Review of the July 2011 Draft of the Texas Mathematics Standards

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In some ways, the current draft improves upon the "Commissioner's Draft", on which it is based. Many – but not all – cases of imprecise or mathematically questionable language have been dealt with. However, serious problems remain. In the elementary grades, the draft consistently underemphasizes computational facility. On the other hand, the major results of the high school and late middle school curriculum are generally presented only as results to be applied by students, but not meant to be understood. As the National Mathematics Panel has pointed out, conceptual understanding and computational practice develop in tandem. To ask for understanding in the early grades without addressing computation is just as wrong as asking students to merely apply, but not truly understand the binomial theorem and other major theorems in high school.

Even when the language of the draft is sufficiently precise, it often reads awkwardly. I would advise the authors to have the entire document examined by an individual who is not only mathematically competent, but also has stylistic flair. Several "model frameworks" exist that could be used as linguistic models.

Computational facility in the elementary grades

At each grade level, the standards are preceded by a statement of the relevant "Focal Points", with wording that is often borrowed from the NCTM Focal Points. The authors would do well to actually follow the suggestions of the latter!

Automatic recall of number facts serves as the foundation for computational facility. The draft standard 2N10,

> *fluently apply basic fact strategies to add and subtract with sums to 20 and differences from 20,*

and its multiplication counterpart 3N20 send the wrong message. What is needed is fluent recall of the facts, or better yet, automaticity. Once students have fluent recall, they can and will apply it in all computations they do. If they don't have fluent recall, "fluently applying fact strategies" means nothing. I should add that "differences from 20" is not appropriate, either. It should be "addition facts up to 10 plus 10 and the corresponding subtraction facts", or something equivalent.

The NCTM Focal Points, the NMP report, the top rated state frameworks, the Common Core, and the curriculum guidelines of top achieving countries all ask for fluent use of the standard algorithms of arithmetic. In contrast, the wording of the 4th grade standard 4N16,

> *add and subtract whole numbers and decimals to the hundredths place using a variety of methods, including pictorial models, the inverse relationship between operations, concepts of place value, and efficient algorithms,*

signals disdain for the standard algorithms: "efficient algorithms" can be interpreted in many ways, and they are mentioned merely as an afterthought. The appropriate wording is "the standard algorithms", and they should be taught in grade 2 (addition and subtraction), grade 4 (multiplication), and grade 5 (division), respectively, as recommended in the NCTM Focal Points.

The addition of fractions – actually doing it, not just "representing addition of fractions" as in 4N17 and 5N06 – serves as an important precursor to Algebra. The *computational* addition of fractions (not just decimals!) is not covered in these standards[1]. That is a serious omission.

In my review of the Commissioner's draft, I mentioned that negative integers are not mentioned explicitly, but only implicitly, when rational numbers are introduced in grades 6 and 7. That is still the case. Likewise I mentioned the omission of the concepts of prime and composite numbers; that, too, has not been remedied.

Underemphasis of understanding in the later grades

In my review of the Commissioner's Draft, I wrote:

> The consensus of framework writers in the last twenty or thirty years is that students should know the Pythagorean theorem, be able to apply it, and to understand its proof – not to be able to reproduce its proof necessarily, but more than just understanding the statement. The CCSM says "Explain a proof of the Pythagorean theorem and its converse" and the Focal Points ask students to "explain why the Pythagorean theorem is valid". Both of these formulations are reasonable. On the other hand, I don't know what it means to "represent, verify, and explain the Pythagorean theorem". How would one *represent* the Pythagorean theorem? And the statement of the theorem is pretty straight-forward, so there is not much to explain. The Draft should adopt language similar to that of CCSM or the Focal Points.

The wording of this standard remains unchanged in the current draft. In effect, the standard 8A06 does *not* ask for an understanding of the Pythagorean Theorem, and the other standards that refer to this theorem are all about applying it.

The standards referring to finding roots of quadratic equations or to the quadratic equations are also slanted away from understanding. Compare the standards A1Q06, A1Q07 to the CCSM standard

[1] The standard 5N07 asks for the fluent solution of problems involving the addition of fractions, not for the fluent addition of fractions!

Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)2 = q$ that has the same solutions. Derive the quadratic formula from this form.

Deriving the quadratic formula develops understanding, whereas merely applying it, as in the July draft, does not. Similarly, the Precalculus standard PA11 only talks about applying the Binomial Theorem, not about understanding it. The CCSM, on the other hand, asks to *know and apply* the Binomial Theorem, *with coefficients determined for example by Pascal's Triangle*; it also mentions in a footnote that the *Theorem can be proved by mathematical induction or by a combinatorial argument*. The Texas Standards should aim as high as the CCSM!

Comments on specific standards

KN?? The un-numbered Kindergarten standards mentioning "one-to-one correspondence", "conservation of number", "cardinality", "hierarchical inclusion", and "subitizing" are unnecessarily technical and pretentious. The corresponding CCSM standards "*When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object", "Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted"*, and "*Understand that each successive number name refers to a quantity that is one larger"* are perfectly clear, non-technical, and unpretentious. The Texas standards should aim for the same clarity and unpretentiousness! The "research" cited to justify the inclusion of the "subitizing" standard does not meet even minimal standards of evidence. It should be eliminated.

1N?? The un-numbered first grade standard mentioning "subitizing" should be eliminated; again, the "research" cited to justify this standard does not meet even minimal standards of evidence.

1N04 "Generate a number" is not the best choice of words. How about "Name a number" instead. Even better, the standard should include the idea that "adding one" or "subtracting one" is one way to get such a number, but that there are many other ways.

1N09 Why "such as 50"? Instead you should say "up to 90".

1N14 See my remarks at the beginning of this review.

1N16 Can be said better. How about "Identify problems involving adding or subtracting …, and solve such problems".

1A03 No reason to strike out the example!

2N?? Should be eliminated; again, the "research" cited to justify this standard on "subitizing" does not meet even minimal standards of evidence.

2N02 Should be “represent numbers up to 999”.

2N03 See my comments about 1N04.

2N?? You cannot “justify that fractional parts are halves and fourths” by “non-examples”. Also, “halves or fourths” is better.

2N10 See my remarks at the beginning of this review.

2N13 See my comments about 1N16.

2N?? “Solve mathematical and real-world problems with unknowns in all positions” – the scope of this standard is completely unclear; it needs to be specified.

2N15 The language is unnecessarily pretentious.

2N16 The language is unnecessarily pretentious.

2A?? “Use relationships”? This is a poor choice of words!

2G02 Why “quadrilaterals, pentagons, and octagons”, but not hexagons or heptagons?

2G03 The “e.g.” is helpful, don’t eliminate it.

3N06 It is the fraction, not the number line, that has “denominators of 2, 3, 4, 6, and 8”.

3N9.5 Insert “of fractions” between “representations” and “with denominators …”.

3N12 “such as comparing”, not “such as compare”!

3N20 See my remarks at the beginning of this review.

3G01 Badly said – the first sentence that was struck out is necessary to explain what is meant.

3M01 “Determine the perimeter …” when “given perimeter …”? What on earth is this supposed to mean?

3M07 It is the fraction, not the number line, that has “denominators of 2, 3, 4, 6, and 8”.

4N05 Badly said – the decimal number 1.0006, for example, is not a “fraction with denominator(s) 10 or 100”.

4N17 “Fractions with common denominators” can be easily misinterpreted as “fractions with denominators 2, 3, 4, 6”. More seriously, expressing the opposite notion in later standards leads to the wording “fractions with uncommon denominators”, which conveys the meaning of “fractions with unusual denominators”. Use “like denominators” and “unlike

denominators", or alternatively "equal denominators" and "unequal denominators" throughout this document.

4N18 See my comments about 4N17.

5N06 See my comments about 4N17.

5N08 This should be a fourth grade standard, and should include fluent use of the standard algorithm of multiplication. See my comments at the beginning of this review.

5N09 This should mention the standard algorithm of division. See my comments at the beginning of this review.

5G01 Unclear and pretentious language.

6P10 It is not clear what the phrase "using concrete and pictorial models" refers to. Is it the statement of the problems, or the "Determine" – i.e., the method of solution. If it is the latter, direct computation must be included.

6A03 Here, as in quite a few other instances in this draft, it should be "or", not "and" (as was the case originally). The only "rule equivalent to y=kx **and** y=x+b" is y=x !!!

7P05 The terms "invariant" and "covariant" are unusual in K-12 mathematics, and should not be used.

7P06 If π is introduced as the ratio of the circumference of a circle to its diameter, as is usually done, then the statement that it is also the ratio of the area inside the circle to the radius squared is a (non-trivial for middle-school mathematics) theorem. It should be described as such.

8N01 It may be clear what is meant by "illustrate a rational approximation of an irrational number", but is said quite inelegantly.

8A06 See my remarks at the beginning of this review.

A1Q07 See my remarks at the beginning of this review.

A1A14 The term "trinomial" is rather quaint by now, and is not used in college mathematics. Both the CCSM and the Focal Points avoid it. So should the draft – say "quadratic polynomial" instead.

A1A18 Avoid the terminology "literal equation", which is not used in college mathematics.

GG08 and GG09 are too ambitious for a High School Geometry course.

PA02 Finite geometric series can always be evaluated. That is important enough to constitute a separate standard. Also, the order of the sentence is misleading; it should be “when possible, evaluate finite sums written in Sigma notation”.

PA11 See my remarks at the beginning of this review.