

FINAL RECOMMENDATIONS
Texas Essential Knowledge and Skills (TEKS)
Mathematics, Elementary

Prepared by the State Board of Education (SBOE) TEKS Review Committees

Final Recommendations, October 2011

In 2010-2011 the Commissioner’s Mathematics Advisory Group was convened to offer recommendations regarding the next generation of mathematics standards in Texas. *The Commissioner’s Draft of the Texas Mathematics Standards* reflects the recommendations of the Commissioner’s Mathematics Advisory Group and a panel of national advisors in mathematics. The SBOE-appointed mathematics TEKS review committees used *The Commissioner’s Draft of the Texas Mathematics Standards* as a starting point for their recommendations for revisions to the TEKS.

These proposed revisions reflect the recommended changes of the committees to the standards in *The Commissioner’s Draft of the Texas Mathematics Standards*. Proposed additions are shown in green font with underlines ([additions](#)) and proposed deletions are shown in red font with strikethroughs (~~deletions~~). Changes recommended based on a vertical alignment review are shown in brown font ([additions](#) or ~~deletions~~).

Comments in the right-hand column provide explanations for the proposed changes. The following notations were used as part of the explanations:

- BSG**—information added, changed, or deleted based on broad-strokes guidance from the SBOE
- CRS**—information added or changed to align with the Texas College and Career Readiness Standards (CCRS)
- ER**—information added, changed, or deleted based on expert reviewer feedback
- IF**—information added, changed, or deleted based on informal feedback
- MV**—multiple viewpoints from within the committee
- SBOE**—information added, changed, or deleted based on SBOE feedback
- VA**—information added, changed, or deleted to increase vertical alignment

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Kindergarten

Mathematical Process Standards Kindergarten

I.	Apply mathematics to problems arising in everyday life, society and the workplace.	VA—Process Standards moved to knowledge and skills statements
II.	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.	
III.	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.	
IV.	Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.	
V.	Create and use representations to organize, record, and communicate mathematical ideas.	
VI.	Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.	

Grade K Focal Areas		
Number and Operations	▲	Understanding counting and cardinality
Number and Operations	●	Understanding addition as joining and subtraction as separating
Measurement and Data	■	Comparing objects by measurable attributes

Supporting Topics for the Focal Areas in Grade K and Grade 1		
Number and Operations	▲ ● ■	Representing, comparing, and ordering whole numbers
Two-Dimensional <u>Shapes</u> and Three-Dimensional <u>Solids</u>	■	Identifying <u>shapes</u> and <u>solids</u> and their attributes
	+	Classifying 2D and 3D figures
	■	Composing 2D <u>shapes</u>
	+	Describing location
Measurement and Data	▲ ● ■	Representing data
Color and symbol shows the connection between Focal Areas and Supporting Topics. + Indicates topic supports Focal Area in Grade 1		

Kindergarten

Introduction

The desire to achieve educational excellence is the driving force behind the Texas Essential Knowledge and Skills for mathematics, guided by the Texas College and Career Readiness Standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

The process standards are integrated at every grade level. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process as well as the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

In Kindergarten, instructional time should focus on three critical areas: understanding counting and cardinality, understanding addition as joining and subtraction as separating, and comparing objects by measureable attributes.

- A. Students develop number and operations through several fundamental concepts. Students know number names and the counting sequence. Counting and cardinality lay a solid foundation for number. Students apply the principles of counting to make the connection between numbers and quantities.
- B. Students use meanings of numbers to create strategies for solving problems and responding to practical situations involving addition and subtraction.
- C. Students identify characteristics of objects that can be measured and directly compare objects according to these measureable attributes.

Mathematical Process Standards.

Knowledge and Skills Statement. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

	apply mathematics to problems arising in everyday life, society, and the workplace	VA—Process Standards moved to knowledge and skills statements
	use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process <u>and the reasonableness of the solution</u>	
	select tools, including such as real objects, manipulatives, paper/pencil, and technology <u>as appropriate, and</u> or techniques, including such as mental math, estimation, and number sense <u>as appropriate</u> , to solve problems	
	communicate mathematical ideas, reasoning, and their implications using <u>multiple representations, including</u> symbols, diagrams, graphs, and language <u>as appropriate</u>	

	create and use representations to organize, record, and communicate mathematical ideas	
	<u>analyze mathematical relationships to connect and communicate mathematical ideas</u>	
	<u>display, Eexplain, display, and justify</u> mathematical ideas and arguments using precise mathematical language in written or oral communications.	

Number and Operations.		KN
Knowledge and Skills Statement. The student applies mathematical process standards to understand how to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system. The student is expected to:		
	<u>count with and without objects forward and backward to at least 20</u>	from MN and MA standards
	<u>read, write, and represent whole numbers from 0 to at least 20, with and without objects or pictures</u>	from MN and MA standards Including “with and without objects” enables students to move from the concrete and pictorial to the abstract.
	<u>count a set of objects, up to at least 20, and demonstrate that the last number said tells the number of objects in the set regardless of their arrangement</u>	
	<u>recognize instantly the quantity of a small group of objects in organized and random arrangements</u>	PD: Recognizable arrangements may include domino and dice patterns and five- or ten-frames.
KN02	represent the number of objects in a set at least up to 40 using spoken words and written numerals.	Subsumed in other standards
KN07	generate a set using objects concrete and or pictorial models that represents a number that is more than, less than, or and equal to a given number, up to 40-20	Language in standard clarified
KN03	generate a determine the number <u>that is one more than before or after one less than</u> another number without having to start back at 1 up to at least 20	From MN standards; Clements & Sarama (2009). <u>Early childhood mathematics education research: Learning trajectories for young children</u> . New York, NY: Routledge.
KN04	represent the numerals up to 40 by associating them to the number of elements in sets consisting of actual objects or pictures of objects, including counting out objects in groups of tens and ones such as one group of 10 and 3	Moved to Grade 1 where students will focus on the development of place value.

KN05	compare collections sets of objects of up to 40 objects <u>at least 20 in each set</u> using <u>comparative language</u> ; one-to-one correspondence	Feedback from expert reviewers and informal sources.
KN06	<u>use comparative language to describe two</u> compare numbers between 1 and 10 <u>up to 20</u> presented as written numerals	SBOE recommendation for clarity Professional development: <i>such as greater than, more than, less than, fewer than, same as, equal to</i>
KN08	<u>compose and decompose numbers up to 10 with objects and pictures</u> ; Compose a given target number less than or equal to 10 by producing two sets of objects that, when combined, contain exactly the target number	<ul style="list-style-type: none"> • KN08 and KN09 standards can effectively be combined. • MN Standards • Nita Copley and John Van de Walle recommend composing and decomposing numbers to support Part/Whole reasoning.
KN09	decompose a given set of objects less than or equal to 10 into multiple sets in a variety of ways, and indicate the corresponding number pairs in each case (e.g., 8 can be decomposed into 4 and 4, 5 and 3, 6 and 2, and 7 and 1)	

Number and Operations.

KN

Knowledge and Skills Statement. The student applies mathematical process standards to develop an understanding of addition and subtraction situations in order to solve problems. The student is expected to:

	<u>model the action of joining to represent addition and the action of separating to represent subtraction</u>	Kindergarten children need to see addition and subtraction represented by the actions of joining and separating.
KN10	combine a set of 10 objects with another number set of objects to make a new number set of size between 10 to 20 and indicate the corresponding number relation (e.g., a set of 10 and a set of 1 can be combined to make 11)	Combining sets of tens and extras is moved to Grade 1.
KN11	separate a set of 10 to 20 objects into a group of 10 objects and some more. (e.g., 18 can be separated into a set of 10 and a set of 8)	Separating sets of tens and extras is moved to Grade 1.

KN12	<u>solve word problems using objects and drawings to find sums up to 10 and differences within 10; solve mathematical and real-world problems involving adding and subtracting within 20 using objects, concrete and pictorial models. These problems should include determining the sum when two addends are given and determining the minuend when the difference and subtrahend are given.</u>	Based on MN and MA standards and developmental appropriateness for Kindergarten. Working within 20 requires an understanding of place value which is more appropriate for Grade 1. <i>PD: include determining the whole when the parts are given and determining the missing part when the whole and one part are given.</i>
KN13	explain the solution process <u>strategies used</u> to <u>solve</u> problems involving adding or <u>and</u> subtracting within 20 <u>10</u> using spoken words, objects, <u>concrete and</u> pictorial models, and number sentences.	Students need to develop the language to explain their reasoning processes.

Knowledge and Skills Statement. The student applies mathematical process standards to identify coins in order to recognize the need for monetary transactions. The student is expected to:

<u>identify U.S. coins by name, including pennies, nickels, dimes, and quarters</u>	Financial Literacy; Recommended by SBOE
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Algebraic Reasoning

Knowledge and Skills Statement. The student applies mathematical process standards to the pattern in the number word list. The student is expected to:

KN01	<u>recite numbers up to at least 100</u> count verbally to at least 100 <u>130</u> by ones and tens, beginning with any given number	MN, MA, and CCSS do not count by ones to 130. ER, Capraro, and IF suggested counting to 100.
	<u>represent addition and subtraction with objects, drawings, acting out situations, verbal explanations, or number sentences</u>	VA

Geometry and Measurement ~~Two-Dimensional and Three-Dimensional Figures.~~

KG

Knowledge and Skills Statement The student applies mathematical process standards to analyze attributes of two-dimensional shapes and three-dimensional solids to develop generalizations about their properties. The student is expected to:

KG01	identify two-dimensional <u>shapes, objects (the shape of including</u> circles, triangles, rectangles, <u>and squares, as special rectangles; rhombuses, and hexagons)</u> and three-dimensional objects (the shape of cylinders, cones, spheres, and cubes) <u>found in the real world.</u>	
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	<u>identify three-dimensional solids in the real world, including cylinders, cones, spheres, and cubes</u>	Separated to emphasize the difference between 2-D and 3-D shapes and solids.
KG02	identify two-dimensional components of three-dimensional <u>objects such shapes</u> (e.g., as the face of a <u>tissue box cube is a rectangle square</u>)	Language in standard clarified
KG03	identify attributes of two-dimensional shapes <u>using informal and formal geometric language interchangeably</u> (e.g., <u>such as</u> number of corners, <u>(or</u> vertices), number of sides, <u>and angles</u>)	Language in standard clarified
KG04	identify attributes of three-dimensional shapes (e.g., number of corners (vertices), number of edges, and sides and number of faces).	Recommended by geometry vertical team.
KG05	classify <u>and sort a variety of regular and irregular</u> two- <u>and three-</u> dimensional <u>shapes figures as circles, triangles, rectangles, including squares and rhombuses; or hexagons</u> regardless of orientation or size	Shapes and solids are defined in KG01 and the unnumbered standard following it.
KG06	classify three-dimensional shapes as cylinders, cones, spheres, or cubes regardless of orientation or size.	Combined with KG05
KG07	<u>create compose</u> two-dimensional shapes <u>and three-dimensional shapes</u> using <u>a variety of</u> materials (e.g., <u>popsicle sticks, straws, molding clay, etc.</u>) <u>or and</u> drawings	Language in standard clarified
KG08	describe the position of one or more shapes in relation to another shape using words such as “above,” “below,” “beside,” “between,” “in front of,” and “in back of”.	ER, Milgram and Weilmuenster. These concepts may be taught in other content areas such as language arts and social studies.

Geometry and Measurement ~~and Data~~

KM

Knowledge and Skills Statement. The student applies mathematical process standards to directly compare measurable attributes. The student is expected to:

KM01	give an example of a measurable attribute of a given object, <u>(including</u> length, capacity, <u>and</u> weight <u>temperature</u>)	ER, Weilmuenster. Temperature is taught in science.
KM02	compare two objects <u>directly</u> with a common measurable attribute <u>(length, capacity, weight, temperature)</u> <u>using language such as “more” and “less” to see which object has more of/less of the attribute and describe the difference</u>	PD: use comparative language such as longer/taller/wider, shorter; holds more, holds less; heavier, lighter; colder, warmer to directly

Measurement and Data Analysis		KM
Knowledge and Skills Statement. The student applies mathematical process standards to collect and organize data to make it useful for interpreting information. The student is expected to:		
KM03	classify and sort a set of objects into categories according to an attribute (e.g., number of sides, angles, color, shape, size, first letter in name, etc.) and resort the same set according to a different attribute.	Geometry standard includes this.
KM04	collect, sort, and organize data into two or three categories count the number of objects in a category or subcategory, summarize the data into a picture graphs and use the picture graph to answer questions about the data (e.g., "Which shape do we have the most of?")	MA standards
	<u>use data to create real-object and picture graphs</u>	MA standards
	<u>draw conclusions from real-object and picture graphs</u>	MA standards

DRAFT

Grade 1

Mathematical Process Standards Grade 1

I.	Apply mathematics to problems arising in everyday life, society and the workplace.	VA—Process Standards moved to knowledge and skills statements
II.	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.	
III.	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.	
IV.	Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.	
V.	Create and use representations to organize, record, and communicate mathematical ideas.	
VI.	Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.	

Grade 1 Focal Areas

Number and Operations	▲	Understanding and applying place value
Number and Operations	●	Solving problems involving addition and subtraction
Two-Dimensional <u>Shapes</u> and Three-Dimensional <u>Solids</u>	■	Composing and decomposing two-dimensional <u>shapes</u> and three-dimensional <u>solids</u>

Supporting Topics for the Focal Areas in Grade 1 and Grade 2

Number and Operations	▲	Determining 10 more or 10 less
	▲●	Comparing and ordering whole numbers up to 100
	▲●	Connecting properties and operations
	▲●	Connecting addition and subtraction
Expressions, Equations, and Relationships	▲●	Fluently producing addition and related subtraction facts with sums to 10 and differences from 10
	●	Representing problems involving addition and subtraction
Two-Dimensional <u>Shapes</u> and Three-Dimensional <u>Solids</u>	■	Distinguishing between attributes of figures
Measurement and Data	▲●■	Representing data

Color and symbol shows the connection between Focal Areas and Supporting Topics.

✦ Indicates topic supports Focal Area in Grade 2

Grade 1

Introduction

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The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

In Grade 1, instructional time should focus on three critical areas: (A) understanding and applying place value, (B) solving problems involving addition and subtraction, and (C) composing and decomposing two-dimensional shapes and three-dimensional solids.

- (A) Students use relationships within the numeration system to understand the sequential order of the counting numbers and their relative magnitude.
- (B) Students extend their use of addition and subtraction beyond the actions of joining and separating to include comparing and combining. Students use properties of operations and the relationship between addition and subtraction to solve problems. By comparing a variety of solution strategies, students use efficient, accurate, and generalizable methods to perform operations.
- (C) Students use basic shapes and spatial reasoning to model objects in their environment and construct more complex shapes. Students are able to identify, name, and describe basic two-dimensional shapes and three-dimensional solids.

Mathematical Process Standards.

Knowledge and Skills Statement. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

apply mathematics to problems arising in everyday life, society, and the workplace

VA—Process Standards moved to knowledge and skills statements

	use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process <u>and the reasonableness of the solution</u>	VA—Process Standards moved to knowledge and skills statements
	select tools, <u>including such as</u> real objects, manipulatives, paper/pencil, and technology <u>as appropriate, and of</u> techniques, <u>including such as</u> mental math, estimation, and number sense <u>as appropriate, to</u> solve problems	
	communicate mathematical ideas, reasoning, and their implications using <u>multiple representations, including</u> symbols, diagrams, graphs, and language <u>as appropriate</u>	
	create and use representations to organize, record, and communicate mathematical ideas	
	<u>analyze mathematical relationships to connect and communicate mathematical ideas</u>	
	<u>display, Explain, display, and</u> justify mathematical ideas and arguments using precise mathematical language in written or oral communications.	

Number and Operations.

1N

Knowledge and Skills Statement The student applies mathematical process standards to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system related to place value. The student is expected to:

	<u>recognize instantly the quantity of structured arrangements such as seen on a die or a ten-frame</u>	ER, Capraro NCTM Focal Points; Clements' Learning Trajectories
1N02	<u>use concrete and pictorial models to compose and decompose numbers up the value of a numeral to 100 120 as a sum of so many hundreds, so many tens, and so many ones in more than one way using objects and pictorial models. For example, 64 can be represented as 6 tens, and 4 ones, or as 5 tens, and 14 ones. (representations may be bundles of an object or pictures of bundles)</u>	ER, Weilmuenster Professional development needs to emphasize examples such as 64 can be represented as 6 tens and 4 ones or as 5 tens and 14 ones.
1N03	<u>use objects, pictures, and expanded and standard forms to represent a two digit number numbers up to 120 as the sum of the values represented by the digits in the combined value of tens and ones using objects, pictures, expanded notation, and numbers. For example, 93 is the sum of 9 tens and 3 ones</u>	ER, Weilmuenster
1N04	generate a two digit number that is greater than, <u>or</u> less than, or equal to a given whole number that is greater than 10 and less than 99 up to 120	Professional development needs to ask students to generate numbers that are 1 more/1 less, 10 more/10 less.

1N05	<u>use place value to compare</u> and order whole numbers up to 100 <u>120 using comparative language</u>	ER, Weilmuenster PD: such as greater than, more than, less than, fewer than, same as, equal to
	<u>order whole numbers to 120 using place value and open number lines</u>	NCTM Focus in Grade 2 (p 52-54), TXRCFP (p 16), Clements <u>Early Childhood Mathematics Education Research</u> (p 92) PD needs to include open number lines.
1N06	represent the comparison of two numbers to 100 using the symbols >, <, or =	Research suggests that first grade students are learning quantities concretely to provide support for the abstract comparative symbols used in grades 2 and beyond.

Number and Operations.

Knowledge and Skills Statement. The student applies mathematical process standards to develop and use strategies for whole number addition and subtraction computations in order to solve problems. The student is expected to:

1N07	determine the difference of between two multiples of 10 in the range from 10-90 using objects and pictures.	ER, Weilmuenster
1N08	generate a two-digit number that is 10 more or 10 less than a given number.	Moved to Expressions, Equations, and Relationships for consistency with Grade 2.
1N09	<u>use concrete and pictorial models to</u> determine the sum of a two-digit number <u>multiple of ten</u> and a one-digit number in mathematical and real-world problems, within 100, using concrete and visual models for solving addition problem situations up to 99	ER, Weilmuenster PD: 90 and 9 is 99; 60 and 8 is 68.
1N10	<u>use objects and pictorial models to</u> solve mathematical and real-world word problems involving combining joining, separating, and comparing sets with sums to within 20 and unknowns <u>as any one of the terms in the problem in all positions, using objects and pictorial models such as $2 + 4 = \square$; $3 + \square = 7$; and $5 = \square - 3$</u>	Combined 1N10, 1N011, 1N012 PD: Embed unknowns in all positions within a context.
1N11	solve mathematical and real-world problems involving separating with differences from 20 and unknowns in all positions using objects and pictorial models	
1N12	solve mathematical and real-world problems involving comparisons within 20 and unknowns in all positions using objects and pictorial models	

1N13	solve mathematical and real-world problems involving sets to 20 and unknowns in all positions using objects and pictorial models	Repetition of 10, 11, and 12
	<u>compose 10 with two or more addends with and without concrete objects;</u>	ER, Askey
1N14	<u>apply basic fact strategies to add and subtract</u> fluently produce addition and subtraction facts with sums to 10 and differences from within 10 with fluency <u>20 using strategies, including making 10 and decomposing a number leading to a 10</u>	ER, Weilmuenster, Askey PD: counting on, counting back, doubles, near doubles and representations such as number lines, ten frames leading to mental math strategies.
1N15	explain <u>strategies used to solve</u> the solution to addition and subtraction problems involving adding or subtracting within <u>up to</u> 20 using spoken words, objects, pictorial models, and number sentences	
1N16	generate <u>and solve</u> problem situations when given a mathematical number sentence involving <u>addition and subtraction</u> adding or subtracting of whole numbers within 20	SBOE PD: The difference between a word problem & a problem situation— For example: For $5 + 2 = \square$, Ann has 5 stickers. Mom gives her 2 more. Now she has 7 stickers; instead of Ann has 5 and gave 2 stickers to Mom. Now she has 3 stickers.

Number and Operations ~~Expressions, Equations and Relationships.~~

1A

Knowledge and Skills Statement. The student applies mathematical process standards to identify coins, their values, and the relationships among them in order to recognize the need for monetary transactions. The student is expected to:

<u>identify U.S. coins, including pennies, nickels, dimes, and quarters, by value and describe the relationships between them</u>	Financial Literacy
<u>write a number with the cent symbol to describe the value of a coin</u>	Financial Literacy
<u>use relationships to skip count by twos, fives, and tens to determine the value of pennies, nickels, and dimes</u>	Financial Literacy

Algebraic Reasoning ~~Expressions, Equations and Relationships.~~

1A

Knowledge and Skills Statement. The student applies mathematical process standards to identify and apply number patterns within properties of numbers and operations in order to describe relationships. The student is expected to:

	<u>recite numbers forward and backward from any given number between 1 and 120</u>	SBOE and ER, Weilmuenster Moved to Algebraic Reasoning
	<u>skip count by twos, fives, and tens to 100</u>	This is a verbal pattern.
1N01	skip count by twos, and fives, and tens to determine the total number of objects (up to 130) <u>120</u> in a set (objects include pennies and nickels)	Moved from Number to “Expressions, equations, and relationships.”
	<u>use relationships to determine the number that is 10 more and 10 less than a given number up to 120</u>	MN standards
1A01	represent mathematical and real-world <u>word</u> problems involving addition and subtraction of whole numbers to 20 using concrete objects, strip diagrams, and pictorial models and number sentences (equations)	SBOE Real-world problems will move to opening paragraphs.
1A02	<u>understand that the equal sign represents a relationship determine if where statements on each side of the equal sign a number sentence for addition or subtraction is are true</u>	SBOE and ER, Capraro and Milgram PD: $7 = 7$; $7 = 8 - 1$; $5 + 2 = 2 + 5$; $3 + 4 = 1 + 6$
1A03	determine the unknown whole number in an addition or subtraction equation relating three whole numbers when the unknown may be any one of the three or four terms in the equation For example, the value 7 for \square makes $12 + \square = 19$ a true equation	Language clarified and recommended by ER, Capraro Include fact families in PD.
	<u>identify relationships between addition facts and related subtraction sentences such as $3 + 2 = 5$ and $5 - 2 = 3$</u>	<u>Adding It Up</u> , page 76 Clements PD: Fact families
	<u>apply properties of operations as strategies to add and subtract such as if $2 + 3 = 5$ is known, then $3 + 2 = 5$</u>	

Geometry and Measurement ~~Two-Dimensional and Three-Dimensional Figures.~~

1G

Knowledge and Skills Statement. The student applies mathematical process standards to analyze attributes of two-dimensional shapes and three-dimensional solids to develop generalizations about their properties. The student is expected to:

	<u>classify and sort regular and irregular two-dimensional shapes based on attributes using informal geometric language;</u>	
1G02	distinguish between attributes that define a two-dimensional or three-dimensional <u>figure shape</u> (e.g., such as a closed figure with three sides is a triangle or a solid with <u>exactly six</u> rectangular faces is a rectangular prism) and attributes that do not define the shape (e.g., such as orientation or color)	Language clarified and recommended by ER, Rath
1G01	<u>create</u> draw two-dimensional figures, including circles, half-circles, quarter-circles, triangles, rectangles, squares <u>as special rectangles</u> , rhombuses, and hexagons	ER, Weilmuenster page 5

	<u>identify two-dimensional shapes, including circles, triangles, rectangles, squares as special rectangles, rhombuses, and hexagons, and describe their attributes using formal language such as vertex and side</u>	MAJOR PD ISSUE!!!!
	<u>identify three-dimensional solids, including spheres, cones, cylinders, rectangular prisms (including cubes), and triangular prisms, and describe their attributes using formal language such as vertex, edge, and face</u>	MAJOR PD ISSUE!!!!
1G03	compose two-dimensional shapes or three-dimensional shapes by joining two, three, or four <u>figures shapes</u> , to produce a target shape in more than one way if possible	Introduction to figures by name is important prerequisite for grade 2 concepts.
	<u>partition two-dimensional figures such as circles and rectangles into two and four fair shares or equal parts and describe the parts using words such as “halves,” “half of,” “fourths,” or “quarters”</u>	Siebert and Gaskin, “Creating, Naming, & Justifying Fractions”, <u>Teaching Children Mathematics</u> , April 2006 Watanabe, “Representations in Teaching & Learning Fractions”, <u>Teaching Children Mathematics</u> , April 2002
	<u>identify examples and non-examples of halves and fourths</u>	Witherspoon. “Fractions: In Search of a Meaning”, <u>Arithmetic Teacher</u> , April 1993.

Geometry and Measurement ~~and Data~~.

1M

Knowledge and Skills Statement. The student applies mathematical process standards to select and use units to describe length and time. The student is expected to:

	<u>use measuring tools such as adding machine tape, ribbon, or string to measure the length of objects to reinforce the continuous nature of linear measurement</u>	Nita Copley and Clements & Sarana, <u>Engaging Young Children in Mathematics</u> , page 301
1M01	<u>demonstrate</u> illustrate that the length of an object is the number of same-size units of length that, when laid end-to-end (with no gaps or overlaps), reach from one end of the object to the other, assuming this is possible	
1M02	<u>measure the same object/distance with units of</u> generalize that when two different <u>units lengths and describe how and why the measurements differ</u> are used to measure the same length, one will need a greater number of smaller units than longer units to measure the length	Original moved to grade 2 to be replaced by revised 1M02.
1M03	<u>describe a length to the nearest whole unit using</u> write a number and <u>a unit such as five craft sticks to describe a length</u>	
1M04	<u>tell</u> determine the time <u>to the</u> in hours and half hours using analog and digital clocks	

Measurement and Data Analysis.

1M

Knowledge and Skills Statement. The student applies mathematical process standards to organize data to make it useful for interpreting information and solving problems. The student is expected to:

1M05	<p><u>collect, sort, and organize data in up to three categories using models/representations such as tally marks or T-charts</u> classify and sort a set of objects or data into up to three categories or subcategories and use numbers to describe and compare these categories</p>	<p>MN standards PD: tally marks and T-chart work well to organize data.</p>
1M06	<p><u>use data to create picture and bar-type graphs</u> summarize a data set, with up to four categories, using a frequency table or a picture graph</p>	<p>MN standards</p>
1M07	<p><u>draw conclusions and generate and answer questions using information from picture and bar-type graphs</u> about categories of objects or data and determine solutions to these questions (e.g., the number in each category and how many more or less are in one category than in another)</p>	

DRAFT

Grade 2

Mathematical Process Standards Grade 2

- I. Apply mathematics to problems arising in everyday life, society and the workplace.
- II. Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.
- III. Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
- IV. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.
- V. Create and use representations to organize, record, and communicate mathematical ideas.
- VI. Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

VA—Process Standards moved to knowledge and skills statements

Grade 2 Focal Areas

Number and Operations	▲	Making comparisons within Base 10
Number and Operations	●	Solving problems with addition and subtraction within 100
Number and Operations	■	Building foundations for multiplication

Supporting Topics for the Focal Areas in Grade 2 and Grade 3

Number and Operations	▲	Applying place value and counting
	+	Building foundations for fractions
Expressions, Equations, and Relationships	● ■	Fluently producing addition facts and related subtraction facts
	● ■	Using multiple representations of problem situations
Two-Dimensional and Three-Dimensional Figures	●	Determining missing values in number sentences
	+	Identifying and classifying 2D and 3D figures
Measurement and Data	■	Composing 2D and 3D figures out of unit measures
	■	Decomposing 2D figures
	+	Measuring lengths and time
	●	Solving problems involving length
	■	Representing location on a number line
	● ■	Representing and interpreting data

Color and symbol shows the connection between Focal Areas and Supporting Topics.

+

Indicates topic supports Focal Area in Grade 2

Grade 2

Introduction

The desire to achieve educational excellence is the driving force behind the Texas Essential Knowledge and Skills for mathematics, guided by the College and Career Readiness Standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

In Grade 2, instructional time should focus on three critical areas: (A) making comparisons within the base-ten numeration system; (B) solving problems with addition and subtraction within 100; (C) building foundations for multiplication

- A. Students develop an understanding of the base-ten numeration system and place value concepts. Their understanding of base-ten numeration includes ideas of counting in units and multiples of thousands, hundreds, tens and ones, as well as a grasp of number relationships, which they demonstrate in a number of ways.
- B. Students identify situations in which addition and subtraction are useful to solve problems. Students develop a variety of strategies to use efficient, accurate, and generalizable methods to add and subtract multi-digit whole numbers.
- C. Students use the relationship between skip counting and equal groups of objects to represent the addition or subtraction of equivalent sets. This builds a strong foundation for multiplication and division.

Mathematical Process Standards.

Knowledge and Skills Statement. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

apply mathematics to problems arising in everyday life, society, and the workplace

use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, ~~and~~ evaluating the problem-solving process and the reasonableness of the solution

VA—Process Standards moved to knowledge and skills statements

	select tools, including such as real objects, manipulatives, paper/pencil, and technology <u>as appropriate, and</u> or techniques, including such as mental math, estimation, and number sense <u>as appropriate, to</u> solve problems	
	communicate mathematical ideas, reasoning, and their implications using <u>multiple representations, including</u> symbols, diagrams, graphs, and language <u>as appropriate</u>	VA—Process Standards moved to knowledge and skills statements
	create and use representations to organize, record, and communicate mathematical ideas	
	<u>analyze mathematical relationships to connect and communicate mathematical ideas</u>	
	<u>display</u> , explain, display , and justify mathematical ideas and arguments using precise mathematical language in written or oral communications.	

Number and Operations.

2N

Knowledge and Skills Statement. The student applies mathematical process standards to understand how to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system related to place value. The student is expected to:

2N01	<u>use concrete and pictorial models to compose and</u> decompose the value of a numeral number up to 1,200 <u>1,000</u> as a sum of so many <u>thousands</u> , hundreds, so many tens, and so many ones, in more than one way <u>using objects and pictorial models</u> . For example, 364 can be represented as 3 hundreds, 6 tens, and 4 ones, or as 2 hundreds, 15 tens, and 14 ones. (representations may be bundles of an object or pictures of bundles)	At 3 rd grade, numbers move from being adjectives requiring concrete objects to being nouns representing quantities abstractly. Therefore, students need more concrete experience with the thousands period prior to moving to 3 rd grade. <i>PD: emphasize examples such as 364 can be represented as 3 hundreds, 6 tens, and 4 ones, or as 2 hundreds, 15 tens, and 14 ones.</i>
2N02	<u>use standard, word, and expanded forms to</u> represent numbers up to a three-digit number <u>1,200</u> as the sum of the values represented by the digits in the combined value of hundreds, tens, and ones places using objects, expanded notation, and numbers. For example, 493 is the sum of 4 hundreds, 9 tens and 3 ones	Language in standard clarified
2N03	generate a three-digit number that is greater than, or less than, or equal to a given whole number that is greater than 100 and less than 999 <u>up to 1,200</u>	Language in standard clarified
2N04	<u>use place value to compare and order</u> whole numbers <u>up to 1,200</u> 1,000 <u>using comparative language, numbers, and symbols (>, <, or =)</u>	ER, Capraro It is recommended that students not order more than 4 whole numbers.
2N05	represent the comparison of two numbers to 1,000 using the symbols >, <, or =.	

2M06	<u>locate</u> represent the <u>position point on a number line of a given whole number on an open number line</u> that corresponds to a given whole number	SBOE Moved to Number from Geometry / Measurement PD: Clarify what is meant by an open number line.
2M07	<u>name</u> determine the <u>corresponding</u> whole number <u>that corresponds to</u> of a specific specified point on a number line	Moved to Number from Geometry / Measurement
	<u>order whole numbers to 1,200 using place value and open number lines</u>	NCTM Focus in Grade 2 (p 52-54), TXRCFP (p 16), Clements <u>Early Childhood Mathematics Education Research</u> (p 92) PD needs to include open number lines.

Number and Operations.		2N
Knowledge and Skills Statement. The student applies mathematical process standards to recognize and represent fractional units and communicates how they are used to name parts of a whole. The student is expected to:		
2N06	<u>partition objects such as strips, lines,</u> decompose a strip diagram or regular polygons, <u>and circles</u> into equal parts and name the parts, including halves, fourths and eighths, using objects and pictorial representations <u>words such as "one-half," "three-fourths"</u>	Clements and Sarama. <u>Engaging Young Children in Mathematics</u> p. 301 PD: TexTEAMS Day 3
	<u>explain that the more fractional parts used to make a whole, the smaller the part and the fewer the fractional parts, the larger the part</u>	Siebert and Gaskin, "Creating, Naming, & Justifying Fractions", <u>Teaching Children Mathematics</u> , April 2006
2N07	identify and name one part of an equipartitioned whole as a fraction $1/b$ (where b is a non-zero whole number) using strips diagrams and area models that include regular polygons.	Avoiding the use of symbolic fraction notations makes a stronger introduction to fractions.

2N08	<p><u>use concrete models to count fractional parts beyond one whole using words such as “one-fourth,” “two-fourths,” “three-fourths,” “four-fourths,” “five-fourths,” or “one and one-fourth,” and recognize how many parts it takes to equal one whole such as four-fourths equals one whole</u> determine the missing value in a number statement where two fractions with like denominators form one whole, represented with a strip diagram. (e.g., $\frac{2}{7} + \square = \frac{7}{7}$. A strip diagram is separated into 7 equal parts. Two of the parts are shaded blue, and the remaining parts are shaded a second color.)</p>	<p>Watanabe, “Representations in Teaching & Learning Fractions”, Teaching Children Mathematics, April 2002</p> <p>To provide the conceptual understanding needed by 3rd graders to manipulate fractions using pictures and fraction notation and as a transition between recognizing that fractions are fair shares or equal pieces called halves or fourths (from 1st grade), we recommend that this student expectation move beyond 2nd grade to be replaced by the proposed SE.</p>
	<p><u>identify examples and non-examples of halves, fourths, and eighths</u></p>	<p>Watanabe, “Representations in Teaching & Learning Fractions”, Teaching Children Mathematics, April 2002</p>
<p>Number and Operations.</p>		
<p>Knowledge and Skills Statement. The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve addition and subtraction problems with efficiency and accuracy. The student is expected to:</p>		
2N09	<p>determine the number that is and 10 or 100 more or less than a given number between 100 and 900.</p>	<p>Moved to Algebraic Reasoning.</p>
2N10	<p>recall basic fluently produce addition and subtraction facts to add and subtract within sums to 20 and differences from 20 with automaticity</p>	<p>ER, Weilmuenster page 8.</p>
	<p><u>use mental strategies, flexible methods, and algorithms based on knowledge of place value and equality to add and subtract two-digit numbers</u></p>	<p>PD: mental strategies, flexible methods, and the use of place value in standard algorithms</p>
2N11	<p>solve one-step and multistep mathematical and real-world word problems involving addition and subtraction of two-digit numbers within 100 using a variety of strategies based on place value, including algorithms properties of operations, and the relationship between addition and subtraction with fluency</p>	<p>PD: clarify open number lines, properties of operations, and place value strategies</p>
2N12	<p>solve mathematical and real-world problems involving addition and subtraction within 1,000 using strategies based on place value properties of operations, and the relationship between addition and subtraction.</p>	<p>Combined with 2N11</p>
2N13	<p>generate <u>and solve</u> problem situations for a given mathematical number sentence involving <u>adding addition and subtracting subtraction</u> of whole numbers within <u>100 1,000</u></p>	<p>SBOE Students need to demonstrate their understanding of a number sentence by creating a word problem or situation to match it. Source: <u>Adding It Up</u>, John Van de Walle, Doug Clements</p>

2N14	Determine whether a number (up to 40) of objects in a set is even or odd.	Moved to Algebraic Reasoning
Number and Operations.		
Knowledge and Skills Statement. The student applies mathematical process standards to determine the value of coins in order to solve monetary transactions. The student is expected to:		
	<u>determine the value of a collection of coins up to one dollar</u>	Financial literacy requirement
	<u>use the cent symbol, dollar sign, and the decimal point to name the value of a collection of coins</u>	Financial literacy requirement

Number and Operations.		2N
Knowledge and Skills Statement. The student applies mathematical process standards to connect repeated addition and subtraction to multiplication and division situations that involve equal groupings and shares. The student is expected to:		
2N15	<u>model, create, and describe contextual multiplication situations in which equivalent sets of concrete objects are joined</u> arrange a given number of objects into rectangular arrays with up to 5 rows and up to 5 columns	SBOE and ER, Capraro Resource: Fosnot recommends that multiplication represented by arrays be delayed until students develop the spatial understanding of rows and columns.
2N16	<u>model, create, and describe contextual division situations in which a set of concrete objects is separated into equivalent sets</u> determine the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns	Clarification recommended by SBOE member and expert reviewer Capraro Resource: Fosnot recommends that multiplication represented by arrays be delayed until students develop the spatial understanding of rows and columns.

Algebraic Reasoning Expressions, Equations and Relationships.		2A
Knowledge and Skills Statement. The student applies mathematical process standards to identify and apply number patterns within properties of numbers and operations in order to describe relationships. The student is expected to:		
	<u>use relationships and objects to determine whether a number up to 40 is even or odd</u>	VA with 3 rd – 5 th ER, Askey
2NO9	<u>use relationships to</u> determine the number that is and 10 or 100 more or less than a given number between 100 and 999 <u>up to 1,200</u>	Moved from Number because it requires understanding of numerical relationships.

2A01	represent mathematical and real-world problems involving addition and subtraction of whole numbers to 100 using strip diagrams and number sentences (equations).	Duplicate of 2N11 and 2N12 ER, Weilmuenster page 7
2A02	represent mathematical and real-world problems for multiplication to a product of 25 using arrays, strip diagrams, and number sentences (equations).	Replaced with standard using equal groups of objects to introduce multiplication & division.
	<u>represent and solve addition and subtraction word problems where unknowns may be any one of the terms in the problem</u>	VA SBOE and ER, Capraro
2A03	determine the unknown whole number in an addition or subtraction equation relating three whole numbers when the unknown may be any one of the three terms in the equation. For example, the value 27 for $[\]$ makes $12 + [\] = 39$ a true equation.	SBOE Repeat of standard above
Geometry and Measurement Two-Dimensional and Three-Dimensional Figures.		2G
Knowledge and Skills Statement. The student applies mathematical process standards to analyze attributes of two- and three-dimensional geometric figures to develop generalizations about their properties. The student is expected to:		
2G01	create build and draw two-dimensional shapes based on given attributes, <u>including e.g.,</u> number of sides (less than or equal to six) or and vertices	Language in standard clarified
2G02	identify <u>attributes of two-dimensional shapes including a</u> quadrilaterals, (including parallelograms), pentagons, and octagons	Language in standard clarified
2G03	classify <u>and sort</u> three-dimensional solids, (including cones, cylinders, spheres, <u>triangular</u> and rectangular prisms <u>including and</u> cubes), based on attributes <u>using formal geometric language such as faces, edges, and vertices (e.g., number of faces, edges, or vertices)</u>	Language in standard clarified; PD: the process of classifying & sorting must include description of classifying rules.
	<u>classify and sort polygons with 12 or fewer sides according to attributes, including identifying the number of sides and number of vertices</u>	VA
2G04	compose two-dimensional shapes and three-dimensional <u>solids shapes</u> with given properties or attributes (e.g., such as build a rectangle out of unit squares; build a rectangular prism out of unit cubes)	Language in standard clarified.
2G05	decompose two-dimensional shapes (e.g., such as cutting out a square from this rectangle; <u>divideing</u> this shape in half; <u>or</u> partitioning a rectangle into identical triangles, <u>and identify the resulting geometric parts</u>)	SBOE
2G06	illustrate the area of a rectangle with whole number side lengths as the number of unit squares (n-square units) needed to cover it with no gaps or overlaps. A “unit square” is a square with side length of 1 unit having “one square unit of area”	Moved to Measurement

Geometry and Measurement and Data.		2M
Knowledge and Skills Statement. The student applies mathematical process standards to select and use units to describe length, area, and time. The student is expected to:		
2M01	find <u>illustrate</u> the length of objects using concrete models for standard units of length <u>such as the edges of inch tiles and centimeter cubes</u>	ER, Milgram
	<u>describe the inverse relationship between the size of the unit and the number of units needed to equal the length of an object such as the longer the unit, the fewer units needed and the shorter the unit, the more units needed</u>	ER, Weilmuenster, page 7
2M05	represent whole numbers as distances from <u>any given location</u> zero on a number line	ER, Milgram and Weilmuenster
2M02	determine the length of an object <u>to the nearest half unit</u> using rulers, yardsticks, meter sticks, or measuring tapes to the nearest marked unit	SBOE
2M03	determine a solution to mathematical and real-world <u>a</u> problems involving length, including estimating lengths and using length as a model for addition and subtraction	
2GO6	<u>use concrete models of square units to find the area of a rectangle by covering it with no gaps or overlaps, counting to find the total number of square units, and describing the measurement using a number and the unit such as 24 square units</u> illustrate the area of a rectangle with whole number side lengths as the number of unit squares (n-square units) needed to cover it with no gaps or overlaps. A "unit square" is a square with side length of 1 unit having "one square unit of area"	Moved from Geometry because it is an introduction to area measurement. Language in standard clarified
2M04	<u>read and write</u> determine time to the nearest <u>five- and one-minute increments</u> using analog and digital clocks <u>and distinguish between a.m. and p.m.</u>	Alignment between 1 st and 3 rd .

Measurement and Data <u>Analysis</u>		2M
Knowledge and Skills Statement. The student applies mathematical process standards to organize data to make it useful for interpreting information and solving problems. The student is expected to:		
2M08	explain that the length of a bar in a bar graph or the number of pictures in a picture <u>pictograph</u> represents the number of data points for a given category	
2M09	<u>organize a collection of data summarize a data set</u> , with up to four categories, using a frequency table, a dot plot, a picture <u>pictographs, or and a bar graphs with intervals of one or more</u> with the vertical axis scaled in increments of one	Language in standard clarified.

2M10	<p><u>write and</u> solve one-step mathematical and real world <u>word</u> problems involving addition or subtraction using categorical data represented with <u>in a frequency table, a dot plot, a picture pictographs, or and</u> a bar graphs with unit intervals <u>of one</u></p>	Language in standard clarified.
	<p><u>draw conclusions and make predictions from information in a graph</u></p>	<p><u>Adding It Up</u>, Van de Walle, NCTM Focus in Grade Two: Teaching with the Curriculum Focal Points</p>

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Grade 3

Mathematical Process Standards Grade 3

- I. Apply mathematics to problems arising in everyday life, society and the workplace.
- II. Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.
- III. Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
- IV. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.
- V. Create and use representations to organize, record, and communicate mathematical ideas.
- VI. Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

VA—Process Standards moved to knowledge and skills statements

Grade 3 Focal Areas

Number and Operations	▲	Solving multi-step addition and subtraction problems with whole numbers within 1000
Number and Operations	●	Solving problems with multiplication and division within 100
Number and Operations	■	Understanding and representing fractions as numbers and equivalent fractions

Supporting Topics for the Focal Areas in Grade 3 and Grade 4

Number and Operations	▲ ▲ ■ ▲■	Applying place value Comparing and ordering whole numbers Representing points on a number line that correspond to a given fraction Connecting multiplication and division
Expressions, Equations, and Relationships	●■ ▲●■ ▲	Using multiple representations of problem situations Determining missing values in number sentences Representing real-world relationships using number pairs in a table and verbal description
Two-Dimensional and Three-Dimensional Figures	▲● + +	Relating area to multiplication and to addition Identifying and classifying 2D according to common attributes Decomposing composite figures formed by rectangles to determine area
Measurement and Data	▲ ▲■ ▲ ■ ▲●■	Determining perimeter of polygons Solving problems involving time Measuring liquid volume (capacity) Representing location on a number line Representing and interpreting data
Color and symbol shows the connection between Focal Areas and Supporting Topics. + Indicates topic supports Focal Area in Grade 4		

Grade 3

Introduction

The desire to achieve education excellence is the driving force behind the Texas Essential Knowledge and Skills for mathematics, guided by the College and Career Readiness Standards. By embedding statistics, probability, and finance, and focusing on fluency and solid understandings, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

The process standards describe ways in which students are expected to engage in the content. The placement of the process skill at the beginning of the draft is intentional. The process skills weave the other knowledge and skills together so that students may be successful problems solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

The standards are not a scope and sequence. When possible, the order does reflect a progression of learning, but the order is not a mandated sequence for instruction. The ordering or sequencing for instruction is a local decision. The kindergarten through eighth grade standards are organized by mathematics topic areas or strands, and the high school standards are organized by customary course titles.

For students to become fluent in mathematics students must develop a robust sense of number. The National Research Council's report, *Adding It Up*, defines *procedural fluency* as "skill in carrying out procedures flexibly, accurately, efficiently and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 3 are expected to perform their work without the use of calculators.

The primary focal areas in Grade 3 are place value, operations of whole numbers, and understanding fractional units. These focal areas are supported throughout the mathematical strands of number and operations, algebraic reasoning, geometry and measurement, and data analysis. In Grades 3-5 the number set is limited to positive rational numbers. In number and operations, students will focus on applying place value, comparing and ordering whole numbers, connecting multiplication and division, and understanding and representing fractions as numbers and equivalent fractions. In algebraic reasoning, students will use multiple representations of problem situations, determine missing values in number sentences, and represent real-world relationships using number pairs in a table and verbal descriptions. In geometry and measurement, students will identify and classify two-dimensional figures according to common attributes, decompose composite figures formed by rectangles to determine area, determine the perimeter of polygons, solve problems involving time, and measure liquid volume (capacity) or weight. In data analysis, students will represent and interpret data.

Mathematical Process Standards		
Knowledge and Skills Statement. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:		VA—Process Standards moved to knowledge and skills statements
	apply mathematics to problems arising in everyday life, society, and the workplace	
	use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process <u>and the reasonableness of the solution</u>	
	select tools, including such as real objects, manipulatives, paper/pencil, and technology <u>as appropriate, and</u> or techniques, including such as mental math, estimation, and number sense <u>as appropriate, to solve problems</u>	
	communicate mathematical ideas, reasoning, and their implications using <u>multiple representations, including symbols, diagrams, graphs, and language as appropriate</u>	
	create and use representations to organize, record, and communicate mathematical ideas	
	<u>analyze mathematical relationships to connect and communicate mathematical ideas</u>	
	<u>display</u> , explain, display, <u>and</u> justify mathematical ideas and arguments using precise mathematical language in written or oral communications	

Number and Operations.		3N
Knowledge and Skills Statement The student applies mathematical process standards to represent and compare whole numbers and understand relationships related to place value. The student is expected to:		
3N01	<u>compose and decompose numbers</u> represent value of a numeral to 10,000 <u>100,000 as a sum of so many ten thousands, so many thousands, so many hundreds, so many tens, and so many ones, in more than one way,</u> using objects, and pictorial models, <u>and numbers, including expanded notation as appropriate</u> that address the notion of bundling (composing and decomposing)	ER—Capraro, 1, Askey, 18
	<u>describe the mathematical relationships found in the base-ten place value system through the 100,000th place</u>	ER—Capraro, 1, Askey, 18
3N02	represent the value of the digit in whole numbers through 10,000 using expanded notation and numerals. For example, for the number 4,093, the 4 in the thousands places is 4,000; the 9 in the tens place is 90; and the 3 in the ones place is three; and 4,093 is the sum of 4 thousands, 0 hundreds, 9 tens, and 3 ones	
	<u>represent a number on a number line as being between two consecutive multiples of 10, 100, 1000, or 10,000 and use words such as “closer to,” “is about,” or “is nearly,” in order to round whole numbers to describe relative size of numbers</u>	

3N03	round whole numbers to the nearest 10, 100, or 1,000	clarification
3N04	compare and order whole numbers up to 10,000 <u>100,000</u> and represent comparisons using the symbols $>$, $<$, or $=$	ER—Capraro, 2
3N05	represent the comparison of two numbers to 10,000 using the symbols $>$, $<$, or $=$	Moved to 3N04
Knowledge and Skills Statement. The student applies mathematical process standards to represent and explain fractional units. The student is expected to:		
3N06	represent fractions greater than zero <u>and less than or equal to one</u> in mathematical and real-world problems using <u>concrete</u> objects and pictorial models, including strip diagrams and number lines, <u>with denominators of 2, 3, 4, 6, and 8</u>	ER—Weilmuenster, Askey, Schmid PD: Strip diagram in glossary
3N07	represent the point on a number line that corresponds to a given fraction greater than 0	Included in 3N06
	<u>determine the corresponding fraction greater than zero and less than or equal to one with denominators of 2, 3, 4, 6, and 8 to a specified point on a number line</u>	aligns to and supports 2nd grade moved back from measurement
3N08	explain that <u>the unit fraction $1/b$</u> represents the quantity formed by one part of a whole that has been partitioned into b equal parts where b is a non-zero whole number	ER—Weilmuenster, 10 clarification
3N09	<u>compose and decompose a fraction a/b, with a numerator greater than zero and less than or equal to b as a sum of parts $1/b$ where a is a whole number and b is a non-zero whole number, represents the quantity formed by a parts of size $1/b$</u>	ER—Schmid, 3, Weilmuenster, 10 clarification
	<u>solve problems involving partitioning an object or a set of objects among two or more recipients using pictorial representations of fractions with denominators of 2, 3, 4, 6, and 8, such as two children share five cookies</u>	ER—Weilmuenster, 10 Resource the IES Fraction Guide
3N10	represent equivalent fractions <u>with denominators of 2, 3, 4, 6, and 8</u> using <u>a variety of</u> objects and pictorial models, including number lines	ER—Schmid, 2 and Weilmuenster, 14
3N11	explain that two fractions are equivalent if and only if they are both represented by the same point on the number line or represent the same portion of a same size whole for an area model	
3N12	compare two fractions <u>having the same numerator or denominator</u> in mathematical and real-world problems by reasoning about their sizes and justifying the conclusion using symbols, words, objects, and pictorial models, including strip diagrams and number lines, (fractions being compared should have the same numerator or the same denominator) <u>such as comparing the size of pieces when sharing a candy bar equally among four people or equally among three people</u>	ER—Schmid, 2 and Weilmuenster, 14
Knowledge and Skills Statement. The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve problems with efficiency and accuracy. The student is expected to:		

3N13	solve one-step and multistep mathematical and real-world problems involving addition and subtraction within 1,000 with fluency using strategies based on place value, properties of operations, and the relationship between addition and subtraction	
	<u>use strategies, including rounding to the nearest 10 or 100 and compatible numbers, to estimate solutions to addition and subtraction problems</u>	
	<u>determine the value of a collection of coins and bills</u>	Personal finance
3N14	determine the total number of objects when equally-sized groups of objects are combined or arranged in arrays <u>up to 10 by 10</u> (e.g., 4 groups, each having 7 objects, combine to make a new group of 28 objects)	VA & MV
3N19	<u>represent multiplication facts by using a variety of approaches such as repeated addition, equal-sized groups, arrays, area models, equal jumps on a number line, and skip counting</u> determine the unknown whole number in multiplication and division equations relating three whole numbers (e.g., $8x=24$, $5=?\div 3$, $7\times 6=?$)	SBOE—Cargill
3N20	<u>quickly recall of facts to multiply up to 10 by 10 and recall the corresponding division facts</u> produce with fluency multiplication and division facts with products to 100 and dividends from 100	ER Rigor has been increased because this used to be a 4 th grade expectation.
3N15	<u>use strategies and algorithms, including the standard algorithm, to multiply a two-digit number by a one-digit number. Strategies may include mental math, partial products, and the commutative, associative, and distributive properties</u> determine products using properties of operations (e.g., $5 \times 8 = 40$, so $8 \times 5 = 40$; $2 \times 3 \times 4 = (2 \times 3) \times 4 = 6 \times 4 = 24$; $6 \times 8 = 6 \times (5+3) = 6 \times 5 + 6 \times 3 = 30 + 18 = 48$)	VA & MV Per SBOE—Cargill
3N16	determine the product of a one-digit whole number and multiples of 10 in the range 10-90 (e.g. 8×90, 7×60) using strategies based on place value and properties of operations	Part of 3N15
3N17	determine the number of objects in each group when a set of objects are is partitioned into equal shares or a set of objects are is shared equally (e.g., the number of objects in each share when 28 objects are partitioned equally into 7 shares, or as a number of shares when 28 objects are partitioned into equal shares with 7 objects each)	Grammatical error (are/is)
	<u>use divisibility rules to determine if a number is even or odd</u>	Adding on to 2 nd grade introduction of even and odd
3N18	determine a quotient using the relationship between multiplication and division <u>such as</u> (e.g., the quotient of $40 \div 8$ can be found by determining what <u>factor</u> makes 40 when multiplied by 8)	
3N21	solve one-step and multistep mathematical and real-world problems involving multiplication and division within 100 using strategies based on objects, <u>properties of operations, recall of facts, or</u> pictorial models, (including arrays, area models, and equal groups), properties of operations, or recall of facts	
3N22	solve one-step and multi-step mathematical and real-world problems involving addition, subtraction, multiplication, or division. (Problems may include operations with whole-number measures of length, capacity, or mass.)	Covered in 3N13 and 3N21

Algebraic Reasoning Expressions, Equations and Relationships.		3A
Knowledge and Skills Statement. The student applies mathematical process standards to analyze and create patterns and relationships. The student is expected to:		
3A01	represent <u>and solve</u> one- and two-step mathematical and real-world problems involving single and addition and subtraction of whole numbers to 1,000 using <u>pictorial models, such as</u> strip diagrams and number <u>lines,</u> and sentences (equations)	Combination of ER and committee
3A02	represent <u>and solve</u> one- and two-step multiplication and division mathematical and real-world problems within 100 using arrays, strip diagrams, and number sentences (equations)	Combination of ER and VA
3A03	describe a multiplication expression as a comparison. For example, such as 3 x 24 represents 3 times as much as 24	
3A04	determine the unknown whole number in a multiplication or division equation relating three whole numbers when the unknown is either a missing factor or product. For example, such as the value 4 for [] makes 3 x [] = 12 a true equation	
3A05	represent real-world relationships using number pairs in a table and verbal descriptions. For example, such as 1 insect has 6 legs, 2 insects have 12 legs, and so forth	

Geometry and Measurement Two-Dimensional and Three-Dimensional Figures.		3G
Knowledge and Skills Statement. The student applies mathematical process standards to analyze attributes of two-dimensional geometric figures to develop generalizations about their properties. The student is expected to:		
3G01	<u>classify and sort two- and three-dimensional solids, including cones, cylinders, spheres, triangular and rectangular prisms and cubes</u> explain why polygons with 12 or fewer sides may share <u>according to based on attributes, identify</u> using formal geometric language <u>such as faces, edges, and vertices</u> the number of sides and number of vertices that define a larger category for classification purposes. For example, rhombuses, parallelograms, rectangles, and squares all have four sides and may be classified as quadrilaterals. (They do not all have four right angles, so they may not all be classified as rectangles.)	
3G02	recognize rhombuses, parallelograms, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories	Moved to 4 th grade
3G03	determine the area of rectangles (with whole number side lengths) in mathematical and real-world problems using multiplication related <u>ing the multiplications</u> to the number of rows times the number unit squares in each row	
3G04	decompose composite figures formed by rectangles into two non-overlapping rectangles to determine the area of the original figure using the additive property of area	

	<u>decompose two congruent two-dimensional figures into parts with equal areas and express the area of each part as a unit fraction of the whole and recognize that equal shares of identical wholes need not have the same shape</u>	Resource CCSS 3g2, 2g3
Knowledge and Skills Statement. The student applies mathematical process standards to select appropriate units, strategies, and tools to solve problems involving customary measurement. The student is expected to:		
	<u>represent fractions of halves, fourths, and eighths as distances from zero on a number line</u>	VA
3M01	determine the perimeter of a polygon or a missing length when given perimeter and remaining side lengths in mathematical and real-world problems	
3M02	determine the solution to mathematical and real-world problems involving addition and subtraction of time intervals in minutes, <u>using pictorial models or tools such as a 15-minute event plus a 30-minute event equals 45 minutes</u>	CCSS Excluding elapsed time, tools such as geared clocks and number lines
3M03	determine when it is appropriate to use measurements of liquid volume (capacity) or mass weight	Length is covered in 3M01
3M04	determine liquid volume (capacity) or mass weight using appropriate units and tools	

Data Analysis Measurement and Data.		3M
Knowledge and Skills Statement. The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:		
3M05	summarize a data set with multiple categories using <u>a frequency table, a dot line plot, a pictograph, or a bar graph with scaled intervals</u> (e.g., each picture or interval represents five data points)	Not needed, wording is confusing.
3M06	solve one and two-step mathematical and real-world problems using categorical data represented with a frequency table, <u>a dot line plot, a pictograph, or a bar graph with scaled intervals</u>	

Grade 4

Mathematical Process Standards Grade 4

- I. Apply mathematics to problems arising in everyday life, society and the workplace.
- II. Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.
- III. Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
- IV. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.
- V. Create and use representations to organize, record, and communicate mathematical ideas.
- VI. Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

VA—Process Standards moved to knowledge and skills statements

Grade 4 Focal Areas

Number and Operations	▲	Solving problems with multi-digit addition, subtraction, multiplication and division of whole numbers
Number and Operations	●	Building fractions from unit fractions and comparing fractions Using decimal notation and comparing decimals
Expressions, Equations, and Relationships	■	Extending measurement to area and perimeter formulas

Supporting Topics for the Focal Areas in Grade 4 and Grade 5

Number and Operations	▲ ● ▲ ▲ ●	Applying place value Identifying prime and composite numbers Representing points on a number line that correspond to a given fraction or terminating decimal
Expressions, Equations, and Relationships	▲ ● ■ ▲ ● ■ ▲ ● ■	Representing multi-step problems involving the four operations with whole numbers with expressions and equations Solving multi-step problems involving the four operations with whole numbers with expressions and equations Generating and analyzing patterns
Two-Dimensional and Three-Dimensional Figures	■	Classifying 2D figures
Measurement and Data	■ ▲ ● ■ ▲	Measuring angles Converting units of measure Representing and interpreting data
Color and symbol shows the connection between Focal Areas and Supporting Topics. + Indicates topic supports Focal Area in Grade 5		

Grade 4

Introduction

The desire to achieve education excellence is the driving force behind the Texas Essential Knowledge and Skills for mathematics, guided by the College and Career Readiness Standards. By embedding statistics, probability, finance, and focusing on fluency and solid understandings, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

The process standards describe ways in which students are expected to engage in the content. The placement of the process skill at the beginning of the draft is intentional. The process skills weave the other knowledge and skills together so that students may be successful problems solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

The standards are not a scope and sequence. When possible, the order does reflect a progression of learning, but the order is not a mandated sequence for instruction. The ordering or sequencing for instruction is a local decision. The kindergarten through eighth grade standards are organized by mathematics topic areas or strands, and the high school standards are organized by customary course titles.

For students to become fluent in mathematics students must develop a robust sense of number. The National Research Council's report, *Adding It Up*, defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 4 are expected to perform their work without the use of calculators.

The primary focal areas in Grade 4 are use of operations, fractions and decimals, and describing and analyzing geometry and measurement. These focal areas are supported throughout the mathematical strands of Number and Operations, Algebraic Reasoning, Geometry and Measurement and Data Analysis. In grades 3-5 the number set is limited to positive rational numbers. In Number and Operations, students will apply place value, and represent points on a number line that correspond to a given fraction or terminating decimal. In Algebraic Reasoning, students will represent and solve multistep problems involving the four operations with whole numbers with expressions and equations and generate and analyze patterns. In Geometry and Measurement, students will classify two-dimensional figures, measure angles, and convert units of measure. In Data Analysis students will represent and interpret data.

Mathematical Process Standards

Knowledge and Skills Statement. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

	apply mathematics to problems arising in everyday life, society, and the workplace	VA—Process Standards moved to knowledge and skills statements
	use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process <u>and the reasonableness of the solution</u>	
	select tools, including such as real objects, manipulatives, paper/pencil, and technology <u>as appropriate, and of</u> techniques, including such as mental math, estimation, and number sense <u>as appropriate, to</u> solve problems	
	communicate mathematical ideas, reasoning, and their implications using <u>multiple representations, including</u> symbols, diagrams, graphs, and language <u>as appropriate</u>	
	create and use representations to organize, record, and communicate mathematical ideas	
	<u>analyze mathematical relationships to connect and communicate mathematical ideas</u>	
	<u>display,</u> explain, display, <u>and</u> justify mathematical ideas and arguments using precise mathematical language in written or oral communications	

Number and Operations.

4N

Knowledge and Skills Statement. The student applies mathematical process standards to represent, compare, and order whole numbers and decimals and understand relationships related to place value. The student is expected to:

4N01	explain the meanings of the tenths and hundredths place value positions using fractions	Been absorbed into 4N03 and 4N05
4N02	interpret the value of each place-value position as 10 times the position to the right <u>and as 1/10 of the value of the place to its left</u>	Added from 5N02, supports the money part of 4N03.
4N04	represent the value of the digit in whole numbers through <u>1,000,000,000</u> 1,000,000 and decimals to the hundredths using expanded notation and numerals. For example, for such as in the number 3.94, the 3 in the ones place is <u>3 three</u> ; the 9 in the tenths place is 0.9; and 4 in the hundredths place is 0.04; and 3.94 is sum of 3 ones, 9 tenths, and 4 hundredths	Moved from 5th grade.
4N07	compare and order whole numbers to one million <u>1,000,000,000</u> and <u>represent comparisons using the symbols >, <, or =</u>	Taken from 4N08

4N06	round whole numbers to <u>a given place value through</u> the nearest 10,000 or 100,000's <u>place</u>	There might be situations where students may estimate or round to other places.
4N03	represent decimals, including tenths and hundredths, using concrete and visual models and money	Senate Bill 290
4N09	compare and order decimals using concrete and visual models <u>to the hundredths</u>	Clarification
4N05	<u>relate decimals to fractions that name tenths and hundredths</u> represent terminating decimals as fractions with denominators of 10 or 100	Clarity ER—Schmid
	represent the comparison of two numbers to one million using the symbols >, <, or =	ER—Capraro, 1 or move to 4N07
	<u>determine the corresponding decimal to the tenths or hundredths place of a specified point on a number line</u>	Aligns to and supports 2nd and 3rd grade. Supports a focal area. Moved back from Measurement
	represent a point on a number line that corresponds to a given fraction or terminating decimal	Moved to Measurement
<p>Knowledge and Skills Statement. The student applies mathematical process standards to represent and generate fractions to solve problems. The student is expected to:</p>		
4N11	represent a fraction a/b as a sum of fractions $1/b$, where a and b are whole numbers and $b > 0$, <u>including when $a > b$</u>	ER—Schmid, 4 Clarification
4N12	decompose a fraction <u>in more than one way</u> into a sum of fractions with the same denominator in more than one way, recording each decomposition using <u>concrete and pictorial models</u> and <u>recording results with symbolic representations such as</u> (e.g., $7/8 = 5/8 + 2/8$; $7/8 = 3/8 + 4/8$; $2\ 7/8 = 1 + 1 + 7/8$; $2\ 7/8 = 8/8 + 8/8 + 7/8$)	ER—Weilmuenster, 13 Clarification
4N14	determine if two given fractions are equivalent <u>using a variety of methods, including multiplying by a fraction equivalent to one or simplifying a fraction to lowest terms</u>	Alignment to connect to 3rd grade <u>Absorbed 4N13</u>
4N15	generate equivalent fractions to create <u>common equal</u> numerators or <u>common equal</u> denominators to compare two fractions with different unequal numerators and different unequal denominators <u>and represent the comparison of two fractions using the symbols >, <, or =</u>	ER—Weilmuenster, 13 Clarification
4N17	represent <u>and solve</u> addition and subtraction of positive fractions with <u>like equal</u> denominators and referring to the same whole, using objects and pictorial models that build to the number line (such as strip diagrams) and properties of operations (includes fractions as decimals with like denominators of tenths or hundredths (e.g., $1/10 + 0.3$)) .	Consistent vocabulary 5N06 <u>ER—Askey</u>

4N19	estimate the reasonableness of answers sums and differences using positive benchmark fractions $\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \text{and } 1\}$, referring to the same whole For example, if $\frac{1}{2}$ is an addend, the sum must be greater than or equal to $\frac{1}{2}$ if added to a positive number	ER & MV
	<u>represent fractions and decimals to the tenths or hundredths as distances from zero on a number line</u>	ER—Weilmuenster, 14 and Askey, 11 Moved 4N10 and changed. Supports a focal area. Moved back from Measurement
	<u>determine fractional and decimal quantities as being close to 0, $\frac{1}{2}$, and 1</u>	Preparation for 4N19
4N13	explain that $\frac{a}{b}$ and $(n \times a)/(n \times b)$ (where a and b are integers) are equivalent fractions using objects and pictorial models	ER Weilmuenster 13 Clarification put in glossary
4N18	solve mathematical and real-world problems involving positive sums and differences of positive fractions, including mixed numbers, with like denominators referring to the same whole, with fluency.	Consistent vocabulary combined with 4N17
Knowledge and Skills Statement. The student applies mathematical process standards to develop and use strategies and methods for whole number computations and decimal sums and differences in order to solve problems with efficiency and accuracy. The student is expected to:		
4N16	add and subtract whole numbers and decimals to the hundredths place using <u>a variety of methods, including pictorial models, <u>the inverse relationship between operations</u>, concepts of place value, and <u>efficient algorithms and properties of addition</u></u>	ER
4N20	determine products of a number and 10 or 100 using properties of operations and place value understandings	
4N21	represent the product of up to a four-digit number by a one-digit number using arrays, area models or equations	Absorbed into 4N23
4N22	represent the product of 2 two-digit numbers using arrays, area models, or equations, <u>including perfect squares through 15×15</u>	<u>*Perfect square in glossary</u>
4N23	<u>use strategies and algorithms, including the standard algorithm, to multiply up to a four-digit number by a one-digit number and to multiply a two-digit number by a two-digit number. Strategies may include mental math, partial products, and the commutative, associative, and distributive properties</u> determine products of up to a four-digit number and a one-digit number or two-digit numbers using properties of operations (e.g., 34×27 is $34 \times (2 \times 10 + 7) = (34 \times 2 \times 10) + 34 \times 7 = 68 \times 10 + 238 = 680 + 238 = 918$)	ER & MV
4N24	represent the quotient of up to a four-digit whole number divided by a one-digit whole number using arrays, area models, or equations	
4N25	<u>use strategies and algorithms, including the standard algorithm, to divide</u> determine quotients of up to a four-digit dividend <u>by</u> and a one-digit divisor using properties of operations, place value understandings (e.g., partial quotients), or the relationship between multiplication and division	

	<u>use strategies, including rounding to the nearest 10, 100, or 1,000 and compatible numbers, to estimate solutions</u>	
4N26	solve one and two-step mathematical and real-world problems involving multiplication (including scalar comparisons) and division, (including interpreting remainders) <u>with fluency</u>	ER & group consensus

Algebraic Reasoning ~~Expressions, Equations and Relationships.~~

4G

Knowledge and Skills Statement. The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:

4A01	represent multistep mathematical and real-world problems involving the four operations with whole numbers using strip diagrams and equations with a letter standing for the unknown quantity	<u>Back to original</u>
4A02	represent mathematical and real-world and problems using <u>a an input-output</u> table and numerical expressions to generate a number pattern that follows a given rule. For example, such as given the rule “Add 3” and the starting number 1, use the expressions 1 + 3, 2 + 3, 3 + 3, and so forth to generate a table to represent the relationship of the values in the resulting sequence and their position in the sequence	ER
4A03	<u>use models to</u> determine the formulas for the perimeter of a rectangle <u>($l + w + l + w$ or $2l + 2w$)</u> , including the special form for perimeter of a square <u>($4s$)</u> and the area of a rectangle <u>($l \times w$)</u>	<u>Formulas for the STAAR chart</u>
4A04	determine solutions to mathematical and real-world <u>solve</u> problems related to perimeter and area <u>of</u> (rectangles). where (D)dimensions are all positive whole numbers.)	

Geometry and Measurement ~~Two-Dimensional and Three-Dimensional Figures.~~

4G

Knowledge and Skills Statement. The student applies mathematical process standards to analyze geometric attributes in order to develop generalizations about their properties. The student is expected to:

4G01	identify points, lines, line segments, rays, angles (right, acute, obtuse) , and perpendicular and parallel lines	<u>Acute, right, obtuse and straight angles.</u>
4G03	<u>identify and</u> draw <u>a one or more lines</u> of symmetry, if it <u>they</u> exist s , for a two-dimensional figure	<u>Added Identify from 4G05</u>
	<u>apply knowledge of right angles to identify acute, right, and obtuse triangles</u>	Essential for future geometric strands. (ER—Askey)
	<u>use attributes to recognize rhombuses, parallelograms, trapezoids, rectangles, and squares as examples of quadrilaterals and draw examples of quadrilaterals that do not belong to any of these subcategories</u>	Moved from 3G02 and added trapezoids

4G02	classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size (The classification of triangles is limited to those that are right triangles and those that are not)	Because we will classify triangles by acute, obtuse and right. We will not classify by sides, yet.
4G04	identify two-dimensional shapes that have a line of symmetry	Combined with 4G04.

Knowledge and Skills Statement. The student applies mathematical process standards to solve problems involving angles less than or equal to 180 degrees. The student is expected to:

4M01	illustrate the measure of an angle as the part of a circle (whose center is at the vertex of the angle) , that is “cut out” by the rays of the angle. (Angle measures are limited to whole numbers)	4M01
4M02	illustrate degrees as the units used to measure an angle, where 1/360 of any circle is 1 degree and an angle that “cuts” $n/360$ out of any circle whose center is at the angle’s vertex has a measure of n degrees. (Angle measures are limited to whole numbers)	4M02
4M03	determine the approximate measures of angles in degrees <u>to the nearest whole number</u> using a protractor to the nearest whole number	4M03 SBOE, Lowe
4M04	draw an angle with a given measure	4M04
4M05	decompose angles <u>such as complementary and supplementary angles</u> into two non-overlapping angles to determine the measure of an unknown angle in mathematical and real-world problems using the additive property of angle measure	

Knowledge and Skills Statement. The student applies mathematical process standards to select appropriate customary and metric units, strategies, and tools to solve problems involving measurement. The student is expected to:

4M06	identify relative sizes of measurement units within the customary <u>and metric</u> systems	
4M07	convert the measurements <u>within the same measurement system, customary or metric, of a</u> smaller unit into a larger unit or a larger unit into a smaller unit within the customary system when given other equivalent measures represented in a table	clarification
4M08	determine a solution to real-world and mathematical <u>solve</u> problems <u>that deal with measurements of length, intervals of time, liquid volumes, masses, and money using</u> involving addition, subtraction, multiplication, and <u>or</u> division <u>as appropriate</u> of measurements of length, intervals of time, liquid volumes, masses, and money	SBOE, Lowe

Measurement and Data Analysis.

4MD

Knowledge and Skills Statement. The student applies mathematical process standards to solve problems by collecting, organizing, displaying and interpreting data. The student is expected to:

4M09	represent data that can be ordered on <u>a frequency table</u> , a <u>dot line</u> plot, or a stem and leaf plot marked with whole numbers and fractions	VA
4M10	solve one and two-step mathematical and real-world problems using data (in whole number, decimal, and fraction form) in a frequency table, a <u>dot line</u> plot, or a stem and leaf plot. For example, determine the difference in length between the tallest and shortest student in a class from data represented using a dot plot	VA

DRAFT

Grade 5

Mathematical Process Standards Grade 5

I.	Apply mathematics to problems arising in everyday life, society and the workplace.	VA—Process Standards moved to knowledge and skills statements
II.	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.	
III.	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.	
IV.	Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.	
V.	Create and use representations to organize, record, and communicate mathematical ideas.	
VI.	Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.	

Grade 5 Focal Areas

Number and Operations	▲	Solving problems with the addition and subtraction of fractions and decimals
Number and Operations	●	Solving problems with multiplication and division of decimals and beginning understandings for the multiplication and division of fractions
Expressions, Equations, and Relationships	■	Extending measurement to area and volume formulas

Supporting Topics for the Focal Areas in Grade 5 and Grade 6

Number and Operations	▲ ● ▲ ●	Applying place value Identifying part-to-whole relationships and equivalence
Expressions, Equations, and Relationships	▲ ● ■ ▲ ● ■ ▲ ● ■ +	Representing problems with expressions and equations Solving problems with expressions and equations Building foundations of functions through patterning Using the order of operations
Two-Dimensional and Three-Dimensional Figures	■	Classifying 2D figures
Measurement and Data	■ ● ■ ■ ■	Connecting geometric attributes and measures of 3D figures Using units of measure Representing location using a coordinate plane Representing and interpreting data

Color and symbol shows the connection between Focal Areas and Supporting Topics.

+

Indicates topic supports Focal Area in Grade 6

Grade 5

Introduction

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The standards are not a scope and sequence. When possible the order does reflect a progression of learning, but the order is not a mandated sequence for instruction. The ordering or sequencing for instruction is a local decision. The Kindergarten through Grade 8 standards are organized by mathematics topic areas or strands and the high school standards are organized by customary course titles.

For students to become fluent in mathematics students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines *procedural fluency* as "skill in carrying out procedures flexibly, accurately, efficiently and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance. Students in Grade 5 are expected to perform their work without the use of calculators.

The primary focal areas in Grade 5 are solving problems involving all four operations with positive rational numbers, determine and generate formulas and solutions to expressions, and extending measurement to area and volume. These focal areas are supported throughout the mathematical strands of Number and Operations, Algebraic Reasoning, Geometry and Measurement, and Data Analysis. In Grades 3-5 the number set is limited to positive rational numbers. In Number and Operations, students will apply place value and identify part-to-whole relationships and equivalence. In Algebraic Reasoning, students will represent and solve problems with expressions and equations, build foundations of functions through patterning, identify prime and composite numbers, and use the order of operations. In Geometry and Measurement, students will classify two-dimensional figures, connect geometric attributes to the measures of three-dimensional figures, use units of measure, and represent location using a coordinate plane. In Data Analysis, students will represent and interpret data.

Mathematical Process Standards		
Knowledge and Skills Statement. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:		VA—Process Standards moved to knowledge and skills statements
	apply mathematics to problems arising in everyday life, society, and the workplace	
	use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process <u>and the reasonableness of the solution</u>	
	select tools, including such as real objects, manipulatives, paper/pencil, and technology <u>as appropriate, and</u> or techniques, including such as mental math, estimation, and number sense <u>as appropriate, to</u> solve problems	
	communicate mathematical ideas, reasoning, and their implications using <u>multiple representations, including</u> symbols, diagrams, graphs, and language <u>as appropriate</u>	
	create and use representations to organize, record, and communicate mathematical ideas	
	<u>analyze mathematical relationships to connect and communicate mathematical ideas</u>	
	<u>display,</u> explain, display, <u>and</u> justify mathematical ideas and arguments using precise mathematical language in written or oral communications	

Number and Operations.		5N
Knowledge and Skills Statement. The student applies mathematical process standards to represent, compare, and order positive rational numbers and understand relationships as related to place value. The student is expected to:		
5N02	interpret the value of each place-value position as 1/10 of the value of the place to its left <u>or as 10 times the value of the place to its right</u>	Resource CCSS
5N01	represent the value of the digit in whole numbers through 1,000,000,000 and decimals through the thousandths using expanded notation and numerals	Moved to fourth grade since this is not our focus
5N04	compare and order two decimals to thousandths <u>and represent comparisons using the symbols >, <, or =</u>	ER—Capraro, 1
5N03	round decimals to tenths or hundredths	

5N05	represent the comparison of two decimal numbers to thousandths using the symbols $>$, $<$, or $=$	Moved to 5N04
<p>Knowledge and Skills Statement. The student applies mathematical process standards to develop and use strategies and methods for positive rational number computations in order to solve problems with efficiency and accuracy. The student is expected to:</p>		
5N20	estimate to determine solutions to mathematical and real-world problems involving addition, subtraction, multiplication, or division.	
5N07	solve mathematical and real-world problems involving positive sums and differences of positive rational numbers with fluency, including decimals to the hundredths and mixed numbers	<u>Fluency will not occur yet in this grade per VA discussion.</u>
5N08	<u>use strategies and algorithms, including the standard algorithm, to multiply</u> determine products of up to a three-digit number <u>by and</u> a two-digit number with fluency	
5N09	<u>use strategies and algorithms, including the standard algorithm, to solve for</u> determine quotients of up to a four-digit dividend and a two-digit divisor using properties of operations, place value understandings (e.g., partial quotients), or the relationship between multiplication and division <u>with fluency</u>	
5N10	represent multiplication of decimals <u>with products</u> to <u>the</u> hundredths using objects and pictorial models, including area models	Horizontal alignment and clarity
5N11	extend the definitions of, properties of and relationship between multiplication of whole numbers to multiplication of decimals to hundredths	Stated in 5N12
5N12	<u>solve for</u> determine products of decimals to hundredths, <u>including situations involving money</u> , using strategies based on place value understandings, properties of operations, and the relationship to the multiplication and division of whole numbers	
5N13	represent quotients to hundredths, <u>up to</u> (four-digit dividends and two-digit <u>whole number</u> divisors), using objects and pictorial models, including area models	
5N14	extend the definitions of, properties of, and relationship between division with whole numbers to division of decimals	Stated in 5N15
5N15	<u>solve for</u> determine quotients to hundredths, <u>up to</u> (four-digit dividends and two-digit <u>whole number</u> divisors), using strategies <u>and algorithms, including the standard algorithm such as partial quotients, the properties of operations, and the relationship between multiplication and division</u>	ER & MV
5N06	represent <u>and solve</u> addition and subtraction of positive fractions with <u>unlike unequal</u> denominators and referring to the same whole using objects and pictorial models that build to the number line (such as strip diagrams) and properties of operations This includes fractions as decimals with common denominators of tenths or hundredths (e.g., $1/5 + 0.3$)	Consistent vocabulary with 4N17
5N16	represent <u>and solve</u> multiplication of a positive fraction and a whole number <u>referring and a fraction that refers</u> to the same whole using objects and pictorial models, including area models	

5N17	extend the definitions of, properties of, and relationship between multiplication with whole numbers to multiplication of a fraction and a whole number	Clarifying Understood in 5N16
5N18	represent division of a unit fraction by a whole number and the division of a whole number by a unit fraction such as [e.g., $1/3 \div 7$ and $7 \div (1/3)$] , using objects and pictorial models, including area models	
5N19	extend definitions of, properties of, and relationship between division with whole numbers to division with unit fractions and whole numbers	Moved to 6th
5N21	solve mathematical and real-world problems involving division of multidigit whole numbers with up to four-digit dividends and two-digit divisors	Absorbed in 5N15
5N22	determine solutions to mathematical and real-world problems involving products to hundredths or quotients to hundredths (four-digit dividends and two-digit divisors) with fluency	ER & MV fluency is at 6th grade
5N23	determine solutions to mathematical and real-world problems involving products of positive fractions and whole numbers or positive quotients of positive unit fractions and whole numbers referring to the same whole [e.g., $1/3 \div 7$ and $7 \div (1/3)$], with fluency. (Within problems requiring division, remainders may be expressed as fractions	Moved to 6th

Algebraic Reasoning Expressions, Equations and Relationships.		5A
Knowledge and Skills Statement. The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:		
	identify prime and composite numbers using patterns in factor pairs	needed
5A01	represent and solve multistep mathematical and real-world problems involving the four operations with whole numbers and positive fractions using equations with a letter standing for the unknown quantity	Added for specificity
5A02	generate a numerical pattern when given a rule (The rules should be in the form $y = ax$ or $y = x + a$) and graph for a mathematical or real-world problem situation	
5A03	distinguish between two rules verbally, numerically, graphically, and symbolically. (The rules should be in form $y = ax$ or $y = x + a$.)	? VA
	recognize the difference between additive and multiplicative numerical patterns given in a table or graph	To replace the original 5A03
5A04	describe explain the meaning of including parentheses and brackets in a numeric expression verbally. such as [A student should be able to explain that 4 (14 + 5) is 4 times as large as (14 + 5) without simplifying the expressions.]	Formatting & Clarity SBOE, Lowe
5A05	simplify numerical expressions, including up to two levels of grouping excluding exponents such as $(3 + 7) / (5 - 3)$	Whole number solutions

5A06	<u>use concrete objects and pictorial models to develop</u> determine the formulas for the volume of a rectangular prism, including the special form for a cube ($V = l \times w \times h$, $V = s \times s \times s$, and $V = Bh$)	Committee decision for horizontal alignment (5M02) Formulas for STAAR chart
5A07	<u>represent and solve</u> determine solutions to mathematical and real-world problems related to perimeter, area such as (rectangles including squares and composite figures formed by rectangles), and <u>related to</u> volume <u>such as</u> (rectangular prisms)	SBOE, Lowe
5A08	write equations that represent mathematical and real-world problems including those involving perimeter, area (rectangles, including squares), and volume (rectangular prisms)	Absorbed in to 5A07

Geometry and Measurement Two-Dimensional and Three-Dimensional Figures.		5G
Knowledge and Skills Statement. The student applies mathematical process standards to classify two-dimensional figures by attributes and properties. The student is expected to:		
5G01	classify two-dimensional figures in a hierarchy <u>of sets and subsets using graphic organizers</u> based on their attributes and properties. such as (All rectangles have the property that opposite sides are parallel. Therefore, every rectangle is a parallelogram.)	<u>Per recommendation from 6th grade.</u>
Knowledge and Skills Statement. The student applies mathematical process standards to understand, recognize, and quantify volume. The student is expected to:		
5M01	<u>recognize</u> illustrate a cube with side length of 1 unit as a “unit cube” having “one cubic unit of volume” and the volume of a three-dimensional figure as the number of unit cubes “(n cubic units)” needed to fill it with no gaps or overlaps if possible	Changed verb.
5M02	determine the measure <u>volumes of right a rectangular prisms with whole number side lengths in problems related to the number of layers times the number of</u> by counting <u>unit cubes in the area of the base</u> , (cm^3 , in^3 , or ft^3) including cubic centimeters, cubic inches and cubic feet, packed into a three-dimensional figure without gaps or overlaps. (Side lengths are limited to whole numbers.)	
5M03	decompose right rectangular prisms into layers to determine the volume of the original figure using the additive property of volume	Implied in 5A06
Knowledge and Skills Statement. The student applies mathematical process standards to select appropriate units, strategies, and tools to solve problems involving measurement. The student is expected to:		
5M04	<u>solve problems by</u> calculating conversions within a measurement system, (customary or metric) for mathematical and real-world problems	5M04
Knowledge and Skills Statement. The student applies mathematical process standards to identify locations on a coordinate plane. The student is expected to:		

5M05	describe explain the key attributes of the coordinate plane and the process for graphing ordered pairs of numbers in the first quadrant These attributes include: the axes are perpendicular number lines where the intersection (origin) of the two lines coincides with zero on each number line and the given point (0, 0); the x-coordinate, the first number in an ordered pair, indicates movement parallel to the x-axis starting at the origin, and the y-coordinate, the second number, indicates movement parallel to the y-axis starting at the origin	Too instructional.
5G06	graph ordered pairs of numbers arising from mathematical and real-world problems in the first quadrant of the coordinate plane, <u>including those generated by number patterns or found in an input-output table</u>	<u>Leave mathematical and real-world problem in</u>

Measurement and Data Analysis.

5M

Knowledge and Skills Statement. The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:

5M07	represent categorical and numerical data <u>with bar graphs or frequency tables and continuous numerical data</u> , including data sets of measurements in fractions or decimals, with bar graphs <u>dot line</u> plots, or stem and leaf plots	Per ER—Capraro VA
5M08	represent discrete paired data on a scatter plot	
5M09	solve one- and two-step mathematical and real-world problems using data from a frequency table, a <u>dot line</u> plot, a bar graph, a stem and leaf plot, or scatter plot	VA