Prepared by the State Board of Education (SBOE) TEKS Review Committees

Final Recommendations, October 2011

In 2010-2011 the Commissioner's Mathematics Advisory Group was convened to offer recommendations regarding the next generation of mathematics standards in Texas. *The Commissioner's Draft of the Texas Mathematics Standards* reflects the recommendations of the Commissioner's Mathematics Advisory Group and a panel of national advisors in mathematics. The SBOE-appointed mathematics TEKS review committees used *The Commissioner's Draft of the Texas Mathematics Standards* as a starting point for their recommendations for revisions to the TEKS.

These proposed revisions reflect the recommended changes of the committees to the standards in *The Commissioner's Draft of the Texas Mathematics Standards.* Proposed additions are shown in green font with underlines (additions) and proposed deletions are shown in red font with strikethroughs (~~deletions~~). Changes recommended based on a vertical alignment review are shown in brown font (additions or ~~deletions~~).

Comments in the right-hand column provide explanations for the proposed changes. The following notations were used as part of the explanations:

BSG—information added, changed, or deleted based on broad-strokes guidance from the SBOE

CRS—information added or changed to align with the Texas College and Career Readiness Standards (CCRS)

ER—information added, changed, or deleted based on expert reviewer feedback

IF—information added, changed, or deleted based on informal feedback

MV—multiple viewpoints from within the committee

SBOE—information added, changed, or deleted based on SBOE feedback

VA—information added, changed, or deleted to increase vertical alignment

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Algebra I

Algebra I (one credit)		
Prerequisite: Mathematics, Grade 8 (or its equivalent)		
Recommended Grade Level: Grade 8 or Grade 9		
~~Mathematical Process Standards Algebra I~~		
~~I.~~	~~Apply mathematics to problems arising in everyday life, society and the workplace.~~	VA—Process Standards moved to knowledge and skills statements
~~II.~~	~~Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.~~	
~~III.~~	~~Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.~~	
~~IV.~~	~~Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.~~	
~~V.~~	~~Create and use representations to organize, record, and communicate mathematical ideas.~~	
~~VI.~~	~~Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.~~	

Algebra I Focal Areas
Linear functions, equations and inequalities
Quadratic functions, equations and inequalities
Exponential functions, equations and inequalities
Number and algebraic methods

Algebra I

Introduction

The desire to achieve education excellence is the driving force behind the Texas Essential Knowledge and Skills for mathematics, guided by the Texas College and Career Readiness Standards. By embedding statistics, probability, and financial literacy, while focusing on fluency and deep understandings, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

The process standards are integrated at every grade level. When possible, students will apply mathematics to problems arising in everyday life, society and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process as well as the reasonableness of the solution. They will select appropriate tools, including real objects, manipulatives, paper and pencil, and technology and techniques, such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language. They will use mathematical relationships to generate solutions and make connections and predictions. Students will create and use representations to organize, record, and analyze mathematical relationships to connect and communicate mathematical ideas. They will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written and oral communications.

In Algebra I, students will build on grade 6-8 Mathematics Texas Essential Knowledge and Skills (TEKS), which provide a foundation in linear relationships, number and operations, and proportionality. Students will study linear, quadratic, and exponential functions and their related transformations, equations, and associated solutions. Students will connect functions and their associated solutions in both mathematical and real-world situations. Students will use technology to collect and explore data and analyze statistical relationships. In addition, students will study polynomials of degree one and two, radical expressions, sequences, and laws of exponents. Students will generate and solve linear systems with two equations and two variables and will create new functions through transformations.

Mathematical Process Standards		
Knowledge and Skills Statement. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:		
	apply mathematics to problems arising in everyday life, society, and the workplace	VA—Process Standards moved to knowledge and skills statements
	use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution	
	select tools, including ~~such as~~ real objects, manipulatives, paper/pencil, and technology as appropriate, and ~~or~~ techniques, including ~~such as~~ mental math, estimation, and number sense as appropriate, to solve problems	

	communicate mathematical ideas, reasoning, and their implications using multiple representations, including ~~such as~~ symbols, diagrams, graphs, and language as appropriate	
	create and use representations to organize, record, and communicate mathematical ideas	VA—Process Standards moved to knowledge and skills statements
	analyze mathematical relationships to connect and communicate mathematical ideas	
	display, ~~E~~explain, and ~~or~~ justify mathematical ideas and arguments using precise mathematical language in written or oral communication~~s~~	

Linear Functions, Equations, and Inequalities.		**A1L**
Knowledge and Skills Statement. The student applies the Mathematical Process Standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:		
A1L01	determine the domain and range of a linear function in mathematical problems ~~and real-world problems~~ and determine reasonable domain and range values for real-world situations, both continuous and discrete	We wanted to include real-world situations, so we needed to add "continuous" and "discrete." Note that "independent" and "dependent" quantities are addressed in middle school. The phrase "real-world situations" is needed here to provide context for determining reasonableness.
A1L05	write ~~an equation of a line~~ linear equations in two variables in various forms including $y = mx + b$, ~~$ax + by = c$,~~ $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope, and given two points.	We pluralized the subject of the sentence.
A1L02	~~generate~~ write linear equations in two variables ~~for mathematical and real-world problems~~ given a table of values, given a graph, and given a verbal description.	"Mathematical and real-world problems" are included in the Mathematical Process Standards and have been deleted per expert reviewers' comments.
new	write and solve equations involving direct variation.	Moved from Math 8.
new	write linear equations in two variables that contain a given point and are parallel to a given line.	Changes incorporated based on Informal Feedback.
new	write linear equations in two variables that contain a given point and are perpendicular to a given line.	Changes incorporated based on Informal Feedback.
A1L06	write ~~an equation of a line~~ linear equations in two variables ~~is~~ that are parallel and lines that are ~~or~~ perpendicular to the x- ~~or~~ and to the y-axis, ~~including determining~~ and determine whether ~~its slope is~~ their slopes are ~~0~~ zero or undefined.	We pluralized the subject of the sentence and added clarity to the SE.

A1L03	~~generate~~ write linear inequalities in two variables ~~for mathematical and real-world problems~~ given a table of values, given a graph, and given a verbal description.	"Mathematical and real-world problems" are included in the Mathematical Process Standards and have been deleted per expert reviewers' comments.
A1L04	~~generate~~ write systems of two linear equations ~~for mathematical and real-world problems~~ given a table of values, given a graph, and given a verbal description.	"Mathematical and real-world problems" are included in the Mathematical Process Standards and have been deleted per expert reviewers' comments.
~~**Coordinate Geometry**~~		
Knowledge and Skills Statement. The student applies the Mathematical Process Standards when using graphs of linear functions, their key features, and their related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:		
A1L07.1	determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$. ~~express slope as a rate of change for a linear function represented~~ with a table, a graph, and an equation ~~tabularly, graphically, and algebraically~~	We moved this SE to the strand "Linear Functions, Equations, and Inequalities" to emphasize the connection of slope to other representations besides algebraic. Change in language made in consideration of comments provided by Student Assessment.
A1L07.2	calculate the rate of change of a linear function ~~, given as a table, as a graph, or as an equation,~~ represented tabularly, graphically, and algebraically over a specified interval within ~~a~~ mathematical ~~or~~ and real-world problems.	We moved this SE to the strand "Linear Functions, Equations, and Inequalities" to emphasize the connection of slope to other representations besides algebraic.
A1L08	graph ~~a~~ linear functions on the coordinate plane and identify key features including *x*-intercept, *y*-intercept, zeros, and slope in mathematical and real-world problems.	We pluralized the subject of the sentence. Change in language made in consideration of comments provided by Student Assessment.
A1L10	graph the solution set ~~to~~ of ~~a~~ linear inequalit~~y~~ies in two variables on the coordinate plane.	"Set" added per expert recommendation. We pluralized the subject of the sentence.

A1L07	determine the effects on the graph of the parent ~~linear~~ function $f(x) = x$ when $f(x)$ is replaced by $a \cdot f(x)$, $f(x) + d$, and $f(x - c)$, $f(b \cdot x)$ for specific values of a, b, c and d.	We added clarity and incorporated expert reviewer's comments to add in vocabulary "parent functions." Professional development note: we believe that teachers may need additional professional development on the concept of determining the effect of b and c. Transformations are here to align vertically and to prepare students for college and career readiness.
new	graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist	We added this SE because we felt it was missing.
A1L09	~~approximate~~ estimate graphically the solutions ~~graphically~~ to ~~a~~ systems of two linear equations with two variables in ~~mathematical and~~ real-world problems	We made grammatical changes.
A1L11	graph the solution set ~~to~~ of ~~a~~ systems of two linear inequality~~y~~ies in two variables on the coordinate plane	"Set" added per expert recommendation. We pluralized the subject of the sentence.
~~**Linear Functions and Data**~~		
Knowledge and Skills Statement. The student applies the Mathematical Process Standards to formulate statistical relationships and evaluate their reasonableness based on real-world data. The student is expected to:		
A1L12	~~determine~~ calculate, using technology, the correlation coefficient between two quantitative variables and interpret this quantity as a measure of the strength of the linear association ~~between two quantitative variables~~	We made grammatical changes. This SE is introduced in Algebra I to increase rigor and to align to college and career readiness.
A1L13	~~differentiate between~~ compare and contrast association and causation in real-world problems	Notes for professional development: we want teachers to differentiate between association and causation in real-world problems. A strong correlation does not imply a cause and effect relationship.

A1L14	write, with and without technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems ~~determine, when appropriate, a linear equation that provides a reasonable fit to bivariate data in a scatter plot to approximate solutions to real-world problems and make predictions~~	Change made per suggestion of expert reviewer.
~~**Solving Linear Equations, Inequalities, and Systems of Equations**~~		
Knowledge and Skills Statement. The student applies the Mathematical Process Standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:		
A1L15	solve linear equations ~~for mathematical and real-world problems~~ in one variable, including those for which the application of the distributive property is necessary and includes variables on both sides.	This SE increases the rigor from middle school related to solving equations with one variable. In addition, solving one equation with two variables is covered in the SE on literal equations and in the SE on writing linear equations with two variables in various forms.
A1L16	~~determine the reasonableness, including using the appropriate units, of a solution to a linear equation as applied to mathematical and real-world problems.~~	This has been deleted because it is included in the Mathematical Process Standards.
A1L17	solve linear inequalities in ~~two~~ one variable~~s~~, including ~~solving inequalities~~ those for which the application of the distributive property is necessary and ~~involves~~ includes variables on both sides ~~of the inequality~~.	This SE increases the rigor from middle school related to solving equations with one variable. In addition, solving one equation with two variables is covered in the SE on literal equations and in the SE on writing linear equations with two variables in various forms.
A1L18	~~determine the reasonableness, including using the appropriate units, of a solution to linear inequalities as applied to mathematical and real-world problems.~~	This has been deleted because it is included in the Mathematical Process Standards.
A1L19	solve ~~algebraically~~ ,using substitution and Gaussian elimination, systems of two linear equations with two variables for mathematical and real-world problems.	We made grammatical revisions and added language per Student Assessment.
A1L20	~~determine the reasonableness, including using the appropriate units, of a solution to a system of linear equations as applied to mathematical and real-world problems.~~	This has been deleted because it is included in the Mathematical Process Standards.

Quadratic Functions~~,~~ and Equations~~, and Inequalities.~~		A1Q
Knowledge and Skills Statement. The student applies the Mathematical Process Standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. The student is expected to:		
~~Representation~~		
A1Q01	determine the domain and range of ~~a~~ quadratic functions ~~in mathematical and real-world problems~~.	We pluralized the subject of the sentence.
~~A1Q02~~	~~apply the Remainder Theorem to a quadratic function. [For a quadratic polynomial q(x) and a number a, the remainder on division of q(x) by x – a is q(a), so q(a) = 0 if and only if (x – a) is a factor of q(x).]~~	In conversations with the Algebra II team, it was decided that this would be covered in Algebra II.
A1Q03	write equations of quadratic functions given the vertex and another point on the graph, write this equation in vertex form ($f(x) = a(x – h)^2 + k$), and then rewrite this equation from vertex form to standard form ($f(x) = ax^2 + bx + c$). ~~write the equations of a quadratic functions in various standard and vertex forms including $f(x) = ax^2 + bx + c$ and $f(x) = a(x – h)^2 + k$.~~	We pluralized the subject of the sentence. The change from "standard" and "vertex" to the actual equation form was made per expert reviewer comment, and is consistent with wording in the "Linear Functions, Equations, and Inequalities" strand of this document. Clarification as to what properties of quadratic functions should be used to write equations was provided per comment from Student Assessment.
A1Q06.5	~~determine~~ write quadratic functions when given ~~the roots~~ real solutions ~~or~~ and graphs of their related equations	We moved this from the strand "Numerical and Algebraic Methods" to the strand "Quadratic Functions and Equations" so that the SE could be logically connected to solutions and representations of quadratic equations and functions. This has been limited to real solutions per expert and informal feedback.
~~Coordinate Geometry~~		

Knowledge and Skills Statement. The student applies the Mathematical Process Standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:		
A1Q06	graph ~~a~~ quadratic functions on the coordinate plane and use the graph to ~~determine~~ identify key attributes ~~features~~, if possible, including *x*-intercept, *y*-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry ~~in mathematical and real-world problems~~	We pluralized the subject of the sentence. Additional statements were placed per comments from Student Assessment.
A1Q05	~~relate~~ describe the relationship between the linear factors of ~~a~~ quadratic expressions ~~to~~ and the zeros of ~~the~~ their associated quadratic functions	We clarified the verb per expert request. We pluralized the subject of the sentence.
A1Q04	determine the effects on the graph of the parent ~~quadratic~~ function $f(x) = x^2$ when $f(x)$ is replaced by $a \cdot f(x)$, $f(x) + d$, $f(x - c)$, $f(K \cdot x)$ for specific values of a, b, c and d.	Professional development note: we believe that teachers may need additional professional development on the concept of determining the effect of b and c. We pluralized the subject of the sentence.
Knowledge and Skills Statement. The student applies the Mathematical Process Standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:		
~~Solving Equations~~		

A1Q07	solve quadratic equations, having real ~~roots~~ solutions ~~in mathematical and real-world problems,~~ by ~~inspection (e.g., such as $x^2 = a^2$),~~ factoring, taking square roots, completing the square, and applying the quadratic formula.	We changed "roots" to "solutions" to vertically align with Algebra II. Note for Professional Development: teachers should use concrete objectives (such as algebra tiles) to develop students' conceptual understanding of completing the square. "Mathematical and real-world problems" are included in the Mathematical Process Standards and have been deleted per expert reviewers' comments. The phrase "by inspection" has been deleted per expert reviewers' comments because $x^2 = a^2$ is a special case of solving by completing the square.
A1Q08	~~determine the reasonableness, including using the appropriate units, of a solution to a quadratic equation applied mathematical and real-world problems.~~	This has been deleted because it is included in the Mathematical Process Standards.
new	write, using technology, quadratic functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.	We added this because it is missing and because parallel SEs exist in the linear section and in the exponential section.

~~Other~~ Exponential Functions~~,~~ and Equations~~, and Inequalities~~.	**A1E**
Knowledge and Skills Statement. The student applies the Mathematical Process Standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations, and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data. The student is expected to:	
~~Representation~~	
A1E01	determine the domain and range of ~~an~~ exponential functions of the form $f(x) = a \cdot b^x$ ~~in mathematical and real-world problems.~~
	We pluralized the subject of the sentence.

A1E02	interpret the meaning of the values of a and b in ~~an~~ exponential functions of the form $f(x) = a \cdot b^x$ in ~~mathematical and~~ real-world problems.	We pluralized the subject of the sentence. An expert suggested that we include the meaning of the variable x. We feel that professional development activities should discuss the meaning of x and $f(x)$, and make connections to law of exponents and domain and range.
A1E03	~~generate~~ write ~~an~~ exponential functions in the form $f(x) = a \cdot b^x$ (where b is a rational number) to describe problems arising from mathematical and real-world situations including growth and decay.	We pluralized the subject of the sentence.
A1E04	graph ~~an~~ exponential functions that models growth ~~or~~ and decay and ~~determine~~ identify key features, including ~~x intercept,~~ y-intercept~~,~~ and asymptotes, in mathematical and real-world problems.	We pluralized the subject of the sentence and added clarity.
A1E05	write, using technology, exponential functions that provide a reasonable fit to data and make predictions for real-world problems ~~identify an exponential function that approximately fits data graphed on a scatter plot to approximate solutions for real-world problems~~.	Change made per suggestion of expert reviewer.

Number and Algebraic Methods.		**A1A**
Knowledge and Skills Statement: The student applies the Mathematical Process Standards and algebraic methods to rewrite in equivalent forms, and perform operations on, polynomial expressions. The student is expected to:		
A1A11	add and subtract determine ~~evaluate the sum, and difference, and product~~ ~~of~~ polynomials of degree one ~~or~~ and degree two.	We split A1A11 so that sum and difference would be considered separately from product of polynomials, as per expert recommendation.
A1A11.5	multiply ~~evaluate determine the sum, difference, and product of~~ polynomials of degree one ~~or~~ and degree two.	
A1A12	determine ~~evaluate~~ the quotient of a polynomial of degree ~~of~~ one ~~or~~ and polynomial of degree two when divided by a polynomial of degree one ~~or~~ and polynomial of degree two.	We made revisions to add clarity and to incorporate expert suggestions.
A1A17	rewrite ~~transform~~ polynomial expressions ~~with~~ of degree ~~of~~ one ~~or~~ and degree two ~~to~~ in equivalent forms using the distributive property, such as rewriting $(4x)(x - 2)$ as $(4x)(x) - (4x)(2)$, and then writing it as $4x^2 - 8x$, or $4x^2 - 8x$ to $(4x)(x) - (4x)(2)$, and then factoring the result as $(4x)(x - 2)$.	We made revisions to add clarity and to incorporate expert suggestions.
A1A13	~~determine the factors of a polynomial of degree one or two and write the polynomials in factored form~~.	This has been removed as it is now covered in A17 and A14.

A1A14	factor, if possible, ~~determine the factors of simple~~ trinomials with real factors in the form $ax^2 + bx + c$, including perfect square trinomials of degree two.	We made revisions to add clarity and to incorporate expert suggestions.
A1A16	decide ~~determine~~ if a binomial can be written as the difference of two squares and, if possible, use the structure of a difference of two squares to rewrite it, ~~transform it to illustrate this structure,~~ such as rewriting the expression $49x^4 - y^4$ ~~=~~ to $(7x^2)^2 - (y^2)^2$, ~~=~~ and then factoring it as $(7x^2 + y^2)(7x^2 - y^2)$.	We made revisions to add clarity and to incorporate expert suggestions.
Knowledge and Skills Statement: The student applies the Mathematical Process Standards and algebraic methods to rewrite algebraic expressions into equivalent forms. The student is expected to:		
A1A01	simplify numerical ~~transform~~ radical expressions involving square roots ~~to solve mathematical and real-world problems~~.	Specified numerical based on conversations with geometry and expert reviewer.
A1A09	simplify numeric and ~~transform~~ algebraic expressions using the laws of ~~integer~~ exponents, including integral and rational exponents.	09 and 10 were combined and 10 deleted per expert review Numeric added to align with Algebra II.
~~A1A10~~	~~extend previous understandings of the laws of integral exponents to the corresponding laws for rational exponents.~~	
Knowledge and Skills Statement: The student applies the Mathematical Process Standards and algebraic methods to write, solve, analyze, and evaluate equations, relations and functions. The student is expected to:		
A1A02	decide ~~determine~~ whether ~~a~~ relations represented ~~with words, a table, graph, or symbols~~ verbally, tabularly, graphically, and symbolically define~~s~~ a function.	In professional development, note that "symbolically" includes algebraic, mapping and set notation representations.
A1A06	evaluate ~~determine the value of a linear, quadratic, or exponential~~ function~~s~~, expressed in function notation, given ~~an~~ one or more element~~s~~ in ~~its~~ their domains. ~~such as finding f(2) if f(x) = x + 4.~~	We pluralized the subject of the sentence and added clarity; the example is not necessary.
A1A07	identify terms of ~~an~~ arithmetic ~~or~~ and geometric sequence~~s~~ when the sequence~~s~~ ~~is~~ are given in function form ~~or~~ and given in recursive form.	We pluralized the subject of the sentence and added clarity.
A1A08	write ~~find a formula for the general~~ a formula for the nth term of ~~an~~ arithmetic ~~or~~ and geometric sequence~~s~~, given the value of several of ~~its~~ their terms.	We pluralized the subject of the sentence and added verb clarification. Additional revision made per comment from expert reviewer.
A1A18	solve ~~a literal equation~~ mathematic and scientific formulas, and other literal equations, for a specified variable.	Revision made per comment from expert reviewer.

~~A1A03~~	~~calculate the rate of change of a linear function , given as a table, as a graph, or as an equation, represented tabularly, graphically, or algebraically over a specified interval within a mathematical or real-world problem.~~	We moved this to the strand "Linear Functions, Equations, and Inequalities."
~~A1A04~~	~~express slope as a rate of change for a linear function represented with a table, a graph, and an equation tabularly, graphically, or algebraically.~~	We moved this to the strand "Linear Functions, Equations, and Inequalities."
~~A1A05~~	~~determine the slope of a line given the standard form of a line.~~	We moved this to the strand "Linear Functions, Equations, and Inequalities."
~~A1A15~~	~~determine a quadratic function when given the roots or graph of its related equation.~~	We moved to A1Q02b.

Algebra II

Mathematical Process Standards Algebra II		
I.	Apply mathematics to problems arising in everyday life, society and the workplace.	VA—Process Standards moved to knowledge and skills statements
II.	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.	
III.	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.	
IV.	Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.	
V.	Create and use representations to organize, record, and communicate mathematical ideas.	
VI.	Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.	

Algebra II Focal Areas
Attributes of functions and their inverses
Systems of equations and inequalities
Quadratic and square root functions, equations and inequalities
Exponential and logarithmic functions and equations and inequalities
Cubic, cube root, absolute value, and rational functions, equations and inequalities
Number and algebraic methods
Data analysis

Algebra II

Introduction

General requirements. Students shall be awarded one-half to one credit for successful completion of this course. Prerequisite: Algebra I.

The desire to achieve educational excellence is the driving force behind the Texas Essential Knowledge and Skills for mathematics guided by the Texas College and Career Readiness Standards. By embedding statistics, probability, finance, and focusing on fluency and solid understandings, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

The process standards are integrated at every grade level. When possible students will apply mathematics to problems arising in everyday life, society and the workplace. Students will use a problem solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process. They will select tools such as real objects, manipulatives, paper and pencil, and technology or techniques such as mental math, estimation, reasonableness, and number sense to solve problems. Emphasis will be on communication of mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, language, and various notation including interval and inequality notation. Students will create and use representations to organize, record, and communicate mathematical ideas. They will explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

In Algebra II students build on the foundations from K-8 and Algebra I. Students broaden their knowledge of quadratic functions, exponential functions and systems of equations. They study logarithmic, square root, cubic, cube root, absolute value, rational functions and their related equations. Students connect functions to their inverses and to their associated equations and solutions in both mathematical and real world situations. In addition, students extend their knowledge of data analysis and numeric and algebraic methods.

Mathematical Process Standards		
Knowledge and Skills Statement. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:		
	apply mathematics to problems arising in everyday life, society, and the workplace	VA—Process Standards moved to knowledge and skills statements
	use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, ~~and~~ evaluating the problem-solving process and the reasonableness of the solution	
	select tools, including ~~such as~~ real objects, manipulatives, paper/pencil, ~~and~~ technology as appropriate, and ~~or~~ techniques, including ~~such as~~ mental math, estimation, and number sense as appropriate, to solve problems	
	communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	

	create and use representations to organize, record, and communicate mathematical ideas	
	analyze mathematical relationships to connect and communicate mathematical ideas	
	display, ~~E~~explain, ~~display,~~ and justify mathematical ideas and arguments using precise mathematical language in written or oral communication~~s~~	

Attributes of Functions and their Inverses.		**A2F**
Knowledge and Skills Statement. The student applies mathematical processes to understand that functions have distinct key attributes and to understand the relationship between a function and its inverse. The student is expected to:		
A2F01	graph the functions ~~$f(x) = b^x, and f(x) = log_b(x),$~~ $f(x) = \|x\|, f(x) = \sqrt{x}, f(x) = \frac{~~a~~ 1}{~~x~~ x}, f(x) = x^3,$ $f(x) = \sqrt[3]{x}$, $f(x) = b^x, and f(x) = log_b(x)$ where b is 2, 10 and e, and when applicable ~~determine~~ analyze the key attributes such as domain, range, intercepts , symmetries, ~~and~~ asymptotic behavior, and relative ~~maximums and minimums~~ maxima and minima given an interval. ~~For the functions $f(x) = b^x$ and f(x)=logb(x), b is 2, 10 or and e.~~	ER, Clarity
A2F04	graph and write the inverse of a function~~, if it exists, by reflection across the line $y = x$.~~	Implied
A2F03	~~explain~~ describe and analyze the relationship between a function and its inverse (quadratic and square root, logarithmic and exponential) ~~if it exists,~~ including the restrictions on domains and ranges. ~~(Include quadratic, square root, logarithmic and exponential functions.)~~	Clarity
A2F02	use ~~determine~~ the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other.	Clarity
A2F05	~~graph step and other piecewise-defined functions, including the greatest integer function, and when applicable determine key attributes such as domain, range and symmetry in mathematical and real-world problems.~~	Moved to Pre-Calculus; depth vs breadth focus on mastery of significant non-linear functions including quadratic, exponential, logarithmic, square root, cubic, cube root, rational and absolute value; breaking these functions into pieces will be mastered in Pre-Calculus

Systems of Equations and Inequalities.	**A2L**
Knowledge and Skills Statement. The student applies mathematical processes to formulate systems of equations and inequalities, to use a	

variety of methods to solve, and to analyze reasonableness of solutions. The student is expected to:		
A2L01	~~generate~~ formulate systems of equations ~~for mathematical and real-world problems~~, including systems consisting of three linear equations in three ~~unknowns~~ variables and systems consisting of two equations, the first linear and the second quadratic.	ER; Consistency
A2L03	~~represent a system of linear equations using a matrix in mathematical and real-world problems. ,and explain why it might be an advantage to replace the system by the matrix~~.	ER; subsumed in A2L01
A2L04	solve systems of three linear equations ~~with~~ in three variables ~~algebraically~~ by methods such as elimination, using technology with matrices, and substitution. ~~in mathematical and real-world problem. (Include the use of algebraic methods and matrices.)~~.	Clarity; feedback; AIL19 specifically states "using algebraic methods, including substitution and Gaussian elimination." A2 requires a different level of algebraic manipulation not restricted to any specific method
A2L08	solve, algebraically, systems of two equations in two variables ~~made up~~ consisting of a linear equation and a quadratic equation ~~in mathematical and real-world problems~~.	Clarity
A2L05	~~determine the reasonableness, including using the appropriate units, of solutions to systems of three linear equations in three variables. in mathematical and real-world problems.~~	Reasonableness of SE
A2L09	determine the reasonableness~~, including using the appropriate units,~~ of solutions to systems of a linear equation and a quadratic equation in two variables ~~in mathematical and real-world problems~~.	Clarity
A2L02	~~generate~~ formulate systems of at least two linear inequalities in two variables ~~to solve mathematical and real-world problems.~~	Clarity
A2L06	solve systems of two or more linear inequalities ~~with~~ in two variables ~~in mathematical and real-world problems both algebraically and using matrices~~	Clarity
A2L07	determine ~~the reasonableness, including using the appropriate units, of~~ possible solutions in the solution set of ~~to~~ systems of two or more linear inequalities in two variables ~~in mathematical and real-world problems~~.	Clarity

Quadratic~~,~~ and Square Root~~, Cubic and Cube Root~~ Functions, Equations, and Inequalities.	**A2Q**
Knowledge and Skills Statement. The student applies mathematical processes to understand that quadratic and square root functions and quadratic inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:	
A2Q01	~~generate~~ write the ~~a~~ quadratic function ~~with graph having a given vertex and axis of symmetry, and~~given three specified points in the plane ~~generate a quadratic function with a graph that contains two or more specified points in the plane~~.	Clarity – encompassed in A2Q07

A2Q07	~~generate~~ write the equation of a parabola using given attributes ~~that may include~~ including vertex, focus, directrix, axis of symmetry, and direction of opening ~~and focal width in mathematical and real-world problems~~.	ER
A2Q06	determine the effect on the graph~~s~~ of $f(x) = \sqrt{x}$~~, $f(x) = x^3$, and $f(x) = \sqrt[3]{x}$~~ when *f(x)* is replaced by *a f(x), f(x) + d, f(bx),* ~~or~~ and *f(~~x+c~~ x-c)* for specific positive and negative values of *a,b,c,* and *d.*	Moved to new strand
A2Q05	~~Rewrite~~ transform a quadratic function $f(x) = ax^2 + bx + c$ ~~in~~ to the form $f(x) = a(x-h)^2 + k$ ~~to reveal~~ to identify the different ~~properties~~ attributes of *f(x)* ~~in mathematical and real-world problems~~.	Clarity – reveal is too vague, transform and attribute maintains consistency
~~A2Q02~~	~~generate square root functions for mathematical and real-world problems.~~	Embedded in A2Q04
A2Q04	~~generate~~ formulate quadratic~~,~~ and square root~~, cubic, and cube root~~ equations ~~for real-world problems~~.	Separated strands
A2Q08	solve quadratic and square root equations ~~that may have real or complex roots in mathematical and real-world problems.~~	Consistency
A2Q11	~~determine the reasonableness, including using the appropriate units, of a~~ identify extraneous solutions of ~~to a~~ square root ~~or cube root~~ equations ~~in mathematical and real-world problems~~.	Moved to new strand; clarity
A2Q03	~~generate~~ solve quadratic inequalities ~~for mathematical and real-world problems~~.	Clarity
A2Q09	~~give examples showing how extraneous solutions may arise with quadratic equations in real-world problems.~~	MV; Reasonableness of SE
A2Q10	~~solve cube root equations that have real solutions or complex roots in mathematical and real-world problems.~~	Moved to new strand

Exponential and Logarithmic Functions~~,~~ and **Equations**~~, and Inequalities~~**.**		**A2E**
Knowledge and Skills Statement. The student applies mathematical processes to understand that exponential and logarithmic functions can be used to model situations and solve problems. The student is expected to:		
A2E02	determine the effects on the key attributes on the graphs of $f(x) = b^x$ and $f(x) = log_b(x)$ where b is 2, 10 and e when *f(x)* is replaced by *a f(x), f(x) + d,* ~~f(bx), or~~ and *f(~~x+c~~ x-c)* for specific positive and negative values of *a,~~b,~~c* and *d.*	ER; Consistency with AI and PreCalculus
A2E01	~~generate~~ formulate exponential and logarithmic equations that model real-world situations.	Clarity

A2A08	rewrite ~~transform~~ exponential ~~expressions~~ equations as ~~to~~ their corresponding logarithmic ~~expressions~~ equations and logarithmic equations ~~expressions~~ as ~~to~~ their corresponding exponential ~~expressions~~ equations ~~in mathematical and real-world problems.~~	Incorrect terminology
A2E03	solve exponential equations of the form $y = a \cdot b^x$ where a is a nonzero real number and b is greater than zero and not equal to 1and single logarithmic equations that have real ~~roots~~ solutions ~~in mathematical and real-world problems~~.	Consistency
A2E04	~~determine the reasonableness, including using the appropriate units, of a solution to an exponential equation in mathematical and real-world problems.~~	Implied in solving
A2E05	determine the reasonableness~~, including using the appropriate units,~~ of a solution to a logarithmic equation ~~in mathematical and real-world problems~~.	Clarity

~~Quadratic, Square Root,~~ Cubic, ~~and~~ Cube Root, Absolute Value and Rational Functions, Equations, and Inequalities.	**A2**
Knowledge and Skills Statement. The student applies mathematical processes to understand that cubic, cube root, rational, and absolute value functions and inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:	
~~A2Q06~~ A2C02	~~determine~~ analyze the effect on the graphs of ~~$f(x) = \sqrt{x}$,~~ $f(x) = x^3$, ~~and~~ $f(x) = \sqrt[3]{x}$ when *f(x)* is replaced by *a· f(x),* ~~f(x) + d,~~ *f(bx),* ~~or f(x+c~~ x-c*)*, and f(x) + d for specific positive and negative values of *a, b, c,* and *d.*	Moved to new strand, Specificity
~~A2Q04~~	~~generate quadratic, square root, cubic, and cube root equations for real-world problems.~~	ER – emphasize quadratic/linear systems (RA)
~~A2Q10~~ A2C04	solve cube root equations ~~that have real or complex roots in mathematical and real-world problems~~.	ER
~~A2Q11~~	~~determine the reasonableness, including using the appropriate units, of a solution to a square root or cube root equation in mathematical and real-world problems~~.	Informal feedback, contained in process standards
~~A2Q08~~ A2C03	~~solve quadratic and square root equations that may have real or complex roots solutions in mathematical and real-world problems.~~	Moved to new strand
~~A2E07~~ A2C07	~~determine~~ analyze the effect on the graphs of $f(x) = \|x\|$ when *f(x)* is replaced by *a· f(x),* ~~f(x) + d,~~ *f(bx),* ~~or f(x+c~~ x-c*)*, and f(x) + d for specific positive and negative values of *a, b, c* and *d.*	Consistency, Specificity
~~A2E06~~ A2C06	~~generate~~ formulate absolute value linear equations that model ~~mathematical and~~ real-world situations.	ER

~~A2E08~~ A2C08	solve absolute value linear equations ~~that have real or complex roots in mathematical and real-world problems~~.	Reasonableness of SE
~~A2E09~~ A2C09	solve absolute value linear inequalities ~~in mathematical and real-world problems~~.	Process Standards
~~A2E11~~ A2C11	~~determine~~ analyze the effect on the graphs of $f(x) = \frac{a}{x}\frac{1}{x}$, when *f(x)* is replaced by *a· f(x),* ~~f(x) + d,~~ *f(bx),* ~~or~~ *f(~~x+c~~* x-c*),* and f(x) + d for specific positive and negative values of *a, b, c,* and *d.*	ER, Consistency, specificity
~~A2E10~~ A2C10	~~generate~~ formulate rational equations that model ~~mathematical and~~ real-world situations.	Clarity
~~A2E12~~ A2C12	solve rational equations that have real solutions ~~or complex roots in mathematical and real-world problems~~.	Reasonableness of SE
~~A2E13~~ A2C13	determine the reasonableness~~, including using the appropriate units,~~ of a solution to a rational equation ~~in mathematical and real-world problems~~.	Clarity
~~A2E14~~ A2C14	~~generate examples showing how extraneous solutions may arise with rational equations in real-world problems.~~ determine the restrictions on the domain of a rational function	Reasonableness of SE
new	formulate and solve ~~inverse variation~~ equations involving inverse variation	VA

Number and Algebraic Methods.		**A2A**
Knowledge and Skills Statement. The student applies mathematical processes to simplify and perform operations on expressions and to solve equations. The student is expected to:		
A2A02	~~apply the properties of matrix addition, matrix subtraction, scalar multiplication of a matrix and matrix multiplication in mathematical and real-world problems.~~	ER; Feedback; Expert testimony
A2A01	~~use the relation i2 = –1 and the commutative, associative, and distributive properties to~~ add, subtract, and multiply complex numbers.	Clarity
A2A04	~~determine the sum, difference, and product of~~ add, subtract, and multiply polynomials. ~~in mathematical and real-world problems~~.	Supporting skill for generating and solving equations.

		apply the Binomial Theorem with coefficients determined by Pascal's Triangle to expand binomials of the form $(x + y)^n$, where n is a positive integer less than or equal to 5. Four expert reviewers agreed Binomial Theorem fit best in Pre-Calculus. The Binomial Theorem is an advanced topic for the math standards for MA, Common Core, CA and Singapore. For the National Math Panel, the Binomial Theorem is grouped with finite probability, Pascal's Triangle and binomial coefficients which reside in TX Pre-Calculus.
A2A05	~~determine the quotient of a polynomial divided by a binomial in mathematical and real-world problems, including quotients with remainders.~~ determine the quotient of a polynomial of degree three and of degree four when divided by a polynomial of degree one and of degree two.	VA Algebra I; MV
A2A06	~~apply the Remainder Theorem to~~ determine the linear factors of a polynomial function of degree three and of degree four using algebraic methods such as the Remainder Theorem	ER; VA Algebra I
A2A10	determine linear and quadratic factors of a polynomial expression of degree three and of degree four, including factoring the sum and difference of two cubes and factoring by grouping~~, when suitable factorizations are available~~.	Parallels with A2A05
A2A09	determine the sum, difference, product and quotient of ~~simple~~ rational expressions with integral exponents of degree one and of degree two ~~and including determining the restrictions on the domain. in mathematical and real-world problems.~~	Specificity; Supporting skill for generating and solving equations.
A2A07	~~transform~~ rewrite radical expressions that contain variables to equivalent forms. ~~in mathematical and real-world and problems.~~	Supporting skill for generating and solving equations.
A2A03	~~transform~~ solve equations involving rational exponents. ~~algebraic expressions~~	VA Algebra I
~~A2A08~~	~~transform exponential expressions to their corresponding logarithmic expressions and logarithmic expressions to their corresponding exponential expressions. in mathematical and real-world problems.~~	Moved to new strand
new	write the domain and range of a function in interval notation	VA

Data.		**A2D**
Knowledge and Skills Statement. The student applies mathematical processes to analyze data, select appropriate models, write corresponding functions, and make predictions. The student is expected to:		
~~A2D01~~	~~when appropriate, use the mean and standard deviation of a data set to fit a normal distribution and to approximate normal population percentages using tools such as calculators, spreadsheets and tables.~~	moved to MMA
~~A2D02~~	~~recognize that there are data sets for which it is not appropriate to model with a normal distribution.~~	moved to MMA
~~A2D03~~	~~determine whether data from generating process~~ such as simulation ~~are consistent with a specified model.~~	moved to MMA
~~A2D04~~	~~distinguish the purposes and differences among sample surveys, experiments and observation studies including explaining the role of randomization in each type of study and the scope of inference from each type of study.~~	moved to MMA
~~A2D05~~	~~use data from a sample survey to estimate population mean or population proportion.~~ including developing the margin of error through the use of simulation models for random sampling.	moved to MMA
~~A2D06~~	~~use data from a randomized experiment to compare two treatments and use simulation to decide if the observed differences are statistically significant.~~	moved to MMA
~~A2D07~~	~~determine the strengths and weaknesses of reports based on data. when solving problems in real-world situations.~~	moved to MMA
new	analyze data to select the appropriate model from among linear, quadratic and exponential models	ER; moved from MMA
new	use regression methods available through technology to write a linear function, a quadratic function and an exponential function from a given set of data	ER; moved from MMA
new	predict and make decisions and critical judgments from a given set of data using linear, quadratic, and exponential models	ER

Geometry

~~Mathematical Process Standards Geometry~~		
~~I.~~	~~Apply mathematics to problems arising in everyday life, society and the workplace.~~	VA—Process Standards moved to knowledge and skills statements
~~II.~~	~~Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.~~	
~~III.~~	~~Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.~~	
~~IV.~~	~~Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.~~	
~~V.~~	~~Create and use representations to organize, record, and communicate mathematical ideas.~~	
~~VI.~~	~~Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.~~	

Geometry Focal Areas
~~Representations connecting algebra and geometry~~ Coordinate and Transformational Geometry
~~Logical argument, proof, congruence and constructions~~ Logical argument and constructions
Proof and congruence
~~Similarity and Trigonometry~~ Similarity, Proof, and Trigonometry
~~Problem solving with surface area and volume~~ Two- and Three-dimensional figures
~~Basic theorems about circles~~ Circles
Probability

Geometry (1 credit)

General requirements: prerequisite Algebra I

Introduction

The College and Career readiness standards are the driving force behind the Texas Essential Knowledge and Skills for mathematics. Maintaining a focus on fluency and solid understandings and by embedding statistics and finance, Texas will lead the way in mathematics education to prepare all Texas students for the challenges they will face in the 21st century.

The process standards integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

In Geometry, students build on the foundations from K-8 and Algebra I to strengthen their mathematical reasoning skills in geometric contexts. Within the course, students will begin to focus on more precise terminology, symbolic representations, and the development of proofs. Students will explore concepts covering coordinate and transformational geometry; logical argument and constructions; proof and congruence; similarity, proof, and trigonometry; two- and three- dimensional figures; circles; and probability. Students will connect previous knowledge from Algebra I to Geometry through the coordinate and transformational geometry strand. In the logical arguments and constructions strand, students are expected to create formal constructions using a straight edge and compass. Though this course is primarily Euclidean geometry, students should complete the course with an understanding that non-Euclidean geometries exist. In proof and congruence, students will use deductive reasoning to justify, prove and apply theorems about geometric figures. Throughout the standards, to "prove" means a formal proof to be shown in a paragraph, flow chart, or two-column formats. Proportionality is the unifying component of the similarity, proof and trigonometry strand and students will use their proportional reasoning skills to prove and apply theorems and solve problems in this strand. The two- and three-dimensional figure strand focuses on the application of formulas in multi-step situations because students have developed their background knowledge in two-and three-dimensional figures. Utilizing patterns to identify geometric properties, students will apply theorems about circles to determine relationships between special segments and angles in circles. Due to the emphasis of probability and statistics in the College and Career Readiness Standards, standards dealing with probability have been added to the Geometry curriculum to ensure students have proper exposure to these topics before pursuing their post-secondary education.

These standards are meant to provide clarity and specificity in regards to the content covered in the high school Geometry course. These standards are not meant to limit the methodologies utilized to convey this knowledge to students. Though the standards are written in a particular order, they are not necessarily meant to be taught in the given order. In the standards, the phrase "to solve problems" includes both contextual and non-contextual problems unless specifically stated.

Mathematical Process Standards		
Knowledge and Skills Statement. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:		
	apply mathematics to problems arising in everyday life, society, and the workplace	VA—Process Standards moved to knowledge and skills statements
	use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution	
	Select tools, including ~~such as~~ real objects, manipulatives, paper/pencil, and technology as appropriate, and ~~or~~ techniques, including ~~such as~~ mental math, estimation, and number sense as appropriate, to solve problems	
	communicate mathematical ideas, reasoning, and their implications using multiple representations, including ~~such as~~ symbols, diagrams, graphs, and language as appropriate	
	create and use representations to organize, record, and communicate mathematical ideas	
	analyze mathematical relationships to connect and communicate mathematical ideas	
	display, ~~E~~explain, and ~~or~~ justify mathematical ideas and arguments using precise mathematical language in written or oral communication~~s~~	
Coordinate and Transformational Geometry ~~Representations: Connecting Algebra and Geometry.~~		**GA**
Knowledge and Skills Statement. The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to:		
GA01	determine the coordinates of a point that is a given fractional distance less than one from one end of a line segment to the other in one- and two-dimensional coordinate systems ~~the coordinate plane~~, including finding the midpoint	ER - Askey
GA03	derive and use the distance, slope, and midpoint formulas to verify ~~prove~~ geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of lines~~, using coordinates and algebraic methods~~	MV, ER- identified specific techniques to verify geometric relationships Proof have been moved to a different section of the standards
GA02	determine an equation ~~with graph~~ of a line parallel or perpendicular to a given line ~~and~~ that passes through a given point	ER ER - John
~~GA04~~	~~determine the equation of a parabola given its focus and directrix~~	This SE is addressed in Algebra II. Per ER, more appropriately placed there.
~~GA05~~	~~solve problems with geometric contexts arising from mathematical and real-world situations that include symbolic representations~~	ER - This is more of a process standard

Knowledge and Skills Statement. The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:		
GG07	describe and perform ~~identify~~ transformations of figures in a plane using coordinate ~~function~~ notation, i.e. (x, y) →(-x, y)~~, taking points in the plane as inputs and giving other points as outputs~~	Moved from Logical Arguments and Constructions ER – Verb changes MV & IF
GG08	determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations ~~(translation, reflection, rotation) or~~, a composition of non-rigid transformations, and a composition of both, ~~(dilations that preserve similarity and reductions and enlargements that do not preserve similarity). (Sequences~~ include~~e~~ing dilations where the center can be any point in the plane~~)~~	Moved from Logical Argument and Constructions MV ER - Askey
GG09	identify the sequence of ~~Euclidean~~ transformations ~~including rotations and reflections~~ that will carry ~~the image of~~ a given ~~figure~~ pre-image onto an image on and off the coordinate plane ~~itself in a given number of steps~~	Moved from Logical Argument and Constructions ER – Schmid, Askey
New SE	identify and distinguish between reflectional and rotational symmetry in a plane figure	CRS – B2A ER - Ross

Logical Argument~~,~~ ~~Proof, Congruence~~ and Constructions.		**GG**
Knowledge and Skills Statement. The student uses the process skills with inductive reasoning to understand geometric relationships. The student is expected to:		
GG01	distinguish between undefined terms, definitions, postulates, conjectures, and theorems ~~using mathematical induction and deductive reasoning~~	
GG02	identify and determine the validity of the converse, inverse, and contrapositive of a conditional statement and recognize the connection between a biconditional statement and a true conditional statement with a true converse	MV, CRS
GG03	verify that a conjecture is false using a counterexample~~s~~.	Grammar "s" removed per Askey
GG06	compare ~~identify key differences between~~ geometric relationships ~~within~~ between Euclidean and spherical geometries~~. (Include~~ , including parallel lines and the sum of the angles in a triangle~~)~~	MV

Knowledge and Skills Statement. The student uses constructions to validate conjectures about geometric figures. The student is expected to:		
GG04	investigate patterns to make conjectures about geometric relationships, including angles formed by parallel lines cut by a transversal, criteria required for triangle congruence, special segments of triangles, diagonals of quadrilaterals, interior and exterior angles of polygons, and special segments and angles of circles ~~represent formal geometric constructions~~ choosing from a variety of tools such as compass and straightedge, paper folding and dynamic geometric software ~~(Constructions include duplicating a line segment; duplicating an angle; constructing an angle bisector; finding the midpoint of a line segment; finding a line parallel or perpendicular to a given line through a point not on the line; and constructing the perpendicular bisector of a line segment.)~~	CRS, Constructions were moved to GG05 ER MV
GG05	construct congruent segments, congruent angles, a segment bisector, an angle bisector, perpendicular lines, the perpendicular bisector of a line segment, and a line parallel to a given line through a point not on a line using a compass and a straightedge ~~represent the construction of an equilateral triangle, a square or a regular hexagon inscribed in a circle choosing from a variety of tools such as compass and straightedge, paper folding and dynamic geometric software~~	MV Common Core
GG05.5	use the constructions of congruent segments, congruent angles, angle bisectors, and perpendicular bisectors to make conjectures about geometric relationships	Extension of GG05
GG12	verify the Triangle Inequality theorem using constructions and apply the theorem to solve problems ~~use the fact that the sum of the measures of the lengths of any two sides of a triangle is greater than the measure of the length of the third side (Triangle Inequality theorem) in mathematical and real-world problems~~	ER IF

~~Logical Argument,~~ Proof~~,~~ and Congruence ~~and Constructions.~~		**GG**
Knowledge and Skills Statement. The student uses the process skills with deductive reasoning to prove and apply theorems by utilizing a variety of methods (coordinate, transformational, axiomatic) and formats (two-column, paragraph, flow chart). The student is expected to:		
GG13	prove theorems about ~~the relationships between line segments, lines, and~~ angles ~~that are~~ formed by the intersection of lines and line segments, including vertical angles, angles formed by parallel lines cut by a transversal, and equidistance between the endpoints of a segment and points on its perpendicular bisector, ~~choosing from various formats of proof such as paragraph, flow, two-column, coordinate or transformational~~, and ~~use~~ apply these relationships to solve problems ~~(Theorems include vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; and points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.)~~	Moved from Logical Argument and Constructions ER
GG11	prove ~~whether~~ two triangles are congruent by applying the Side-Angle-Side, Angle-Side-Angle, ~~AAS or~~ Side-Side-Side, Angle-Angle-Side, and Hypotenuse-Leg ~~triangle~~ congruence conditions	Moved from Logical Argument and Constructions ER & IF
GG10	apply the definition of congruence, in terms of rigid transformations, to identify congruent figures and their corresponding sides and angles ~~using the definition of congruence in terms of rigid motions~~	Moved from Logical Argument and Constructions ER

GG14	prove theorems about the ~~angle~~ relationships in triangles, including the sum of interior angles, base angles of isosceles triangles, midsegments, and medians ~~choosing from various formats of proof such as paragraph, flow, two-column, coordinate or transformational,~~ and apply ~~use~~ these relationships to solve problems ~~(Theorems include measures of interior angles of a triangle sum to 180°; base angles of an isosceles triangle are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.)~~	Moved from Logical Argument and Constructions ER MV
GG15	prove a quadrilateral is a parallelogram, rectangle, square, or rhombus using opposite sides, opposite angles, or diagonals ~~theorems about parallelograms choosing from various formats of proofs such as paragraph, flow, two-column, coordinate or transformational,~~ and apply ~~use~~ these relationships to solve problems ~~(Theorems include opposite sides are congruent; opposite angles are congruent; the diagonals of a parallelogram bisect each other; and rectangles are parallelograms with congruent diagonals~~	Moved from Logical Argument and Constructions ER - Askey MV & IF

Similarity, Proof, and Trigonometry.	**GS**
Knowledge and Skills Statement. The student uses the process skills in applying similarity to solve problems. The student is expected to:	
GS01 apply the definition of similarity in terms of a dilation to identify similar figures and their ~~similarity transformation to determine whether two figures are similar including identifying~~ proportional sides and the congruent corresponding angles	MV
GS02 apply the Angle-Angle criterion to verify similar triangles and apply the proportionality of the corresponding sides to solve ~~problems mathematical and real-world and mathematical~~ problems.	ER IF
Knowledge and Skills Statement. The student uses the process skills with deductive reasoning to prove and apply theorems by utilizing a variety of methods (coordinate, transformational, axiomatic) and formats (two-column, paragraph, flow chart). The student is expected to:	
GS03 prove theorems about similar triangles, including the Triangle Proportionality theorem, ~~choosing from various formats of proof such as paragraph, flow, two-column, coordinate or transformational,~~ and apply ~~use~~ these theorems to solve problems ~~(Theorems include a line parallel to one side of a triangle divides the other two proportionally and conversely and the Pythagorean theorem proved using triangle similarity)~~	ER MV
GS04 identify and apply the relationships that exist when an ~~prove the theorem that the length of the~~ altitude is drawn to the hypotenuse of a right triangle, including ~~is~~ the geometric mean, ~~between the lengths of the segments on the hypotenuse, choosing from various formats of proof such as paragraph, flow, two-column, coordinate, or transformational, and use this theorem~~ to solve problems	ER - Askey MV IF
Knowledge and Skills Statement. The student uses the process skills to understand and apply relationships in right triangles. The student is expected to:	
GS04.5 use similarity of right triangles and the Pythagorean theorem to develop the relationships between the angles and the sides of right triangles, leading to the definitions of the trigonometric ratios sine, cosine and tangent	ER – Askey, JW, Rath, Ross,
GS05 determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine and tangent ~~in mathematical and real-world~~ to solve problems	ER MV IF

GS06	apply the relationships in special right triangles (30°-60°-90° and 45°-45°-90°) and the Pythagorean theorem, including Pythagorean triples, ~~in mathematical and real-world~~ to solve problems	MV IF

~~Measurement~~ **Two-dimensional and three-dimensional figures.**		**GM**
Knowledge and Skills Statement. The student uses the process skills to recognize characteristics and dimensional changes of two- and three-dimensional figures. The student is expected to:		
~~GM01~~	~~use appropriate units of measure to solve real-world problems, including conversions between measurement systems~~	Embedded in standards GM04, GM05, GM06, GM07
GM02	identify the shapes of two-dimensional cross-sections of prisms, pyramids, cylinders, cones and spheres and identify three-dimensional objects ~~and identify three-dimensional objects~~ generated by rotations of two-dimensional shapes ~~objects~~	Grammar IF – TASM
GM03	determine and describe how changes in the linear dimensions of a shape affect its perimeter, area, surface area, or volume, including proportional and non-proportional dimensional change ~~in mathematical problems~~	Clarify IF
Knowledge and Skills Statement. The student uses the process skills in the application of formulas to determine measures of two- and three-dimensional figures. The student is expected to:		
GM04	apply the formula for ~~determine~~ the area of regular polygons ~~and the area of composite two-dimensional figures in mathematical and real-world~~ to solve problems using appropriate units of measure	MV, added new standard GM07
new SE	determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure	Created new standard from GM04
GM05	apply the formulas for ~~determine~~ the total and lateral surface area ~~(where applicable)~~ of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres and composite figures, ~~in mathematical and real-world~~ to solve problems using appropriate units of measure ~~(These figures include prisms, pyramids, cones, cylinders, spheres and composite figures. Dimensions may be labeled with single variables.)~~	MV This standard, though mostly covered in 7th and 8th grade, is included because students haven't used the formulas for surface area of pyramids, cones or spheres until HS geometry.
GM06	apply the formulas for ~~determine~~ the volume of three-dimensional figures, including prisms, pyramids, cones, cylinders, spheres and composite figures, ~~in mathematical and real-world~~ to solve problems using appropriate units of measure ~~(These figures include prisms, pyramids, cylinders, cones, spheres, and composite figures. Dimensions may be labeled with single variables.)~~	MV This standard, though mostly covered in 7th and 8th grade, is included because composite figures, polygonal prisms, and pyramids are new figures for students to compute volume.

Circles.	**GC**
Knowledge and Skills Statement. The student uses the process skills to understand geometric relationships and apply theorems and equations about circles. The student is expected to:	

GC01	apply ~~prove~~ theorems about circles, including relationships among ~~inscribed~~ angles, radii, chords, tangents, and secants ~~lines, and line segments, and use these relationships~~ to solve non-contextual problems	CRS, ER These theorems are developed in GG04 when students use patterns to make conjectures about geometric relationships.
GC02	apply the proportional relationship between the measure of an arc length of a circle and the circumference of the circle to solve ~~in mathematical and real-world~~ problems~~. This includes the ratio of the length of an arc intercepted by a central angle and the radius of the circle and the radian measure of an angle~~	Grammar IF
GC03	apply the proportional relationship between the measure of the area of a sector of a circle and the area of the circle to solve ~~in mathematical and real-world~~ problems	IF
GC002.5	describe radian measure of an angle as the ratio of the length of an arc intercepted by a central angle and the radius of the circle	IF
GC04	show that the equation of a circle with center at the origin and radius r is $x^2 + y^2 = r^2$ and determine the equation for the graph of a circle with radius r and center (h, k), $(x - h)^2 + (y - k)^2 = r^2$ ~~and justify the derivation of this equation using the Pythagorean theorem and properties of translations~~	ER, Present in Pre- Calculus Askey

Probability.	**GD**
Knowledge and Skills Statement. The student uses the process skills to understand probability in real world situations and how to apply independence and dependence of events. The student is expected to:	
GD08	~~apply the formulas for permutations and combinations to solve real-world problems.~~ ~~Recognize differences in permutations and combinations and develop strategies for solving each.~~ develop strategies to use permutations and combinations to solve contextual problems	Recommendation by SBOE Chairperson
GD01	determine probabilities based on area ~~in mathematical and real-world~~ to solve contextual problems ~~[Obtain the probability measure by taking the measure (area) of a subset and dividing it by the measure (area) of the entire set]~~	Relates to CRS ER, VA IF
~~GD02~~	~~represent events as subsets of a sample space using the characteristics of the outcomes or as unions, intersections or complements of other events in mathematical and real-world problems~~	Relates to CRS IF Merged with GD03
GD03	identify whether two events are independent and ~~give an example of how~~ compute the probability of the two events occurring together with or without replacement ~~is the product of their probabilities~~	Relates to CRS ER – Rath VA
~~GD04~~	~~interpret results in a two-way frequency table of data when the two variables are related~~	ER, MV,VA – not present in CRS Moved to AQR
~~GD05~~	~~treating a two-way frequency table as a sample space, identify whether two events are independent and determine conditional probabilities~~	ER, MV,VA – not present in CRS Moved to AQR

GD06	apply conditional probability ~~of A given B and independence~~ in contextual ~~real-world~~ problems	Relates to CRS, clarity
GD06.5	apply independence in ~~real-world~~ contextual problems	ER - Rath
~~GD07~~	~~use the Addition rule, *P* (*A* or *B*) = *P* (*A*) + *P* (*B*) in mathematical and real-world problems~~	ER, MV,VA – not present in CRS Moved to AQR

Precalculus

~~Mathematical Process Standards Precalculus~~		
~~I.~~	~~Apply mathematics to problems arising in everyday life, society and the workplace.~~	VA—Process Standards moved to knowledge and skills statements
~~II.~~	~~Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.~~	
~~III.~~	~~Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.~~	
~~IV.~~	~~Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.~~	
~~V.~~	~~Create and use representations to organize, record, and communicate mathematical ideas.~~	
~~VI.~~	~~Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.~~	

Precalculus Focal Areas
Functions
~~**Geometric reasoning**~~ **Relations and Geometric Reasoning**
~~**Measurement**~~ **Number and Measure**
~~**Number and algebraic methods**~~ **Algebraic Reasoning**

Precalculus

Introduction

The prerequisites for Precalculus are that students have successfully completed two years of algebra and one year of geometry.

The College and Career Readiness Standards are the driving force behind the Texas Essential Knowledge and Skills for mathematics. By embedding statistics and finance and focusing on fluency and deep understandings, Texas will lead the way in mathematics education and prepare all Texas students for the technological challenges they will face in the 21st century.

The process standards are integrated at every grade level. When possible, students will apply mathematics to problems arising in everyday life, society and the workplace. Students will use a problem solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving problem process. They will select tools such as real objects, manipulatives, paper and pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems. Communication of mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs and language will be emphasized. Students will create and use representations to organize, record, and communicate mathematical ideas. They will explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

Precalculus is the preparation for calculus. The course takes a functional point of view towards topics and is designed to strengthen and enhance conceptual understanding and mathematical reasoning used when modeling and solving mathematical and real-world problems. Students systematically work with functions and their multiple representations. The study of the topics, concepts, and procedures of precalculus deepens students' understanding of algebra and extends their ability to apply algebra concepts and procedures at higher conceptual levels, as a tool for future study in mathematics. Students investigate and explore mathematical ideas, develop multiple strategies for analyzing complex situations, and use technology such as graphing calculators and mathematical software to build understanding, make connections between representations, and provide support in solving problems.

Mathematical Process Standards

Knowledge and Skills Statement. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

	apply mathematics to problems arising in everyday life, society, and the workplace	VA—Process Standards moved to knowledge and skills statements
	use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution	
	select tools, including ~~such as~~ real objects, manipulatives, paper/pencil, and technology as appropriate, and ~~or~~ techniques, including ~~such as~~ mental math, estimation, and number sense as appropriate, to solve problems	
	communicate mathematical ideas, reasoning, and their implications using multiple representations, including ~~such as~~ symbols, diagrams, graphs, and language as appropriate	

	create and use representations to organize, record, and communicate mathematical ideas	
	analyze mathematical relationships to connect and communicate mathematical ideas	VA—Process Standards moved to knowledge and skills statements
	display, explain, ~~display,~~ and justify mathematical ideas and arguments using precise mathematical language in written or oral communication~~s~~	

Functions.		**PF**
Knowledge and Skills Statement. The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real world problems. The student is expected to:		
PF01	use the composition of two functions to model and solve real-world problems	
PF02	~~give an example~~ demonstrate that function composition is not always commutative	Clarity of language
PF03	represent a given function as a composite function of two or more functions, ~~For example, $f(x)=\sqrt{x^2+3}$ can be represented as $f(x)=(g\circ h)(x)$ where $g(x)=\sqrt{x}$ and $h(x)=x^2+3$, or f can be represented as $f(x)=(g\circ w\circ v)(x)$ where $g(x)=\sqrt{x}$, $w(x)=x+3$, and $v(x)=x^2$.~~	Instructional example not needed
PF04	describe symmetry of graphs of even and odd functions ~~in mathematical and real-world problems~~	ER
PF05	determine an inverse function, when it exists, for a given function over its domain or a subset of its domain and represent the inverse ~~graphically and/or algebraically.~~ using multiple representations	Current verbiage is too limiting
PF06	graph exponential ~~functions~~, ~~and~~ logarithmic ~~functions (including base e), trigonometric functions~~, ~~piece-wise defined functions~~, rational ~~functions~~, polynomial, power, ~~and~~ trigonometric, inverse trigonometric ~~functions~~, and piece-wise defined functions, including step functions	Base e is implied VA Added Algebra 2 SE A2FO5
PG03	graph functions, including exponential, logarithmic, sine, cosine, rational, polynomial, and power functions ~~logarithmic functions with various bases, including the natural log function,~~ and their transformations including $a \cdot f(x)$, $f(x) + d$, $f(x - c)$, $f((b \cdot x)$ for specific values of a, b, c, and d, in mathematical and real-world problems	Combining SE PG03, PG04, PG07, PG16 for continuity of transformations. The real world problems will be addressed in modeling. SE was moved from Geometric Reasoning Focal Area to Function Focal Area to better delineate the focal areas.

PG17	graph ~~inverse trigonometric functions (~~arcsin *x*~~,~~ and arccos *x*~~)~~ ~~with and without technology~~ ~~including explaining why there is a need for restricted domains and ranges in mathematical and real-world problems.~~ and describe the limitations on the domain	SE clarification on the expectation and eliminated the instructional suggestion SE was moved from Geometric Reasoning Focal Area to Function Focal Area to better delineate the focal areas.
PF07	determine and analyze the key features of exponential ~~functions~~, ~~and~~ logarithmic ~~functions (including base e)~~, ~~trigonometric functions~~, ~~piece-wise defined functions,~~ rational ~~functions~~, polynomial, power, ~~and~~ trigonometric, inverse trigonometric ~~functions~~, and piece-wise defined functions including step functions such as domain, range, symmetry, relative maximum, relative minimum, zeros, asymptotes, and intervals over which the function is increasing or decreasing	Base e is implied VA Added Algebra 2 SE A2FO5
PG04	~~graph power functions (including radical) and their transformations including~~ analyze and describe ~~the concept of~~ end behavior of functions including exponential, logarithmic, rational, polynomial, and power functions using infinity notation to communicate this characteristic in mathematical and real-world problems	The original SE excluded all functions except power functions. SE was moved from Geometric Reasoning Focal Area to Function Focal Area to better delineate the focal areas.
PG05	~~graph~~ analyze characteristics of rational functions ~~and determine characteristics such as domain,~~ and the behavior of the function around the asymptotes including horizontal, vertical and oblique asymptotes. ~~(horizontal, vertical, slant) and describe the differences between the domains of the rational functions (p(x) s(x)) / (q(x) s(x)) and p(x) / q(x) for p, q and s polynomial functions in mathematical and real-world problems~~	Focus the SE on particular characteristics of rational functions SE was moved from Geometric Reasoning Focal Area to Function Focal Area to better delineate the focal areas.
PG06	determine various types of discontinuities in the interval $(-\infty, \infty)$ as they relate to ~~rational~~ functions, such as rational and piecewise defined functions, and explore the limitations of the graphing calculator as it relates to the behavior of the function around discontinuities. ~~in mathematical and real-world problems~~	The original SE was limiting the discussion of discontinuities to rational functions. ER SE was moved from Geometric Reasoning Focal Area to Function Focal Area to better delineate the focal areas.
	describe the left-sided behavior and the right-sided behavior of the graph of a function around discontinuities	Discussion of discontinuities as alternative to limits concept
	analyze situations modeled by functions including exponential, logarithmic, rational, polynomial, and power functions to solve real-world problems such as problems involving growth and decay and optimization	SE was created to address the real-world problems for multiple functions that were in PG05, PG07, PA10

PG15	~~determine whether a situation can be modeled by a sinusoidal function, develop a mathematical model to describe the situation, and use the model to solve mathematical and real-world problems.~~ develop and use a sinusoidal function that models a situation in mathematical and real-world problems	Clarification of expectation SE was moved from Geometric Reasoning Focal Area to Function Focal Area to better delineate the focal areas.
PM02	determine the values of the trigonometric functions at the special angles ~~(30°, 45°, 60°) and the angles, such as the half-angles,~~ and relate~~d to~~ them in mathematical and real-world problems	Removed the examples SE was moved from Measurement Focal Area to Function Focal Area to better delineate the focal areas.

Relations and Geometric Reasoning ~~Geometric Reasoning~~.		**PG** More inclusive and descriptive focal area name
Knowledge and Skills Statement. The student uses the process standards in mathematics to model and make connections between algebraic and geometric relations. The student is expected to:		
PG01	graph a set of parametric equations	Moved SE within Focal Area with common content
PG02	convert parametric equations into rectangular relations and convert rectangular relations into parametric equations. ~~to solve mathematical and real-world problems~~	As written the standard is too limiting so it is moved to a SE PA15
PA15	use parametric equations to model and solve ~~problems involving motion in~~ mathematical and real-world problems	Moving the solve from SE PG02 and removing limiting feature SE was moved from Number and Algebraic Methods Focal Area to Relations and Geometric Reasoning Focal Area to better delineate the focal areas.
PG07	~~graph exponential functions and their transformations, including for specific values of *a*, *b*, *c*, and *d*, to solve problems in mathematical and real-world problems~~	SE was addressed in PG03
PG08	graph points in the polar coordinate system and convert between ~~the~~ rectangular coordinates and polar coordinate~~s~~ ~~systems in mathematical and real-world problems.~~	ER
PG09	graph polar equations such as cardiods, limaçons, or lemniscates by plotting points and using technology~~. using symmetry, using zeros and maximum values including recognizing special polar graphs~~	concepts are beyond introduction level of polar
PG13	determine the conic section formed when a plane intersects a double napped cone	Moved SE within Focal Area for dilineation

PG10	make connections between the locus definition of conic sections and their equations in rectangular coordinates. ~~derive, in rectangular coordinates, the equation of a circle, parabola, ellipse, and hyperbola from their locus definitions~~	Clarification of expectation
PG11	use the characteristics of an ellipse to write the equation of an ellipse with center (*h*,*k*). ~~and determine the foci and eccentricity in mathematical and real-world problems.~~	Clarification of expectation ER
PG12	use the characteristics of a hyperbola to write the equation of a hyperbola with center (*h*,*k*) ~~and determine the foci, eccentricity and the equations of the asymptotes in mathematical and real-world problems.~~	Clarification of expectation ER
PG16	~~graph the sine and cosine functions and apply one or more transformations to these functions, including *a* · *f(x), f(x) + d, f(x – c), f(b · x)* for specific values of *a*, *b*, *c* and *d* in mathematical and real-world problems~~	SE was combined with PG03
PG18	~~estimate the limit of a function at a point, including one-sided limits, using graphs and tables.~~	Duplicates the content of Calculus Replace with discussion of discontinuities
PG19	~~illustrate cases in which a limit of a function fails to exist at a point or as *x* grows without bound, including unequal left-hand and right-hand limits at a point, unbounded behavior, and oscillating behavior.~~	Duplicates the content of Calculus
PG20	~~use knowledge of the limiting process to describe the behavior of a function including end behavior.~~	Duplicates the content of Calculus
PG21	~~explain, informally, why a limit fails to exist at a point or as *x* grows without bound, including unequal left-hand and right-hand limits at a point, unbounded behavior, and oscillating behavior.~~	Duplicates the content of Calculus
PG22	~~solve problems requiring an understanding of the limiting process in mathematical and real-world problems.~~	Duplicates the content of Calculus

Number and Measure~~ment~~.		**PM** More inclusive and descriptive focal area name
Knowledge and Skills Statement. The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real world problems. The student is expected to:		
PM01	determine the relationship between the unit circle~~, the wrapping function (W(x)=(cos x, sin x)),~~ and the definition of a periodic function to evaluate trigonometric functions in mathematical and real world problems	eliminated the instructional suggestion
PM06	~~Identify radian measure of a central angle of a unit circle as the length of the arc subtended by that angle.~~ describe the relationship between degree and radian measure on the unit circle	Clarification of expectation
PM07	represent angles in radians ~~and~~ or degrees based on the concept of rotation and find the measure of reference angles and angles in standard position ~~with a common terminal side in mathematical and real-world problems involving arc length, linear and angular speeds and area of the sector of a circle~~	Separate SE into two SE's

	represent angles in radians or degrees based on the concept of rotation in mathematical and real-world problems, including linear and angular velocity	Separate SE PM07
PM03	determine~~, using reference angles,~~ the value of trigonometric ratios of ~~any~~ angles, ~~including~~ and ~~solving~~ solve problems involving trigonometric ratios ~~involving points on the terminal side of an angle~~ in mathematical and real world problems	Clarification of expectation Moved SE within Focal Area with common content
PM08	use trigonometry ~~to determine directional bearing and harmonic motion~~ in mathematical and real world problems, including directional bearing.	Rephrasing of the SE
PM04	use the Law of Sines in mathematical and real-world problems.	Moved SE within Focal Area with common content
PM05	use the Law of Cosines in mathematical and real-world problems.	Moved SE within Focal Area with common content
PG23	use vectors to model situations involving magnitude and direction.	SE was moved from Geometric Reasoning Focal Area to Number and Measure Focal Area to better delineate the focal areas.
PG24	represent the addition of vectors and the multiplication of a vector by a scalar geometrically and symbolically.	SE was moved from Geometric Reasoning Focal Area to Number and Measure Focal Area to better delineate the focal areas.
PG25	apply vector addition and multiplication of a vector by a scalar in mathematical and real-world problems.	SE was moved from Geometric Reasoning Focal Area to Number and Algebraic Focal Area to better delineate the focal areas.

Algebraic Reasoning ~~Number and Algebraic Methods~~.		**PA**
Knowledge and Skills Statement. The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:		
PA01	represent finite sums and infinite series using sigma notation.	
	expand finite sums and infinite series written in sigma notation	Separate SE PA02 for clarification
PA02	evaluate finite sums and geometric series when possible written in sigma notation. ~~calculate the value, when it exists, of an expression written in sigma notation.~~	ER
PA03	represent arithmetic sequences and geometric sequences ~~and series~~ using ~~recursion~~ recursive formulas. ~~and sigma notation.~~	Clarification of SE and combine series with SE PA05

PA04	calculate the nth term and the nth partial sum of an ~~determine the nth terms and the sum of a finite~~ arithmetic series in mathematical and real-world problems.	Consistent wording with SE PA06
PA05	represent arithmetic series and geometric ~~sequences and~~ series using ~~a recursion formula and~~ sigma notation.	Clarification of SE and combine sequence with SE PA03
PA06	calculate the n^{th} term of a geometric series, the n^{th} partial sum of a geometric series, and sum of ~~a~~ an infinite geometric series when it ~~this sum~~ exists.	ER and clarification of SE
PA11	~~use the Binomial Theorem to write the expression $(a + b)^n$ (n a positive integer) in expanded form.~~ apply the Binomial Theorem for the expansion of $(a+ b)^n$ in powers of a and b for a positive integer n, where a and b are any numbers	Reworded for specificity Moved SE within Focal Area with common content
PA07	~~determine the trigonometric form of a complex number and relate to polar coordinates~~	SE is not a prerequisite for Calculus
PA08	~~determine the product and quotient of complex numbers in trigonometric form~~	SE is not a prerequisite for Calculus
PA09	~~determine powers and all the n^{th} roots of complex numbers~~	SE is not a prerequisite for Calculus
PA10	use the properties of logarithms to evaluate or transform logarithmic expressions ~~requiring the change of base formula in both mathematical and real-world problems~~	ER and the change of base statement leads to limitations Moved SE within Focal Area with common content
PA12	~~use Pascal's Relation (triangle) to give a recursive definition of the coefficient $a^p b^{n-p}$ in the expansion of $(a+b)n$~~	ER
PA13	generate and solve logarithmic equations ~~including those requiring change of base~~ in mathematical and real-world problems	ER
PA14	generate and solve exponential equations in mathematical and real-world problems	
PA17	solve polynomial equations with real coefficients by applying a variety of techniques such as factoring, graphical methods or technology ~~including the Fundamental Theorem of Algebra, factoring, Descartes Rule of Signs, and knowing that complex zeros occur in conjugate pairs~~ in mathematical and real-world problems	ER Added examples for clarification
PA18	solve polynomial ~~and rational~~ inequalities with real coefficients by applying a variety of techniques such as factoring, graphical methods or technology ~~using critical numbers, by testing intervals~~ and write ~~writing~~ the solution set of the polynomial inequality in interval notation in mathematical and real-world problems	ER and separate SE into two SEs Added examples for clarification
	solve rational inequalities with real coefficients by applying a variety of techniques such as factoring, graphical methods or technology and write the solution set of the rational inequality in interval notation in mathematical and real-world problems	Separate SE PA18

PG14	use trigonometric identities such as reciprocal, quotient, Pythagorean, cofunctions, even/odd, and sum and difference identities for cosine and sine to simplify trigonometric expressions. ~~in mathematical and real-world problems~~	ER SE was moved from Geometric Reasoning Focal Area to Algebraic Reasoning Focal Area to better delineate the focal areas.
PA16	generate and solve trigonometric equations in mathematical and real-world problems	Consistent language with SE PA14 Moved SE within Focal Area with common content

Mathematical Models with Applications (MMA)

~~Mathematical Process Standards Mathematical Models with Applications (MMA)~~		
I.	~~Apply mathematics to problems arising in everyday life, society and the workplace.~~	VA—Process Standards moved to knowledge and skills statements
II.	~~Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.~~	
III.	~~Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.~~	
IV.	~~Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.~~	
V.	~~Create and use representations to organize, record, and communicate mathematical ideas.~~	
VI.	~~Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.~~	

MMA Focal Areas	
	1 Math Modeling Process Standards
~~Numeric reasoning~~	2 Math Modeling in Personal Finance
~~Expressions, equations and generalized relationships~~	3 Math Modeling in Science and Engineering
~~Geometric reasoning~~	4 Math Modeling in Fine Arts
~~Probabilistic and statistical reasoning~~	5 Math Modeling in Social Sciences

Mathematical Models with Applications (One-Half to One Credit)

(a) General requirements.

The provisions of this section shall be implemented beginning the 2013-2014 school year. Students can be awarded one-half to one credit for successful completion of this course. **Prerequisite:** Algebra I. This course must be taken before receiving credit for Algebra II.

(b) Introduction

(1) The desire to achieve education excellence is the driving force behind the Texas Essential Knowledge and Skills for mathematics, guided by the College and Career Readiness Standards. By embedding statistics, probability, and finance, and focusing on fluency and solid understandings, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) Mathematical Models with Applications is designed to build on knowledge and skills from Kindergarten to Grade 8 and Algebra I. This math course provides a path for students to succeed in Algebra II and prepares them for various post-secondary choices. Students learn to apply mathematics through experiences in personal finance, science, engineering, fine arts, and social science. Students use algebraic, graphical, and geometric reasoning to recognize patterns and structure, model information, solve problems, and communicate solutions. Students will select from tools such as physical objects, manipulatives, technology (including graphing calculators, data collection devices, and computers), paper/pencil, and from methods such as algebraic techniques, geometric reasoning, patterns, and mental math to solve problems.

(4) In this course students will use a mathematical modeling cycle to analyze problems, understand problems better, and improve decisions. A basic mathematical modeling cycle is summarized below.* The student will:

(A) Represent:

(i) identify the variables in the problem and select those that represent essential features,

(ii) formulate a model by creating and selecting from representations such as geometric, graphical, tabular, algebraic, or statistical that describe the relationships between the variables,

(B) Compute: analyze and perform operations on these relationships to draw conclusions,

(C) Interpret: interpret the results of the mathematics in terms of the original problem,

(D) Revise: confirm the conclusions by comparing them with the problem and then revise as necessary,

(E) Report: report on the conclusions and the reasoning behind them.

***Note: See page 9 for a graphical representation of the above Modeling Cycle for professional development.**		
Mathematical Process Standards		
M.1 Knowledge and Skills Statement. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:		
	apply mathematics to problems arising in everyday life, society, and the workplace.	VA—Process Standards moved to knowledge and skills statements
	use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution	
	select tools, including ~~such as~~ real objects, manipulatives, paper/pencil, and technology as appropriate, ~~or~~ and techniques, including ~~such as~~ mental math, estimation, and number sense as appropriate, to solve problems	
	communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	
	create and use representations to organize, record, and communicate mathematical ideas	
	analyze mathematical relationships to connect and communicate mathematical ideas	
	display, ~~E~~explain, ~~display~~ and justify mathematical ideas and arguments using precise mathematical language in written or oral communication~~s~~	
~~Numeric Reasoning.~~		**~~MMAN~~**
~~MMAN01~~	~~compare and analyze various methods for solving a real-life problem~~	ER deleted based on Expert Reviewer's recommendation to include in process standards
~~MMAN02~~	~~use multiple approaches (algebraic, graphical, and geometric methods) to solve problems from a variety of disciplines~~	ER deleted based on Expert Reviewer's recommendation to include in process standards
~~MMAN03~~	~~select a method to solve a problem, defend the method, and justify the reasonableness of the results~~	ER deleted based on Expert Reviewer's recommendation to include in process standards

Mathematical Modeling in Personal Finance ~~Algebraic Reasoning (Expressions, Equations, and Generalized Relationships)~~.	**MMAA**
M.2 Knowledge and Skills Statement. The student uses mathematical processes with graphical and numerical techniques to study patterns and	

analyze data related to personal finance. The student is expected to:		
(A) MMAA01	use rates~~,~~ and linear functions, ~~and direct variation~~ to solve problems involving personal finance and budgeting, including compensations and deductions	ER—Askey, direct variation is the same as proportional relationship and not needed
(B) MMAA02	solve problems involving personal taxes	
(C) MMAA03	analyze data to make decisions about banking, including options for online banking, checking accounts, overdraft protection, processing fees, and debit card/ATM fees	Information added for clarification and updated methods of banking
M.3 Knowledge and Skills Statement. The student uses the mathematical processes with algebraic formulas, graphs, and amortization modeling to solve problems involving credit. The student is expected to:		
(A)	use formulas to generate tables to display series of payments for loan amortizations resulting from financed purchases	ER—Askey
(B) MMAA04	analyze ~~methods of payment available~~ personal credit options in retail purchasing and compare relative advantages and disadvantages of each option	Clarification of wording
(C) MMAA05	use technology to create amortization models to investigate home financing and compare buying a home to ~~and~~ renting a home	ER—Clarify that students are not required to amortize with paper/pencil
(D) MMAA06	use technology to create amortization models to investigate automobile financing and compare buying ~~and~~ a vehicle to leasing a vehicle	ER—Clarify that students are not required to use amortization tables with paper/pencil
M.4 Knowledge and Skills Statement. The student uses mathematical processes with algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning. The student is expected to:		
(A) MMAA08	analyze and compare coverage options and rates in insurance	
(B) MMAA09	investigate and compare investment options, including stocks, bonds, annuities, certificates of deposit, and retirement plans	Clarifying and updating information
(C) MMAA07	analyze types of savings options involving simple and compound interest and compare relative advantages of these options	

Math Modeling in Science and Engineering

M.5 Knowledge and Skills Statement. The student applies mathematical processes with algebraic techniques to study patterns and analyze data as it applies to science. The student is expected to:

	~~use data from a statistical study to describe patterns or departures from patterns such as observed differences between a control and a treatment , and describe if practical significance exists~~	This standard was eliminated from Algebra 2. It was considered for this course, but we feel the content is JUST science.
(A) MMAA10	use proportionality ~~direct~~ and inverse variation to describe physical laws such as Hook's Law, Newton's Second Law of Motion, and Boyle's Law~~s~~	ER—Askey, direct variation is the same as proportional Clarification as to which laws to use – only Newton's law involves direct variation
(B) MMAG01	use ~~geometric~~ exponential models available through technology to model growth and decay in areas such as population, biology, ~~and~~ ecology, and chemistry, including radioactive decay	Clarification of the model to be used and added one more area of science to be modeled. This standard mirrors SE A2D03, which has been eliminated from the Algebra 2 course standards.
(C)	use quadratic functions to model motion such as an object dropped, bounced, thrown, or kicked	Standard added to specify the inclusion of quadratic models
M.6 Knowledge and Skills Statement. The student applies mathematical processes with algebra and geometry to study patterns and analyze data as it applies to architecture and engineering. The student is expected to:		
(A) MMAG03	use similarity, geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure ~~in art and~~ architecture	Split this standard between this strand and the Fine Arts to focus on architecture in this strand. Also, added the word "similarity" as this is included when working with transformations
(B)	use scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and non-proportional changes in surface area and volume as applied to fields such as engineering drawing, architecture, and construction	Added standard to support a weak area in geometric thinking. Use within applications in this strand brings more meaning to the mathematics
(C)	use the Pythagorean Theorem and special right-triangle relationships to calculate distances	Added standard to support a weak area in geometric thinking. Use within applications in this strand brings more meaning to the mathematics

(D)	use trigonometric ratios to calculate distances and angle measures as applied to fields such as surveying, navigation, and orienteering	Split from MMAG02: moved the use of ratios to calculate distances to apply to Science and Engineering fields; left trigonometric functions to model periodic motion in art and music in
Math Modeling in Fine Arts ~~Geometric Reasoning~~.		**MMAG**
M.7 Knowledge and Skills Statement. The student uses mathematical processes with algebra and geometry to study patterns and analyze data as it applies to fine arts. The student is expected to:		
~~MMAG01~~	~~use geometric models available through technology to model growth and decay in areas such as population, biology, and ecology~~	Moved to Math Modeling in science and engineering.
(A) MMAG02	use trigonometric ratios and functions available through technology to ~~calculate distances and~~ model periodic behavior in art and music	Moved "calculate distances" to Science and Engineering fields; kept trigonometric ratios and functions to model periodic motion in art and music in Fine Arts. (behaviors) ER—Ross
(B) MMAG03	use similarity, geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and photography ~~and architecture~~	Split this standard between this strand and the Science and Engineering to focus on art in this strand. Added the word "similarity" as this is included when working with transformations Added photography for more specificity and depth
(C) MMAG04	use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music	
(D)	use scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and non-proportional changes in surface area and volume as applied to fields such as painting, sculpture, and photography	Added standard to support a weak area in geometric thinking. Use within applications in this strand brings more meaning to the mathematics

Math Modeling in Social Sciences ~~Probabilistic and Statistical Reasoning.~~	**MMAD**
M.8 Knowledge and Skills Statement. The student applies mathematical processes to determine the number of elements in a finite sample	

space and compute the probability of an event. The student is expected to:		
(A)	determine the number of ways an event, such as a sports tournament, may occur using combinations, permutations, and the Fundamental Counting Principle	CCRS—this expectation is not addressed in any other of the secondary math student expectations.
~~MMAD07~~	~~determine the appropriateness of a model for making predictions from a given set of data~~	This is covered in MMAD04.
(C) MMAD08	compare theoretical ~~and~~ to empirical probability such as determining if a particular game of chance is fair	Added the example for clarification
(D) MMAD09	use experiments to determine the reasonableness of a theoretical model such as binomial, geometric, etc.	
M.9 Knowledge and Skills Statement. The student applies mathematical processes and mathematical models to analyze data as it applies to social sciences. The student is expected to:		
(A) MMAD01	interpret information from various graphs, including line graphs, bar graphs, circle graphs, histograms, scatterplots, dot plots, stem and leaf plots, and box and whisker plots, to draw conclusions from the data and determine the strengths and weaknesses of the conclusions	This standard includes the requirements of Algebra 2, SE A2D07
(B) MMAD02	analyze numerical data using measures of central tendency (mean, median, and mode) and variability (range, IQR, and standard deviation) ~~and correlation~~ in order to make inferences with normal distributions	ER—Askey suggested changing "measures of central tendency" to "summary Statistics". The committee decided to go with the SBOE's Broad Strokes Guidance and keep terminology familiar to teachers and parents. Moved correlation to SE MMAD04 below. This standard includes Algebra 2, SE A2D01.
(C)	distinguish the purposes and differences among types of research, including surveys, experiments, and observational studies	This standard was moved from Algebra 2, SE A2D04 and has been edited for a better fit with this course.
(D)	use data from a sample to estimate population mean or population proportion	This standard was moved from Algebra 2, SE A2D05.
(E) MMAD03	analyze marketing claims based on graphs and statistics from electronic and print media and justify ~~journals, newspapers, and other sources to determine~~ the validity of stated or implied conclusions ~~arguments~~	Added more specificity and updated types of media
(F) MMAD04	use regression methods available through technology to model linear and exponential functions, ~~describe various models for data such as linear, quadratic, exponential, etc., select the most appropriate model, and use the model to~~ interpret ~~information~~ correlations, and make predictions	Deleted the quadratic regression; Algebra II is adding a similar standard to include quadratic and other functions.

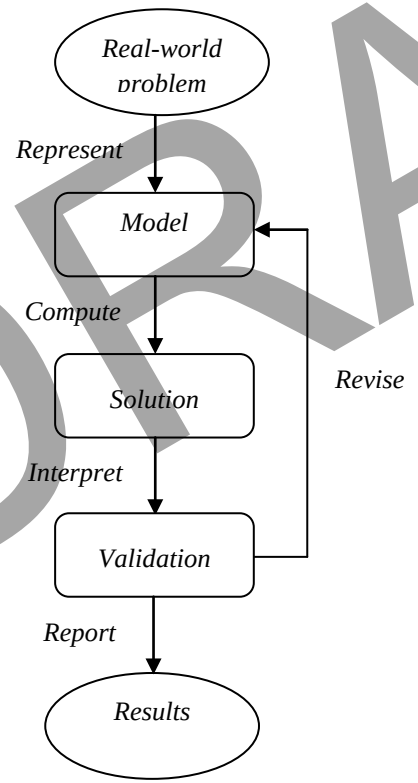
M.10 Knowledge and Skills Statement. The student applies mathematical processes to design a study and use graphical, numerical, and analytical techniques to communicate the results of the study. The student is expected to:		
(A) MMAD05	formulate a meaningful question, determine the data needed to answer the question, gather the appropriate data, analyze the data, and draw reasonable conclusions	
(B) MMAD06	communicate methods used, analyses conducted, and conclusions drawn for a data-analysis project through the use of one or more of a ~~by~~ written report, a visual display, an oral report, or a multimedia presentation	Reworded to clarify that a student may use more than one medium for presentation.

The following model is referenced in our introduction and is recommended for use in teacher professional development.

A basic mathematical modeling cycle is summarized below. The student will:
(A) Represent:

(i) identify the variables in the problem situation and select those that represent essential features,
(ii) formulate a model by creating and selecting from representations such as geometric, graphical, tabular, algebraic, or statistical that describes the relationships between the variables,

(B) Compute: analyze and perform operations on these relationships to draw conclusions,
(C) Interpret: interpret the results of the mathematics in terms of the original problem situation,
(D) Revise: confirm the conclusions by comparing them with the problem situation and then revise as necessary,
(E) Report: report on the conclusions and the reasoning behind them.



Advanced Quantitative Reasoning (AQR)

~~Mathematical Process Standards Advanced Quantitative Reasoning (AQR)~~		
~~I.~~	~~Apply mathematics to problems arising in everyday life, society and the workplace.~~	VA—Process Standards moved to knowledge and skills statements
~~II.~~	~~Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.~~	
~~III.~~	~~Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.~~	
~~IV.~~	~~Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.~~	
~~V.~~	~~Create and use representations to organize, record, and communicate mathematical ideas.~~	
~~VI.~~	~~Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.~~	

AQR Focal Areas
Expressions, equations and generalized relationships
Geometric reasoning
Probabilistic and statistical reasoning

Advanced Quantitative Reasoning

General Requirements

Students shall be awarded one-half to one credit for successful completion of this course. **Prerequisites:** Geometry and Algebra II

Introduction

The desire to achieve education excellence is the driving force behind the Texas Essential Knowledge and Skills for mathematics, guided by the College and Career Readiness Standards. By embedding statistics, probability, and finance, while focusing on fluency and deep understandings, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

In Advanced Quantitative Reasoning, students develop and apply skills necessary for college, careers, and life. Course content consists primarily of applications of high school math concepts to prepare students to become well-educated and highly informed 21st century citizens. The student develops and applies reasoning, planning, and communication to make decisions and solve problems in applied situations involving numerical reasoning, probability, statistical analysis, finance, mathematical selection, and modeling with algebra, geometry, trigonometry, and discrete mathematics.

Mathematical Process Standards		
Knowledge and Skills Statement. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:		VA—Process Standards moved to knowledge and skills statements
	apply mathematics to problems arising in everyday life, society, and the workplace	
	use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, ~~and~~ evaluating the problem-solving process and the reasonableness of the solution	
	select tools, including ~~such as~~ real objects, manipulatives, paper/pencil, ~~and~~ technology as appropriate, and ~~or~~ techniques, including ~~such as~~ mental math, estimation, and number sense as appropriate, to solve problems	
	communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	

	create and use representations to organize, record, and communicate mathematical ideas	VA—Process Standards moved to knowledge and skills statements
	analyze mathematical relationships to connect and communicate mathematical ideas	
	display, explain, ~~display,~~ and justify mathematical ideas and arguments using precise mathematical language in written or oral communication~~s~~	

Numeric Reasoning.		**AQRN**
Knowledge and Skills Statement. The student applies the process standards in mathematics to generate new understandings by extending existing knowledge. The student generates new mathematical understandings through problems involving numerical data that arise in everyday life, society, and the workplace. The student extends existing knowledge and skills to analyze real-world situations. The student is expected to:		
Develop and Apply Skills Used in College and Careers		
~~AQRN01~~	~~gather data, conduct investigations, and apply mathematical concepts and models to solve problems in mathematics and other disciplines~~	This SE is subsumed within the mathematical process standards.
~~AQRN02~~	~~demonstrate reasoning skills in developing, explaining, and justifying sound mathematical arguments, and analyze the soundness of mathematical arguments of others~~	This SE is subsumed within the mathematical process standards.
~~AQRN03~~	~~communicate with and about mathematics orally and in writing as part of independent and collaborative work, including making accurate and clear presentations of solutions to problems~~	This SE is subsumed within the mathematical process standards.
Analyze Numerical Data		
	compare and contrast precision and accuracy in real-life situations such as in measurements and significant figures	ER
AQRN04	apply~~,~~ and analyze ~~compare, and contrast~~ published ~~ratios, rates,~~ ratings, ~~averages,~~ weighted averages, and indices to make informed decisions	IF—simplified language and removed middle school content
AQRN05	solve problems involving ~~large~~ quantities that are not easily measured using proportionality such as packing problems, crowd estimation, and white blood cell count	ER—Askey
	solve geometric problems involving indirect measurement, including similar triangles, Pythagorean Theorem, Law of Sines, Law of Cosines, and the use of dynamic geometry software	Moved from AQRG04
	solve problems involving large quantities using combinatorics such as numbers of unique license plates and phone numbers	IF & ER—Askey
AQRN06	use arrays to ~~efficiently~~ manage efficiently large collections of data and add, subtract, and multiply matrices to solve applied problems, including geometric transformations	Grammar
~~AQRN07~~	~~apply algorithms and identify errors in recording and transmitting identification numbers~~	IF & ER—Askey

Use Ranking and Selection		
~~AQRN08~~	~~apply and analyze various ranking algorithms to determine an appropriate method for a given situation~~	ER
AQRN09	analyze various voting and selection processes to ~~determine an appropriate method for a~~ compare results in given situations such as at-large versus single-member districts and plurality versus majority voting	ER
	select and apply an algorithm of interest to solve real-life problems, such as: problems using recursion or iteration involving population growth or decline, fractals, and compound interest; the validity in recorded and transmitted data using checksums and hashing; sports rankings, weighted class rankings, and search engine rankings; and problems involving scheduling or routing situations using vertex-edge graphs, critical paths, Euler paths, and minimal spanning trees; then communicate the application of the algorithm in precise mathematical and nontechnical language to peers	ER—Rath & Ross Merging AQRN07, AQRN08, AQRN10, AQRA06
Use Network Models		
AQRN10	~~solve problems involving scheduling or routing situations that can be represented by a vertex-edge graph, and find critical paths, Euler paths, or minimal spanning trees.~~	Technical edit
AQRN11	~~construct, analyze, and interpret flow charts in order to develop and describe problem-solving procedures.~~	ER: suggested inclusion of conditionals and loops. See comment below for justification of deletions.

Algebraic Reasoning (Expressions, Equations, and Generalized Relationships).	**AQRA**
Knowledge and Skills Statement. The student applies the process standards in mathematics to create and analyze mathematical models of everyday situations to make informed decisions related to earning, investing, spending, and borrowing money by appropriate, proficient, and efficient use of tools, including technology. The student uses mathematical relationships to make connections and predictions. The student judges the validity of a prediction and uses mathematical models to represent, analyze, and solve dynamic real-world problems. The student is expected to:	

Model Data		
~~AQRA01~~	~~determine whether or not there is a linear relationship in a set of bivariate data by finding and interpreting the correlation coefficient for the data~~	VA Already addressed in Algebra I and Grade 8 Mathematics.
AQRA02	collect numerical bivariate data~~; use the data~~ to create a scatterplot~~;~~, select a function to model the data, justify the model selection, and use the model to interpret results and make predictions	Clarification
	describe the degree to which uncorrelated variables may or may not be related and analyze situations where correlated variables do or do not indicate a cause and effect relationship	ER & VA to extend A1L13

Model Change and Relationships		
AQRA03	determine or analyze an appropriate growth or decay model for problem situations, including linear, exponential, and logistic functions	
AQRA04	determine or analyze an appropriate cyclical model for problem situations that can be modeled with ~~trigonometric~~ periodic functions	ER
AQRA05	determine or analyze an appropriate piecewise model for problem situations	
~~AQRA06~~	~~solve problems using recursion or iteration, including those involving population growth or decline, and compound interest~~	ER
Model Financial Situations		
AQRA07	create ~~determine~~, represent, and analyze mathematical models for various types of income calculations to determine the best option for a given situation	BSG
AQRA08	create ~~determine~~, represent, and analyze mathematical models for expenditures, including those involving credit, to determine the best option for a given situation	BSG
AQRA09	create ~~determine~~, represent, and analyze mathematical models and appropriate representations, including formulas and amortization tables, for various types of loans and investments to determine the best ~~loan or investment plan~~ option for a given situation such as cell phone plans and buying versus leasing a car	ER & BSG

Geometric Reasoning.		**AQRG**
Model with Geometric Tools		
~~AQRG01~~	~~create and use two- and three-dimensional representations of authentic situations using geometric models or dynamic geometric environments for computer-aided design and other applications.~~	Combined with AQRG04
~~AQRG02~~	~~use vectors to represent and solve applied problems.~~	Reasonableness of SE
~~AQRG03~~	~~use matrices to represent geometric transformations and solve applied problems~~.	Moved to AQRN06
~~AQRG04~~	~~solve geometric problems involving inaccessible distances.~~	Move to Numeric Reasoning
Probabilistic and Statistical Reasoning.		**AQRD**

Knowledge and Skills Statement. The student utilizes the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to effectively communicate the results of student-generated statistical studies and the critical analysis of published statistical studies. The student is expected to:		
Analyze and Evaluate Risk and Return in the Context of Everyday Situations		
	use a two-way frequency table as a sample space to identify whether two events are independent and to interpret the results	
	use the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, in mathematical and real-world problems	Deleted from Geometry and inserted in AQR
AQRD01	calculate ~~determine and interpret~~ conditional probabilities and probabilities of compound events ~~by constructing and analyzing representations, including~~ using tree diagrams, Venn diagrams, ~~and~~ area models, and formulas such as Bayes' Theorem ~~to make decisions in problem situations~~	ER We felt determining probabilities was a separate process from interpreting probabilities. We also thought that formulas were important. See comments below for justification of deletions.
	interpret conditional probabilities and probabilities of compound events by analyzing representations to make decisions in problem situations	This new SE contains parts that were deleted from the previous SE.
AQRD02	use probabilities to make and justify decisions about risks in everyday life such as the lottery, weather forecasts, and insurance costs	Clarification
AQRD03	calculate expected value to analyze mathematical fairness, payoff, and risk	
Critique Applications of Statistics		
	determine the validity of logical arguments that include compound conditional statements by constructing truth tables	
AQRD04	identify limitations ~~or~~ and lack of relevant information in studies reporting statistical information, especially when studies are reported in condensed form	To insure that students are exposed to both limitations and lack of relevant information.
AQRD05	interpret and compare ~~the results of polls,~~ statistical results using appropriate technology given a margin of error in situations such as polls, quality control, and measurements	ER
AQRD06	identify ~~uses and~~ potential misuses of ~~statistical analyses in studies reporting statistics or using~~ statistics to justify particular conclusions, including assertions of a cause and effect relationship rather than an association, and missteps or fallacies in logical reasoning such as confounding variables and hasty generalizations	Technical edit & ER

AQRD07	describe strengths and weaknesses of sampling techniques, data and graphical displays, and interpretations of summary statistics and other results appearing in a study, including reports published in the media	
Conduct Statistical Analyses		
AQRD08	determine the need for and purpose of a statistical investigation and what type of statistical analysis can be used to answer a specific question or set of questions	
AQRD09	identify the population of interest for a statistical investigation, select an appropriate sampling technique, and collect data	Clarification
AQRD10	identify the variables to be used in a study	
AQRD11	determine possible sources of statistical bias in a study and how ~~such~~ bias may affect the ~~ability to generalize~~ validity of the results	ER
AQRD12	create data displays for given data sets to investigate, compare, and estimate center, shape, spread, and unusual features of the data	
AQRD13	analyze ~~determine~~ possible sources of data variability ~~of data, both~~, including those that can be controlled and those that cannot be controlled	Clarification
Communicate Statistical Information		
AQRD14	report results of statistical studies to a particular audience, including selecting an appropriate presentation format, creating graphical data displays, and interpreting results in terms of the question studied	
AQRD15	justify the design and the conclusion(s) of statistical studies, including the methods used ~~for each~~	Technical edit
AQRD16	communicate statistical results in ~~both~~ oral and written formats using appropriate statistical and nontechnical language	Technical edit

Independent Study in Mathematics

General Requirements

Students shall be awarded one-half to one credit for successful completion of this course. **Prerequisites:** Geometry and Algebra II

Introduction

The desire to achieve education excellence is the driving force behind the Texas Essential Knowledge and Skills for mathematics, guided by the College and Career Readiness Standards. By embedding statistics, probability, and finance, and focusing on fluency and deep understandings, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

In Independent Study in Mathematics, students will extend their mathematical understanding beyond the Algebra II level in a specific area or areas of mathematics such as theory of equations, number theory, non-Euclidean geometry, linear algebra, advanced survey of mathematics, or history of mathematics.

General Requirements		
(A)	~~General requirements. Students can be awarded one-half to one credit for successful completion of Independent Study in Mathematics. Required prerequisites: Algebra II, Geometry.~~ Students may repeat this course with different course content for up to three credits.	Deleted information is provided in the introduction above.
(B)	~~Content requirements. Students will extend their mathematical understanding beyond the Algebra II level in a specific area or areas of mathematics, such as theory of equations, number theory, non-Euclidean geometry, advanced survey of mathematics, or history of mathematics.~~ The requirements for each course must be approved by the local district before the course begins.	Deleted information is provided in the introduction above.
(C)	If this course is being used to satisfy requirements for the Distinguished Achievement Program, student research/products must be presented before a panel of professionals or approved by the student's mentor.	

Mathematical Process Standards		
Knowledge and Skills Statement. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:		VA—Process Standards moved to knowledge and skills statements
	apply mathematics to problems arising in everyday life, society, and the workplace	
	use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution	
	select tools, including ~~such as~~ real objects, manipulatives, paper/pencil, ~~and~~ technology as appropriate, and ~~or~~ techniques, including ~~such as~~ mental math, estimation, and number sense as appropriate, to solve problems	
	communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate	
	create and use representations to organize, record, and communicate mathematical ideas	
	analyze mathematical relationships to connect and communicate mathematical ideas	
	display, explain, ~~display,~~ and justify mathematical ideas and arguments using precise mathematical language in written or oral communication~~s~~	