111.6. Grade 4, Adopted 2012.

(a)   Introduction.

(1)   The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency , and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2)   The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, [and] number sense , and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3)   For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 4 are expected to perform their work without the use of calculators.

(4)   The primary focal areas in Grade 4 are use of operations, fractions, and decimals and describing and analyzing geometry and measurement. These focal areas are supported throughout the mathematical strands of number and operations, algebraic reasoning, geometry and measurement, and data analysis. In Grades 3-5, the number set is limited to positive rational numbers. In number and operations, students will apply place value and represent points on a number line that correspond to a given fraction or terminating decimal. In algebraic reasoning, students will represent and solve multi-step problems involving the four operations with whole numbers with expressions and equations and generate and analyze patterns. In geometry and measurement, students will classify two-dimensional figures, measure angles, and convert units of measure. In data analysis, students will represent and interpret data.

(5)   Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b)   Knowledge and skills.

(1)   Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A)   apply mathematics to problems arising in everyday life, society, and the workplace;

(B)   use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C)   select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D)   communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E)   create and use representations to organize, record, and communicate mathematical ideas;

(F)   analyze mathematical relationships to connect and communicate mathematical ideas; and

(G)   display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2)   Number and operations. The student applies mathematical process standards to represent, compare, and order whole numbers and decimals and understand relationships related to place value. The student is expected to:

(A)   interpret the value of each place-value position as 10 times the position to the right and as one-tenth of the value of the place to its left;

(B)   represent the value of the digit in whole numbers through 1,000,000,000 and decimals to the hundredths using expanded notation and numerals [~~such as in the number 3.94, the 3 in the ones place is 3; the 9 in the tenths place is 0.9; and the 4 in the hundredths place is 0.04; and 3.94 is the sum of 3 ones, 9 tenths, and 4 hundredths~~] ;

(C)   compare and order whole numbers to 1,000,000,000 and represent comparisons using the symbols >, <, or =;

      (D)   round whole numbers to a given place value through the hundred thousands [100,000s] place;

      (E)   represent decimals, including tenths and hundredths, using concrete and visual models and money;

      (F)   compare and order decimals using concrete and visual models to the hundredths;

      (G)   relate decimals to fractions that name tenths and hundredths; and

      (H)   determine the corresponding decimal to the tenths or hundredths place of a specified point on a number line.

(3)   Number and operations. The student applies mathematical process standards to represent and generate fractions to solve problems. The student is expected to:

      (A)   represent a fraction a/b as a sum of fractions $1/b$, where a and b are whole numbers and $b > 0$, including when a > b;

      (B)   decompose a fraction in more than one way into a sum of fractions with the same denominator using concrete and pictorial models and recording results with symbolic representations [such as 7/8 = 5/8 + 2/8; 7/8 = 3/8 + 4/8; 2 7/8 = 1 + 1 + 7/8; 2 7/8 = 8/8 + 8/8 + 7/8] ;

      (C)   determine if two given fractions are equivalent using a variety of methods [, including multiplying by a fraction equivalent to one or simplifying a fraction to lowest terms] ;

      (D)   [generate equivalent fractions to create equal numerators or equal denominators to] compare two fractions with different [unequal] numerators and different [unequal] denominators and represent the comparison [of two fractions] using the symbols >, =, or ≤ [, or =] ;

      (E)   represent and solve addition and subtraction of fractions with equal denominators [and referring to the same whole] using objects and pictorial models that build to the number line [such as strip diagrams] and properties of operations;

      (F)   evaluate [estimate] the reasonableness of sums and differences of fractions using benchmark fractions 0, 1/4, 1/2, 3/4, and 1, referring to the same whole; and

      (G)   represent fractions and decimals to the tenths or hundredths as distances from zero on a number line . [; and]

      [(H)   determine fractional and decimal quantities as being close to 0, 1/2, and 1.]

(4)   Number and operations. The student applies mathematical process standards to develop and use strategies and methods for whole number computations and decimal sums and differences in order to solve problems with efficiency and accuracy. The student is expected to:

      (A)   add and subtract whole numbers and decimals to the hundredths place using the standard algorithm [a variety of methods, including pictorial models, the inverse relationship between operations, concepts of place value, and efficient algorithms] ;

      (B)   determine products of a number and 10 or 100 using properties of operations and place value understandings;

      (C)   represent the product of 2 two-digit numbers using arrays, area models, or equations, including perfect squares through 15 by 15;

      (D)   use strategies and algorithms, including the standard algorithm, to multiply up to a four-digit number by a one-digit number and to multiply a two-digit number by a two-digit number. Strategies may include mental math, partial products, and the commutative, associative, and distributive properties;

      (E)   represent the quotient of up to a four-digit whole number divided by a one-digit whole number using arrays, area models, or equations;

      (F)   use strategies and algorithms, including the standard algorithm, to divide up to a four-digit dividend by a one-digit divisor;

      (G)   round [use strategies, including rounding] to the nearest 10, 100, or 1,000 or use [and] compatible numbers [,] to estimate solutions involving whole numbers ; and

      (H)   solve with fluency one- and two-step problems involving multiplication and division, including interpreting remainders.

(5)   Algebraic reasoning. The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:

      (A)   represent multi-step problems involving the four operations with whole numbers using strip diagrams and equations with a letter standing for the unknown quantity;

      (B)   represent problems using an input-output table and numerical expressions to generate a number pattern that follows a given rule representing [such as given the rule "Add 3" and the starting number 1, use the expressions 1 + 3, 2 + 3, 3 + 3, and so forth to generate a table to represent] the relationship of the values in the resulting sequence and their position in the sequence;

      (C)   use models to determine the formulas for the perimeter of a rectangle ($l + w + l + w$ or $2l + 2w$), including the special form for perimeter of a square ($4s$) and the area of a rectangle (l x w); and

      (D)   solve problems related to perimeter and area of rectangles where dimensions are whole numbers.

(6)   Geometry and measurement. The student applies mathematical process standards to analyze geometric attributes in order to develop generalizations about their properties. The student is expected to:

      (A)   identify points, lines, line segments, rays, angles, and perpendicular and parallel lines;

      (B)   identify and draw one or more lines of symmetry, if they exist, for a two-dimensional figure;

      (C)   apply knowledge of right angles to identify acute, right, and obtuse triangles; and

      (D)   classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.

(7)   Geometry and measurement. The student applies mathematical process standards to solve problems involving angles less than or equal to 180 degrees. The student is expected to:

      (A)   illustrate the measure of an angle as the part of a circle whose center is at the vertex of the angle that is "cut out" by the rays of the angle. Angle measures are limited to whole numbers;

      (B)   illustrate degrees as the units used to measure an angle, where 1/360 of any circle is one degree and an angle that "cuts" n/360 out of any circle whose center is at the angle's vertex has a measure of n degrees. Angle measures are limited to whole numbers;

      (C)   determine the approximate measures of angles in degrees to the nearest whole number using a protractor;

      (D)   draw an angle with a given measure; and

(E) [decompose angles such as complementary and supplementary angles into two non-overlapping angles to] determine the measure of an unknown angle formed by two non-overlapping adjacent angles given one or both angle measures .

(8) Geometry and measurement. The student applies mathematical process standards to select appropriate customary and metric units, strategies, and tools to solve problems involving measurement. The student is expected to:

    (A) identify relative sizes of measurement units within the customary and metric systems;

    (B) convert measurements within the same measurement system, customary or metric, from a smaller unit into a larger unit or a larger unit into a smaller unit when given other equivalent measures represented in a table; and

    (C) solve problems that deal with measurements of length, intervals of time, liquid volumes, mass [masses] , and money using addition, subtraction, multiplication, or division as appropriate.

(9) Data analysis. The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:

    (A) represent data on a frequency table, dot plot, or stem-and-leaf plot marked with whole numbers and fractions; and

    (B) solve one- and two-step problems using data in whole number, decimal, and fraction form in a frequency table, dot plot, or stem-and-leaf plot.

(10) Personal financial literacy. The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:

    (A) distinguish between fixed and variable expenses;

    (B) calculate profit in a given situation;

    (C) compare the advantages and disadvantages of various savings options; [and]

    (D) describe how to allocate a weekly allowance among spending : [,] saving, including for college; and sharing ; and [.]

    (E) describe the basic purpose of financial institutions, including keeping money safe, borrowing money, and lending.

§111.7. Grade 5, Adopted 2012.

(a) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on computational thinking, mathematical fluency , and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select

appropriate tools such as real objects, manipulatives, algorithms, paper and pencil, and technology and techniques such as mental math, estimation, [and] number sense , and generalization and abstraction to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, computer programs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3)   For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 5 are expected to perform their work without the use of calculators.

(4)   The primary focal areas in Grade 5 are solving problems involving all four operations with positive rational numbers, determining and generating formulas and solutions to expressions, and extending measurement to area and volume. These focal areas are supported throughout the mathematical strands of number and operations, algebraic reasoning, geometry and measurement, and data analysis. In Grades 3-5, the number set is limited to positive rational numbers. In number and operations, students will apply place value and identify part-to-whole relationships and equivalence. In algebraic reasoning, students will represent and solve problems with expressions and equations, build foundations of functions through patterning, identify prime and composite numbers, and use the order of operations. In geometry and measurement, students will classify two-dimensional figures, connect geometric attributes to the measures of three-dimensional figures, use units of measure, and represent location using a coordinate plane. In data analysis, students will represent and interpret data.

(5)   Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(b)   Knowledge and skills.

(1)   Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A)   apply mathematics to problems arising in everyday life, society, and the workplace;

(B)   use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C)   select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D)   communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E)   create and use representations to organize, record, and communicate mathematical ideas;

(F)   analyze mathematical relationships to connect and communicate mathematical ideas; and

(G)   display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2)   Number and operations. The student applies mathematical process standards to represent, compare, and order positive rational numbers and understand relationships as related to place value. The student is expected to:

(A)   represent the value of the digit in decimals through the thousandths using expanded notation and numerals;

(B)   compare and order two decimals to thousandths and represent comparisons using the symbols >, <, or =; and

(C)   round decimals to tenths or hundredths.

(3)   Number and operations. The student applies mathematical process standards to develop and use strategies and methods for positive rational number computations in order to solve problems with efficiency and accuracy. The student is expected to:

(A)   estimate to determine solutions to mathematical and real-world problems involving addition, subtraction, multiplication, or division;

(B)   [use strategies and algorithms, including the standard algorithm, to] multiply with fluency a three-digit number by a two-digit number using the standard algorithm ;

(C)   [use strategies and algorithms, including the standard algorithm, to] solve with proficiency [fluency] for quotients of up to a four-digit dividend by [and] a two-digit divisor using strategies and the standard algorithm ;

(D)   represent multiplication of decimals with products to the hundredths using objects and pictorial models, including area models;

(E)   solve for products of decimals to the hundredths, including situations involving money, using strategies based on place-value understandings, properties of operations, and the relationship to the multiplication of whole numbers;

(F)   represent quotients of decimals to the hundredths, up to four-digit dividends and two-digit whole number divisors, using objects and pictorial models, including area models;

(G)   solve for quotients of decimals to the hundredths, up to four-digit dividends and two-digit whole number divisors, using strategies and algorithms, including the standard algorithm;

(H)   represent and solve addition and subtraction of fractions with unequal denominators referring to the same whole using objects and pictorial models [such as strip diagrams] and properties of operations;

(I)   represent and solve multiplication of a whole number and a fraction that refers to the same whole using objects and pictorial models, including area models;

(J)   represent division of a unit fraction by a whole number and the division of a whole number by a unit fraction such as $1/3 \div 7$ and $7 \div 1/3$ using objects and pictorial models, including area models;

(K)   add and subtract positive rational numbers fluently; and

(L)   divide whole numbers by unit fractions and unit fractions by whole numbers.

(4)   Algebraic reasoning. The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:

(A)   identify prime and composite numbers [using patterns in factor pairs] ;

(B)   represent and solve multi-step problems involving the four operations with whole numbers using equations with a letter standing for the unknown quantity;

(C)   generate a numerical pattern when given a rule in the form $y = ax$ or $y = x + a$ and graph;

(D)   recognize the difference between additive and multiplicative numerical patterns given in a table or graph;

(E)   describe the meaning of parentheses and brackets in a numeric expression [~~such as 4 (14 + 5) is 4 times as large as (14 + 5)~~] ;

(F)   simplify numerical expressions that do not involve exponents, including up to two levels of grouping [~~such as (3 + 7) / (5 - 3)~~] ;

(G)   use concrete objects and pictorial models to develop the formulas for the volume of a rectangular prism, including the special form for a cube ($V = l$ x w x h, $V = s$ x s x s, and $V = Bh$); and

(H)   represent and solve problems related to perimeter and/or area [~~such as rectangles and composite figures formed by rectangles~~] and related to volume [~~such as rectangular prisms~~] .

(5)   Geometry and measurement. The student applies mathematical process standards to classify two-dimensional figures by attributes and properties. The student is expected to classify two-dimensional figures in a hierarchy of sets and subsets using graphic organizers based on their attributes and properties [~~such as all rectangles have the property that opposite sides are parallel; therefore, every rectangle is a parallelogram~~] .

(6)   Geometry and measurement. The student applies mathematical process standards to understand, recognize, and quantify volume. The student is expected to:

(A)   recognize a cube with side length of one unit as a unit cube having one cubic unit of volume and the volume of a three-dimensional figure as the number of unit cubes (n cubic units) needed to fill it with no gaps or overlaps if possible; and

(B)   determine the volume of a rectangular prism with whole number side lengths in problems related to the number of layers times the number of unit cubes in the area of the base.

(7)   Geometry and measurement. The student applies mathematical process standards to select appropriate units, strategies, and tools to solve problems involving measurement. The student is expected to solve problems by calculating conversions within a measurement system, customary or metric.

(8)   Geometry and measurement. The student applies mathematical process standards to identify locations on a coordinate plane. The student is expected to:

(A)   describe the key attributes of the coordinate plane , including perpendicular number lines (axes) where the intersection (origin) of the two lines coincides with zero on each number line and the given point (0, 0); the x-coordinate, the first number in an ordered pair, indicates movement parallel to the x-axis starting at the origin; and the y-coordinate, the second number, indicates movement parallel to the y-axis starting at the origin; [~~and the process for graphing ordered pairs of numbers in the first quadrant; and~~]

(B)   describe the process for graphing ordered pairs of numbers in the first quadrant of the coordinate plane; and

(C) [~~(B)~~]graph in the first quadrant of the coordinate plane ordered pairs of numbers arising from mathematical and real-world problems, including those generated by number patterns or found in an input-output table.

(9)   Data analysis. The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:

(A)   represent categorical data with bar graphs or frequency tables and numerical data, including data sets of measurements in fractions or decimals, with dot plots or stem-and-leaf plots;

      (B)   represent discrete paired data on a scatterplot; and

      (C)   solve one- and two-step problems using data from a frequency table, dot plot, bar graph, stem-and-leaf plot, or scatterplot.

(10)   Personal financial literacy. The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:

      (A)   define income tax, payroll tax, sales tax, and property tax;

      (B)   explain the difference between gross income and net income;

      (C)   identify the advantages and disadvantages of different methods of payment, including check, credit card, debit card, and electronic payments;

      (D)   develop a system for keeping and using financial records;

      (E)   describe actions that might be taken to balance a budget when expenses exceed income; and

      (F)   balance a simple budget.