The Commissioner's Draft of the Texas Mathematics Standards

Office of the Commissioner
Texas Education Agency

The Commissioner's Draft of the Texas Mathematics Standards (CDTMS)

April 14, 2011

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PREFACE

Introduction

Texas's focus on education reform and accountability for the past three decades leaves no doubt that Texas takes a leadership role in relation to other states in seeking to educate its citizens. As the Texas Education Agency (TEA) continues to support students and teachers in reaching higher academic standards, Texas has a targeted focus on college and career readiness for all students. Beginning in the 2011-2012 school year, the State of Texas Assessments of Academic Readiness (STAAR) program replaces the Texas Assessment of Knowledge and Skills (TAKS) with increased cognitive complexity within assessment items (TEA, 2011).

The mathematics Texas Essential Knowledge and Skills (TEKS) specify the expectations for what Texas students should **know and be able to do** in mathematics. The current mathematics TEKS have served Texas citizens well, identifying clear expectations and setting high standards; however, the world is changing and many careers now require more and different mathematics, including an emphasis on mathematical reasoning and complex problem-solving. The Texas College and Career Readiness Standards, published by the Texas Higher Education Coordinating Board and the TEA in 2009, set a high bar for preparing all Texas students for success in the workplace. The time has come for a new generation of TEKS for mathematics. The mathematics education research and resources currently available offer an unprecedented opportunity for informing a revision of the TEKS for mathematics.

Process

In anticipation of the State Board of Education's scheduled 2011-2012 revision of the TEKS for Mathematics, the Commissioner of Education convened a group of advisors to review current research and resources and to offer suggestions regarding the upcoming TEKS revision and future professional development. The Commissioner's Mathematics Advisory Group, established in the fall of 2010, includes mathematics educators and mathematicians from Texas. The recommendations of the Commissioner's Mathematics Advisory Group regarding the next generation of mathematics standards in Texas were compiled and then reviewed by a panel of national advisors in mathematics, known as the National Review Team. This publication is the final draft document known as "The Commissioner's Draft of the Texas Mathematics Standards." Members of the Commissioner's Mathematics Advisory Group and the members of the National Review Team are listed on the next page. The resources used by the Commissioner's Mathematics Advisory Group in their work are also listed later in this document. It is important to note that the collaborative process used to create this draft document does not reflect each professional view of every participant in the process. The document is a synthesis of the suggestions of the Commissioner's Mathematics Advisory Group and the National Review Team.

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Resources Used

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Mathematical Processes

The advisors paid careful attention to the balance of procedural skills and the requirement that students solve routine as well as non-routine problems. The draft begins with a set of mathematical processes that are expected at every grade level from kindergarten to grade 12. The placement of the process skills at the beginning of the draft is intentional. The process skills weave the other knowledge and skills together so that students may be successful problems solvers and use mathematics efficiently and effectively in daily life.

Local Control

The TEKS for mathematics identify what students should know and be able to do as proficient students and users of mathematics. The mathematics TEKS do not inform teachers how to teach the content. The advisors deliberately avoided instructional language in this draft of standards as well. Curriculum and instructional decisions are left to local districts and schools.

Organization of the Draft Standards

The standards are not a scope and sequence. When possible, the order does reflect a progression of learning, but the order is not a mandated sequence for instruction. The ordering or sequencing for instruction is a local decision. The kindergarten through eighth grade standards are organized by mathematics topic areas or strands, and the high school standards are organized by customary course titles. Because of the rise of the importance of data analysis and statistics in the current century, all grade levels and courses contain data and statistics standards. Although most of the standards may be easily identified with a particular topic area, there are standards that could be placed in more than one topic area. In these situations, a decision was made that may seem to some like an artificial separation from one topic area but a good fit in another topic area. The standards are placed at the grade level where mastery is expected. This does not preclude introducing the content at earlier grade levels. Preparatory content can and should be introduced earlier as appropriate. These decisions are local decisions.

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Mathematical Process Standards K-12

I.	Apply mathematics to problems arising in everyday life, society and the workplace.
II.	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.
III.	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
IV.	Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.
V.	Create and use representations to organize, record, and communicate mathematical ideas.
VI.	Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

Kindergarten

Mathematical Process Standards Grade K	
I.	Apply mathematics to problems arising in everyday life, society and the workplace.
II.	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.
III.	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
IV.	Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.
V.	Create and use representations to organize, record, and communicate mathematical ideas.
VI.	Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

Grade K Focal Areas		
Number and Operations	▲	Understanding counting and cardinality
Number and Operations	●	Understanding addition as combining and subtraction as separating
Measurement and Data	■	Comparing objects by measurable attributes

Supporting Topics for the Focal Areas in Grade K and Grade 1		
Number and Operations	▲●■	Representing, comparing, and ordering whole numbers
Two-Dimensional and Three-Dimensional Figures	■	Identifying figures and their attributes
	+	Classifying 2D and 3D figures
	■	Composing 2D figures
	+	Describing location
Measurement and Data	▲●■	Representing data
Color and symbol shows the connection between Focal Areas and Supporting Topics.		
+ Indicates topic supports Focal Area in Grade 1		

Kindergarten

Number and Operations. The student is expected to:	**KN**
KN01	count verbally to 130 by ones and tens, beginning with any given number.
KN02	represent the number of objects in a set at least up to 40 using spoken words and written numerals.
KN03	determine the number before or after another number without having to start back at 1.
KN04	represent the numerals up to 40 by associating them to the number of elements in sets consisting of actual objects or pictures of objects, including counting out objects in groups of tens and ones such as one group of 10 and 3.
KN05	compare collections of up to 40 objects using one-to-one correspondence.
KN06	compare numbers between 1 and 10 presented as written numerals.
KN07	generate a set using objects or pictorial models that represents a number that is more than, less than, or equal to a given number, up to 40.
KN08	compose a given target number less than or equal to 10 by producing two sets of objects that, when combined, contain exactly the target number.
KN09	decompose a given set of objects less than or equal to 10 into multiple sets in a variety of ways, and indicate the corresponding number pairs in each case (e.g., 8 can be decomposed into 4 and 4, 5 and 3, 6 and 2, and 7 and 1).
KN10	combine a set of 10 objects with another number set of objects to make a new number set of size between 10 to 20 and indicate the corresponding number relation (e.g., a set of 10 and a set of 1 can be combined to make 11).
KN11	separate a set of 10 to 20 objects into a group of 10 objects and some more. (e.g., 18 can be separated into a set of 10 and a set of 8).
KN12	solve mathematical and real-world problems involving adding or subtracting within 20 using objects and pictorial models. These problems should include determining the sum when two addends are given and determining the minuend when the difference and subtrahend are given.
KN13	explain the solution process to problems involving adding or subtracting within 20 using spoken words, objects, pictorial models, and number sentences.

Kindergarten

Two-Dimensional and Three-Dimensional Figures. The student is expected to:		**KG**
KG01	identify two-dimensional objects (the shape of circles, triangles, rectangles, squares, rhombuses, and hexagons) and three-dimensional objects (the shape of cylinders, cones, spheres, and cubes) found in the real world.	
KG02	identify two-dimensional components of three-dimensional shapes (e.g., as the face of a cube is square).	
KG03	identify attributes of two-dimensional shapes (e.g., number of corners (vertices), number of sides, and angles).	
KG04	identify attributes of three-dimensional shapes (e.g., number of corners (vertices), number of edges, and sides and number of faces).	
KG05	classify two-dimensional shapes as circles, triangles, rectangles, including squares and rhombuses; or hexagons regardless of orientation or size.	
KG06	classify three-dimensional shapes as cylinders, cones, spheres, or cubes regardless of orientation or size.	
KG07	compose two-dimensional shapes and three-dimensional shapes using materials (e.g., popsicle sticks, straws, molding clay, etc.) or drawings.	
KG08	describe the position of one or more shapes in relation to another shape using words such as “above,” “below,” “beside,” “between,” “in front of,” and “in back of”.	

Kindergarten

Measurement and Data. The student is expected to:		**KM**
KM01	give an example of a measurable attribute of a given object (length, capacity, weight, temperature).	
KM02	compare two objects directly with a common measurable attribute (length, capacity, weight, temperature) using language such as “more” and “less”.	
KM03	classify and sort a set of objects into categories according to an attribute (e.g., number of sides, angles, color, shape, size, first letter in name, etc.) and resort the same set according to a different attribute.	
KM04	count the number of objects in a category or subcategory, summarize the data into a picture graphs and use the picture graph to answer questions about the data (e.g., “Which shape do we have the most of?”).	

Grade 1

Mathematical Process Standards Grade 1	
I.	Apply mathematics to problems arising in everyday life, society and the workplace.
II.	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.
III.	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
IV.	Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.
V.	Create and use representations to organize, record, and communicate mathematical ideas.
VI.	Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

Grade 1 Focal Areas		
Number and Operations	▲	Understanding and applying place value
Number and Operations	●	Solving problems involving addition and subtraction
Two-Dimensional and Three-Dimensional Figures	■	Composing and decomposing two-dimensional and three-dimensional

Supporting Topics for the Focal Areas in Grade 1 and Grade 2		
Number and Operations	▲	Determining 10 more or 10 less
	▲●	Comparing and ordering whole numbers up to 100
	▲●	Connecting properties and operations
	▲●	Connecting addition and subtraction
	▲●	Fluently producing addition and related subtraction facts with sums to 10 and differences from 10
Expressions, Equations, and Relationships	●	Representing problems involving addition and subtraction
Two-Dimensional and Three-Dimensional Figures	■	Distinguishing between attributes of figures
Measurement and Data	▲●■	Representing data
Color and symbol shows the connection between Focal Areas and Supporting Topics. + Indicates topic supports Focal Area in Grade 2		

Grade 1

Number and Operations. The student is expected to:	**1N**
1N01	skip count by twos and fives to determine the total number of objects (up to 130) in a set (objects include pennies and nickels).
1N02	decompose the value of a numeral to 100 as a sum of so many tens and so many ones, in more than one way using objects and pictorial models. For example, 64 can be represented as 6 tens, and 4 ones, or as 5 tens, and 14 ones (representations may be bundles of an object or pictures of bundles).
1N03	represent a two-digit number as the sum of the values represented by the digits in the combined values of tens and ones using objects, pictures, expanded notation, and numbers. For example, 93 is the sum of 9 tens and 3 ones.
1N04	generate a two-digit number that is greater than, less than, or equal to a given whole number that is greater than 10 and less than 99.
1N05	compare and order whole numbers up to 100.
1N06	represent the comparison of two numbers to 100 using the symbols >, <, or =.
1N07	determine the difference of between two multiples of 10 in the range from 10-90 using objects and pictures.
1N08	generate a two-digit number that is 10 more or 10 less than a given number.
1N09	determine the sum of a two-digit number and one-digit number in mathematical and real-world problems, within 100, using concrete and visual models for solving addition problem situations.
1N10	solve mathematical and real-world problems involving combining with sums to 20 and unknowns in all positions, using objects and pictorial models.

1N11	solve mathematical and real-world problems involving separating with differences from 20 and unknowns in all positions using objects and pictorial models.
1N12	solve mathematical and real-world problems involving comparisons within 20 and unknowns in all positions using objects and pictorial models.
1N13	solve mathematical and real-world problems involving sets to 20 and unknowns in all positions using objects and pictorial models.
1N14	fluently produce addition and subtraction facts with sums to 10 and differences from 10 with fluency.
1N15	explain the solution to addition and subtraction problems involving adding or subtracting within 20 using spoken words, objects, pictorial models, and number sentences.
1N16	generate problem situations when given a mathematical number sentence involving adding or subtracting of whole numbers within 20.

Grade 1

Expressions, Equations and Relationships. The student is expected to:		**1A**
1A01	represent mathematical and real-world problems involving addition and subtraction of whole numbers to 20 using concrete objects, strip diagrams, and number sentences (equations).	
1A02	determine if a number sentence for addition or subtraction is true.	
1A03	determine the unknown whole number in an addition or subtraction equation relating three whole numbers when the unknown may be any one of the three terms in the equation. For example, the value 7 for [] makes 12 + [] = 19 a true equation.	

Grade 1	
Two-Dimensional and Three-Dimensional Figures. The student is expected to:	**1G**
1G01	draw two-dimensional shapes including circles, half-circles, quarter-circles, triangles, rectangles, squares, rhombuses, and hexagon.
1G02	distinguish between attributes that define a two-dimensional or three-dimensional shape (e.g., a closed figure with 3 sides is a triangle a solid with rectangular faces is a rectangular prism) and an attribute that does not define the shape (e.g., orientation or color).
1G03	compose two-dimensional shapes or three-dimensional shapes by joining two, three, or four shapes, to produce a target shape in more than one way if possible.

Grade 1

Measurement and Data. The student is expected to:		**1M**
1M01	illustrate that the length of an object is the number of same-size units of length that, when laid end-to-end (with no gaps or overlaps), reach from one end of the object to the other, assuming this is possible**.**	
1M02	generalize that when two different units are used to measure the same length, one will need a greater number of smaller units than longer units to measure the length.	
1M03	write a number and unit to describe a length.	
1M04	determine the time in hours and half hours using analog and digital clocks.	
1M05	classify and sort a set of objects or data into up to three categories or subcategories and use numbers to describe and compare these categories.	
1M06	summarize a data set, with up to four categories, using a frequency table or a picture graph.	
1M07	generate questions about categories of objects or data and determine solutions to these questions (e.g., the number in each category and how many more or less are in one category than in another).	

Grade 2

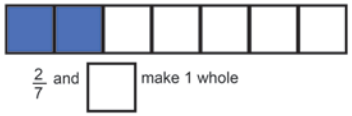
Mathematical Process Standards Grade 2	
I.	Apply mathematics to problems arising in everyday life, society and the workplace.
II.	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.
III.	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
IV.	Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.
V.	Create and use representations to organize, record, and communicate mathematical ideas.
VI.	Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

Grade 2 Focal Areas		
Number and Operations	▲	Making comparisons within Base 10
Number and Operations	●	Solving problems with addition and subtraction within 100
Number and Operations	■	Building foundations for multiplication

Supporting Topics for the Focal Areas in Grade 2 and Grade 3		
Number and Operations	▲	Applying place value and counting
	+	Building foundations for fractions
	●■	Fluently producing addition facts and related subtraction facts
Expressions, Equations, and Relationships	●■	Using multiple representations of problem situations
	●	Determining missing values in number sentences
Two-Dimensional and Three-Dimensional Figures	+	Identifying and classifying 2D and 3D figures
	■	Composing 2D and 3D figures out of unit measures
	■	Decomposing 2D figures
Measurement and Data	+	Measuring lengths and time
	●	Solving problems involving length
	■	Representing location on a number line
	●■	Representing and interpreting data
Color and symbol shows the connection between Focal Areas and Supporting Topics. + Indicates topic supports Focal Area in Grade 2		

Grade 2

Number and Operations. The student is expected to: **2N**

2N01	decompose the value of a numeral to 1000 as a sum of so many hundreds, so many tens and so many ones, in more than one way using objects and pictorial models. For example, 364 can be represented as 3 hundreds, 6 tens, and 4 ones, or as 2 hundreds, 5 tens, and 14 ones (representations may be bundles of an object or pictures of bundles).
2N02	represent a three-digit number as the sum of the values represented by the digits in the combined value of hundreds, tens, and ones places using objects, expanded notation, and numbers. For example, 493 is the sum of 4 hundreds, 9 tens and 3 ones.
2N03	generate a three-digit number that is greater than, less than, or equal to a given whole number that is greater than 100 and less than 999.
2N04	compare and order whole numbers up to 1,000.
2N05	represent the comparison of two numbers to 1,000 using the symbols >, <, or =.
2N06	decompose a strip diagram or regular polygon into equal parts using objects and pictorial representations. 
2N07	identify and name one part of an equipartitioned whole as a fraction 1/*b* (where *b* is a non-zero whole number) using strip diagrams and area models that include regular polygons.
2N08	determine the missing value in a number statement where two fractions with like denominators form one whole, represented with a strip diagram. (e.g., 2/7 + ☐ = 7/7. A strip diagram is separated into 7 equal parts. Two of the parts are shaded blue, and the remaining parts are shaded a second color.) 
2N09	determine the number that is and 10 or 100 more or less than a given number between 100 and 900.
2N10	fluently produce addition and subtraction facts with sums to 20 and differences from 20.

2N11	solve one-step and multi-step mathematical and real-world problems involving addition and subtraction within 100 using strategies based on place value, properties of operations, and the relationship between addition and subtraction with fluency.
2N12	solve mathematical and real-world problems involving addition and subtraction within 1,000 using strategies based on place value (with and without objects and pictorial models), properties of operations, and the relationship between addition and subtraction.
2N13	generate problem situations for a given mathematical number sentence involving adding or subtracting of whole numbers within 1,000.
2N14	determine whether a number (up to 40) of objects in a set is even or odd.
2N15	arrange a given number of objects into rectangular arrays with up to 5 rows and up to 5 columns.
2N16	determine the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns.

Grade 2

Expressions, Equations and Relationships. The student is expected to: **2A**

2A01	represent mathematical and real-world problems involving addition and subtraction of whole numbers to 100 using strip diagrams and number sentences (equations).
2A02	represent mathematical and real-world problems for multiplication to a product of 25 using arrays, strip diagrams, and number sentences (equations).
2A03	determine the unknown whole number in an addition or subtraction equation relating three whole numbers when the unknown may be any one of the three terms in the equation. For example, the value 27 for [] makes 12 + [] = 39 a true equation.

Grade 2

Two-Dimensional and Three-Dimensional Figures. The student is expected to: **2G**

2G01	build and draw two-dimensional shapes based on given attributes, e.g., number of sides (less than or equal to six) or vertices.
2G02	identify two-dimensional shapes including quadrilaterals (including parallelograms), pentagons, and octagons.
2G03	classify three-dimensional shapes (cones, cylinders, spheres, and rectangular prisms including cubes) based on attributes (e.g., number of faces, edges, or vertices).
2G04	compose two-dimensional shapes and three-dimensional shapes with given properties or attributes (e.g., build a rectangle out of unit squares; build a rectangular prism out of unit cubes).
2G05	decompose two-dimensional shapes (e.g., cut out a square from this rectangle; divide this shape in half; partition a rectangle into identical triangles).
2G06	illustrate the area of a rectangle with whole number side lengths as the number of unit squares (*n* square units) needed to cover it with no gaps or overlaps. A “unit square” is a square with side length of 1 unit having “one square unit of area”.

Grade 2

Measurement and Data. The student is expected to:	**2M**
2M01	illustrate the length of objects using concrete models for standard units of length.
2M02	determine the length of an object using rulers, yardsticks, meter sticks, or measuring tapes to the nearest marked unit.
2M03	determine a solution to mathematical and real-world problems involving length, including estimating lengths and using length as a model for addition and subtraction.
2M04	determine time to the nearest minute using analog and digital clocks.
2M05	represent whole numbers as distances from zero on a number line.
2M06	represent the point on a number line that correspond to a given whole number.
2M07	determine the corresponding whole number of a specified point on a number line.
2M08	explain that the length of a bar in a bar graph or the number of pictures in a picture graph represents the number of data points for a given category.
2M09	summarize a data set, with up to four categories, using a frequency table, a dot plot, a picture graph, or a bar graph with the vertical axis scaled in increments of one.
2M10	solve one-step mathematical and real-world problems involving addition or subtraction using categorical data represented with a frequency table, a dot plot, a picture graph, or a bar graph with unit intervals.

Grade 3

Mathematical Process Standards Grade 3	
I.	Apply mathematics to problems arising in everyday life, society and the workplace.
II.	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.
III.	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
IV.	Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.
V.	Create and use representations to organize, record, and communicate mathematical ideas.
VI.	Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

Grade 3 Focal Areas		
Number and Operations	▲	Solving multi-step addition and subtraction problems with whole numbers within 1000
Number and Operations	●	Solving problems with multiplication and division within 100
Number and Operations	■	Understanding and representing fractions as numbers and equivalent fractions

Supporting Topics for the Focal Areas in Grade 3 and Grade 4		
Number and Operations	▲	Applying place value
	▲	Comparing and ordering whole numbers
	■	Representing points on a number line that correspond to a given fraction
	▲■	Connecting multiplication and division
Expressions, Equations, and Relationships	●■	Using multiple representations of problem situations
	▲●■	Determining missing values in number sentences
	▲	Representing real-world relationships using number pairs in a table and verbal description
Two-Dimensional and Three-Dimensional Figures	▲●	Relating area to multiplication and to addition
	+	Identifying and classifying 2D according to common attributes
	+	Decomposing composite figures formed by rectangles to determine area
Measurement and Data	▲	Determining perimeter of polygons
	▲■	Solving problems involving time
	▲	Measuring liquid volume (capacity)
	■	Representing location on a number line
	▲●■	Representing and interpreting data
Color and symbol shows the connection between Focal Areas and Supporting Topics. + Indicates topic supports Focal Area in Grade 4		

Grade 3

Number and Operations. The student is expected to:	**3N**
3N01	represent the value of a numeral to 10,000 using objects and pictorial models that address the notion of bundling (composing and decomposing).
3N02	represent the value of the digit in whole numbers through 10,000 using expanded notation and numerals. For example, for the number 4,093, the 4 in the thousands places is 4,000; the 9 in the tens place is 90; and the 3 in the ones place is three; and 4,093 is the sum of 4 thousands, 0 hundreds, 9 tens, and 3 ones.
3N03	round whole numbers to the nearest 10, 100, or 1,000.
3N04	compare and order whole numbers up to 10,000.
3N05	represent the comparison of two numbers to 10,000 using the symbols >, <, or =.
3N06	represent fractions greater than zero in mathematical and real-world problems using objects and pictorial models, including strip diagrams and number lines.
3N07	represent the point on a number line that corresponds to a given fraction greater than 0.
3N08	explain that 1/*b* represents the quantity formed by one part of a whole that has been partitioned into *b* equal parts where *b* is a non-zero whole number.
3N09	explain that *a/b*, where *a* is a whole number and *b* is a non-zero whole number, represents the quantity formed by *a* parts of size *1/b*.
3N10	represent equivalent fractions using objects and pictorial models, including number lines**.**
3N11	explain that two fractions are equivalent if and only if they are both represented by the same point on the number line or represent the same portion of a same size whole for an area model.
3N12	compare two fractions in mathematical and real-world problems by reasoning about their sizes and justifying the conclusion using symbols, words, objects, and pictorial models, including strip diagrams and number lines (fractions being compared should have the same numerator or the same denominator).

3N13	solve one-step and multi-step mathematical and real-world problems involving addition and subtraction within 1,000 using strategies based on place value, properties of operations, and the relationship between addition and subtraction with fluency.
3N14	determine the total number of objects when equally-sized groups of objects are combined or arranged in arrays (e.g., 4 groups, each having 7 objects, combine to make a new group of 28 objects).
3N15	determine products using properties of operations (e.g., 5 x 8 = 40, so 8 x 5 = 40 ; 2 x 3 x 4 = (2 x 3) x 4 = 6 x 4 = 24; 6 x 8 = 6 x (5+3) =6 x 5 + 6 x 3 = 30 + 18 = 48).
3N16	determine the product of a one-digit whole number and multiples of 10 in the range 10-90 (e.g. 8x90, 7x60) using strategies based on place value and properties of operations.
3N17	determine the number of objects in each group when a set of objects are partitioned into equal shares or a set of objects are shared equally (e.g., the number of objects in each share when 28 objects are partitioned equally into 7 shares, or as a number of shares when 28 objects are partitioned into equal shares with 7 objects each).
3N18	determine a quotient using the relationship between multiplication and division (e.g., the quotient 40÷8 can be found by determining what makes 40 when multiplied by 8).
3N19	determine the unknown whole number in multiplication and division equations relating three whole numbers (e.g., 8x?=24, 5=?÷3, 7x6=?).
3N20	produce with fluency multiplication and division facts with products to 100 and dividends from 100.
3N21	solve one-step and multi-step mathematical and real-world problems involving multiplication and division within 100 using strategies based on objects, pictorial models (including arrays, area models, and equal groups), properties of operations, or recall of facts.
3N22	solve one-step and multi-step mathematical and real-world problems involving addition, subtraction, multiplication, or division. (Problems may include operations with whole-number measures of length, capacity, or mass.)

Grade 3

Expressions, Equations and Relationships. The student is expected to:		**3A**
3A01	represent one- and two-step mathematical and real-world problems involving single and addition and subtraction of whole numbers to 1000 using strip diagrams and number sentences (equations).	
3A02	represent one- and two-step multiplication and division mathematical and real-world problems within 100 using arrays, strip diagrams, and number sentences (equations).	
3A03	describe a multiplication expression as a comparison. For example, 3 x 24 represents 3 times as much as 24.	
3A04	determine the unknown whole number in a multiplication or division equation relating three whole numbers when the unknown is either a missing factor or product. For example, the value 4 for [ ] makes 3 x [ ] = 12 a true equation.	
3A05	represent real-world relationships using number pairs in a table and verbal descriptions. For example, 1 insect has 6 legs, 2 insects have 12 legs, and so forth.	

Grade 3

Two-Dimensional and Three-Dimensional Figures. The student is expected to: **3G**

3G01	explain why polygons with 12 or fewer sides may share attributes that define a larger category for classification purposes. For example, rhombuses, parallelograms, rectangles, and squares all have four sides and may be classified as quadrilaterals. (They do not all have four right angles, so they may not all be classified as rectangles.)
3G02	recognize rhombuses, parallelograms, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
3G03	determine the area of rectangles (with whole number side lengths) in mathematical and real-world problems using multiplication relating the multiplications to the number of rows times the number unit squares in each row.
3G04	decompose composite figures formed by rectangles into two non-overlapping rectangles to determine the area of the original figure using the additive property of area.

Grade 3

Measurement and Data. The student is expected to:		**3M**
3M01	determine the perimeter of a polygon or a missing length when given perimeter and remaining side lengths in mathematical and real-world problems.	
3M02	determine the solution to mathematical and real-world problems involving addition and subtraction of time intervals in minutes.	
3M03	determine when it is appropriate to use measurements of liquid volume (capacity) or mass.	
3M04	determine liquid volume (capacity) or mass using appropriate units and tools.	
3M05	summarize a data set, with multiple categories, using a dot plot, a pictograph, or a bar graph with scaled intervals (e.g., each picture or interval represents five data points).	
3M06	solve one and two-step mathematical and real-world problems using categorical data represented with a frequency table, a dot plot, a pictograph or a bar graph with scaled intervals.	

Grade 4

Mathematical Process Standards Grade 4	
I.	Apply mathematics to problems arising in everyday life, society and the workplace.
II.	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.
III.	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
IV.	Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.
V.	Create and use representations to organize, record, and communicate mathematical ideas.
VI.	Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

Grade 4 Focal Areas		
Number and Operations	▲	Solving problems with multi-digit addition, subtraction, multiplication and division of whole numbers
Number and Operations	●	Building fractions from unit fractions and comparing fractions Using decimal notation and comparing decimals
Expressions, Equations, and Relationships	■	Extending measurement to area and perimeter formulas

Supporting Topics for the Focal Areas in Grade 4 and Grade 5		
Number and Operations	▲●	Applying place value
	▲	Identifying prime and composite numbers
	▲●	Representing points on a number line that correspond to a given fraction or terminating decimal
Expressions, Equations, and Relationships	▲●■	Representing multi-step problems involving the four operations with whole numbers with expressions and equations
	▲●■	Solving multi-step problems involving the four operations with whole numbers with expressions and equations
	▲●■	Generating and analyzing patterns
Two-Dimensional and Three-Dimensional Figures	■	Classifying 2D figures
Measurement and Data	■	Measuring angles
	▲●■	Converting units of measure
	▲	Representing and interpreting data
Color and symbol shows the connection between Focal Areas and Supporting Topics. + Indicates topic supports Focal Area in Grade 5		

Grade 4

Number and Operations. The student is expected to:	**4N**
4N01	explain the meanings of the tenths and hundredths place value positions using fractions.
4N02	interpret the value of each place-value position as 10 times the position to the right.
4N03	represent decimals including tenths and hundredths using concrete and visual models and money.
4N04	represent the value of the digit in whole numbers through 1,000,000 and decimals to the hundredths using expanded notation and numerals. For example, for the number 3.94, the 3 in the ones place is three; the 9 in the tenths place is 0.9; and 4 in the hundredths place is 0.04; and 3.94 is sum of 3 ones, 9 tenths, and 4 hundredths.
4N05	represent terminating decimals as fractions with denominators of 10 or 100.
4N06	round whole numbers to the nearest 10,000 or 100,000.
4N07	compare and order whole numbers to one million.
4N08	represent the comparison of two numbers to one million using the symbols >, <, or =.
4N09	compare and order decimals using concrete and visual models.
4N10	represent a point on a number line that corresponds to a given fraction or terminating decimal.
4N11	represent a fraction a/b as a sum of fractions $1/b$, where a and b are whole numbers and $b>0$.
4N12	decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition using pictorial and symbolic representations (e.g., 7/8=5/8+2/8, 7/8=3/8+4/8, 2 7/8=1+1+7/8, 2 7/8=8/8+8/8+7/8).

4N13	explain that a/b and $(n \times a)/(n \times b)$ (where a and b are integers) are equivalent fractions using objects and pictorial models.
4N14	determine if two given fractions are equivalent.
4N15	generate equivalent fractions to create common numerators or common denominators to compare two fractions with different numerators and different denominators.
4N16	add and subtract whole numbers and decimals to the hundredths place using pictorial models, concepts of place value, and properties of addition.
4N17	represent addition and subtraction of positive fractions with like denominators and referring to the same whole, using objects and pictorial models that build to the number line (such as strip diagrams) and properties of operations (includes fractions as decimals with like denominators of tenths or hundredths (e.g., 1/10 + 0.3)).
4N18	solve mathematical and real-world problems involving positive sums and differences of positive fractions, including mixed numbers, with like denominators referring to the same whole, with fluency.
4N19	estimate the reasonableness of answers using positive benchmark fractions (0, ¼, ½, ¾, 1) referring to the same whole. For example, if ½ is an addend, the sum must be greater than or equal to ½ if added to a positive number.
4N20	determine products of a number and 10 or 100 using properties of operations and place value understandings.
4N21	represent the product of up to a four-digit number by a one-digit number using arrays, area models or equations.
4N22	represent the product of two 2-digit numbers using arrays, area models, or equations.
4N23	determine products of up to a four-digit number and a one-digit number or two two-digit numbers using properties of operations (e.g., 34×27 is $34 \times (2 \times 10 + 7) = (34 \times 2 \times 10) + 34 \times 7 = 68 \times 10 + 238 = 680 + 238 = 918$).
4N24	represent the quotient of up to a four-digit whole number divided by a one-digit whole number using arrays, area models, or equations.
4N25	determine quotients of up to a four-digit dividend and a one-digit divisor using properties of operations, place value understandings (e.g., partial quotients), or the relationship between multiplication and division.
4N26	solve one and two-step mathematical and real-world problems involving multiplication (including scalar comparisons) and division (including interpreting remainders).

Grade 4	
Expressions, Equations and Relationships. The student is expected to:	**4G**
4A01	represent multistep mathematical and real-world problems involving the four operations with whole numbers using strip diagrams and equations with a letter standing for the unknown quantity.
4A02	represent mathematical and real-world and problems using a table and numerical expressions to generate a number pattern that follows a given rule. For example, given the rule "Add 3" and the starting number 1, use the expressions 1 + 3, 2 + 3, 3 + 3, and so forth to generate a table to represent the relationship of the values in the resulting sequence and their position in the sequence.
4A03	determine the formulas for the perimeter of a rectangle, including the special form for perimeter of a square and the area of a rectangle.
4A04	determine solutions to mathematical and real-world problems related to perimeter and area (rectangles). (Dimensions are all positive whole numbers.)

Grade 4

Two-Dimensional and Three-Dimensional Figures. The student is expected to:		**4G**
4G01	identify points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines.	
4G02	classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. (The classification of triangles is limited to those that are right triangles and those that are not).	
4G03	draw a line of symmetry, if it exists, for a two-dimensional figure.	
4G04	identify two-dimensional shapes that have a line of symmetry.	

Grade 4

Measurement and Data. The student is expected to: **4M**

4M01	illustrate the measure of an angle as the part of a circle (whose center is at the vertex of the angle) that is "cut out" by the rays of the angle. (Angle measures are limited to whole numbers.)
4M02	illustrate degrees as the units used to measure an angle, where 1/360 of any circle is 1 degree and an angle that "cuts" *n*/360 out of any circle whose center is at the angle's vertex has a measure of *n* degrees. (Angle measures are limited to whole numbers.)
4M03	determine the approximate measures of angles in degrees using a protractor to the nearest whole number.
4M04	draw an angle with a given measure.
4M05	decompose angles into two non-overlapping angles to determine the measure of an unknown angle in mathematical and real-world problems using the additive property of angle measure.
4M06	identify relative sizes of measurement units within the customary system.
4M07	convert the measurement of a smaller unit into a larger unit or a larger unit into a smaller unit within the customary system when given other equivalent measures represented in a table.
4M08	determine a solution to real-world and mathematical problems involving addition, subtraction, multiplication, and division of measurements of length, intervals of time, liquid volumes, masses, and money.
4M09	represent data that can be ordered on a dot plot or a stem and leaf plot marked with whole numbers and fractions.
4M10	solve one and two-step mathematical and real-world problems using data (in whole number, decimal, and fraction form) in a frequency table, a dot plot, or a stem and leaf plot. For example, determine the difference in length between the tallest and shortest student in a class from data represented using a dot plot.

Grade 5

Mathematical Process Standards Grade 5	
I.	Apply mathematics to problems arising in everyday life, society and the workplace.
II.	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.
III.	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
IV.	Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.
V.	Create and use representations to organize, record, and communicate mathematical ideas.
VI.	Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

Grade 5 Focal Areas		
Number and Operations	▲	Solving problems with the addition and subtraction of fractions and decimals
Number and Operations	●	Solving problems with multiplication and division of decimals and beginning understandings for the multiplication and division of fractions
Expressions, Equations, and Relationships	■	Extending measurement to area and volume formulas

Supporting Topics for the Focal Areas in Grade 5 and Grade 6		
Number and Operations	▲●	Applying place value
	▲●	Identifying part-to-whole relationships and equivalence
Expressions, Equations, and Relationships	▲●■	Representing problems with expressions and equations
	▲●■	Solving problems with expressions and equations
	▲●■	Building foundations of functions through patterning
	+	Using the order of operations
Two-Dimensional and Three-Dimensional Figures	■	Classifying 2D figures
Measurement and Data	■	Connecting geometric attributes and measures of 3D figures
	●■	Using units of measure
	■	Representing location using a coordinate plane
	■	Representing and interpreting data

Color and symbol shows the connection between Focal Areas and Supporting Topics.
+ Indicates topic supports Focal Area in Grade 6

Grade 5

Number and Operations. The student is expected to:	**5N**
5N01	represent the value of the digit in whole numbers through 1,000,000,000 and decimals through the thousandths using expanded notation and numerals.
5N02	interpret the value of each place-value position as 1/10 of the value of the place to its left.
5N03	round decimals to tenths or hundredths.
5N04	compare and order two decimals to thousandths.
5N05	represent the comparison of two decimal numbers to thousandths using the symbols >, <, or =.
5N06	represent addition and subtraction of positive fractions with unlike denominators and referring to the same whole, using objects and pictorial models that build to the number line (such as strip diagrams) and properties of operations. This includes fractions as decimals with common denominators of tenths or hundredths (e.g., 1/5 + 0.3).
5N07	solve mathematical and real-world problems involving positive sums and differences of positive rational numbers with fluency, including decimals to the hundredths and mixed numbers.
5N08	determine products of up to a three-digit number and a two-digit number with fluency.
5N09	determine quotients of up to a four-digit dividend and a two-digit divisor using properties of operations, place value understandings (e.g., partial quotients), or the relationship between multiplication and division.
5N10	represent multiplication of decimals to hundredths, using objects and pictorial models, including area models,.
5N11	extend the definitions of, properties of, and relationship between multiplication of whole numbers to multiplication of decimals to hundredths.
5N12	determine products of decimals to hundredths, using strategies based on place value understandings, properties of operations, and the relationship to the multiplication and division of whole numbers.

5N13	represent quotients to hundredths (four-digit dividends and two-digit divisors), using objects and pictorial models, including area models**.**
5N14	extend the definitions of, properties of, and relationship between division with whole numbers to division of decimals.
5N15	determine quotients to hundredths (four-digit dividends and two-digit divisors), using strategies such as partial quotients, the properties of operations, and the relationship between multiplication and division.
5N16	represent multiplication of a positive fraction and a whole number referring to the same whole, using objects and pictorial models, including area models.
5N17	extend the definitions of, properties of, and relationship between multiplication with whole numbers to multiplication of a fraction and a whole number.
5N18	represent division of a unit fraction by a whole number and the division of a whole number by a unit fraction [e.g., 1/3÷7 and 7÷(1/3)], using objects and pictorial models, including area models.
5N19	extend the definitions of, properties of, and relationship between division with whole numbers to division with unit fractions and whole numbers.
5N20	estimate to determine solutions to mathematical and real-world problems involving addition, subtraction, multiplication or division.
5N21	solve mathematical and real-world problems involving division of multi-digit whole numbers with up to four-digit dividends and two-digit divisors.
5N22	determine solutions to mathematical and real-world problems involving products to hundredths or quotients to hundredths (four-digit dividends and two-digit divisors) with fluency.
5N23	determine solutions to mathematical and real-world problems involving products of positive fractions and whole numbers or positive quotients of positive unit fractions and whole numbers referring to the same whole [e.g., 1/3÷7 and 7÷(1/3)], with fluency. (Within problems requiring division, remainders may be expressed as fractions.)

Grade 5

Expressions, Equations and Relationships. The student is expected to: **5A**

5A01	represent multistep mathematical and real-world problems involving the four operations and positive fractions using equations with a letter standing for the unknown quantity.
5A02	generate a numerical pattern when given a rule (The rules should be in form $y=ax$ or $y=x+a$) for a mathematical or real-world problem situation.
5A03	distinguish between two rules verbally, numerically, graphically, and symbolically. (The rules should be in form $y=ax$ or $y=x+a$.)
5A04	explain the meaning of including parentheses and brackets verbally. [A student should be able to explain that 4(14+5) is 4 times as large as (14+5) without simplifying the expressions.]
5A05	simplify numerical expressions including up to two levels of grouping, excluding exponents.
5A06	determine the formulas for the volume of a rectangular prism, including the special form for a cube ($V=l \times w \times h$, $V=s \times s \times s$, and $V=Bh$).
5A07	determine solutions to mathematical and real-world problems related to perimeter, area (rectangles including squares), and volume (rectangular prisms).
5A08	write equations that represent mathematical and real-world problems including those involving perimeter, area (rectangles, including squares), and volume (rectangular prisms).

Grade 5	
Two-Dimensional and Three-Dimensional Figures. The student is expected to:	**5G**
5G01	classify two-dimensional figures in a hierarchy based on their attributes and properties. (All rectangles have the property that opposite sides are parallel. Therefore, every rectangle is a parallelogram.)

Grade 5

Measurement and Data. The student is expected to: **5M**

5M01	illustrate a cube with side length of 1 unit as a "unit cube" having "one cubic unit of volume" and the volume of a three-dimensional figure as the number of unit cubes (*n* cubic units) needed to fill it with no gaps or overlaps if possible.
5M02	measure volumes of right rectangular prisms by counting unit cubes (cm^3, in^3, or ft^3) packed into a three-dimensional figure without gaps or overlaps. (Side lengths are limited to whole numbers.)
5M03	decompose right rectangular prisms into layers to determine the volume of the original figure using the additive property of volume.
5M04	calculate conversions within a measurement system (customary or metric) for mathematical and real-world problems.
5M05	explain the key attributes of the coordinate plane and the process for graphing ordered pairs of numbers in the first quadrant. These attributes include: the axes are perpendicular number lines where the intersection (origin) of the two lines coincides with zero on each number line and the given point (0, 0); the *x*-coordinate, the first number in an ordered pair, indicates movement parallel to the *x*-axis starting at the origin, and the *y*-coordinate, the second number, indicates movement parallel to the *y*-axis starting at the origin.
5M06	graph ordered pairs of numbers arising from mathematical and real-world problems in the first quadrant of the coordinate plane.
5M07	represent categorical and numerical data, including data sets of measurements in fractions or decimals, with bar graphs, dot plots, or stem and leaf plots.
5M08	represent discrete paired data on a scatter plot.
5M09	solve one and two-step mathematical and real-world problems using data from a frequency table, a dot plot, a bar graph, a stem and leaf plot, or scatter plot.

Grade 6

Mathematical Process Standards Grade 6	
I.	Apply mathematics to problems arising in everyday life, society and the workplace.
II.	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.
III.	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
IV.	Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.
V.	Create and use representations to organize, record, and communicate mathematical ideas.
VI.	Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

Grade 6 Focal Areas		
Number and Operations	▲	Solving problems with multiplication and division of rational numbers
Proportionality	●	Understanding and applying ratios and rates
Expressions, Equations, and Relationships	■	Extending measurement to area of triangles and quadrilaterals

Supporting Topics for the Focal Areas in Grade 6 and Grade 7		
Number and Operations	+	Using integer operations
Proportionality	■	Understanding and using percents
Expressions, Equations, and Relationships	▲●■ ▲●■ ▲●■	Using multiple representations of Independent and dependent quantities Writing and evaluating expressions Solving one-step equations and inequalities
Measurement and Data	▲●■	Summarizing data using appropriate graphical representations
Color and symbol shows the connection between Focal Areas and Supporting Topics. + Indicates topic supports Focal Area in Grade 7		

Grade 6

Number and Operations. The student is expected to:		**6N**
6N01	extend previous understandings of number line to include placement and ordering rational numbers and absolute values of rational numbers.
6N02	order a set of rational numbers arising from mathematical or real-world contexts.
6N03	represent multiplication and division of positive rational numbers in mathematical and real-world problems with concrete, verbal, pictorial (including number line and area models), numerical, and algebraic representations.
6N04	use an area model to present fraction and decimal multiplication and division, including the multiplication or division of a fraction and a decimal. For example, generate area models for 1.3 x 2.5, 1 3/10 x 2 ½, and 1.3 x 2 ½, explaining why these multiplications yield equivalent products
6N05	extend representations for division to include fraction notation. For example, the fraction notation a/b represents the same number as $a \div b$.
6N06	determine whether a quantity is increased or decreased when multiplied by a fraction (proper or improper) with and without computation.
6N07	represent integer operations with concrete (such as counters), verbal, pictorial, tabular (such as patterns), and graphical (such as a number line) representations.
6N08	multiply and divide positive rational numbers fluently.
6N09	add, subtract, multiply, and divide integers fluently.

Grade 6

Proportionality. The student is expected to: **6P**

Number

6P01	differentiate between additive and multiplicative relationships.
6P02	give examples of ratios as multiplicative comparisons of two quantities describing the same attribute. For example, the ratio of the length of worm A to worm B is 9 cm to 6 cm, so worm A is 1.5 times as long as worm B.
6P03	give examples of rates as the comparison by division of two quantities having different attributes, including rates as quotients (e.g., 2 inches per 3 seconds is equivalent to 2/3 inches per second).
6P04	represent mathematical and real-world problems involving ratios and rates using scale factors, tables, graphs, and proportions.
6P05	apply qualitative and quantitative reasoning to solve prediction and comparison real-world problems involving ratios and rates.
6P06	solve mathematical and real-world problems involving ratios and rates using scale factors, unit rates, tables, graphs, and proportions.
6P07	determine conversions within a measurement system, including the use of proportions and unit rates in mathematical and real-world problems
6P08	represent benchmark percent equivalents (1%, 10%, 25%, 33 1/3% and multiples of these values) using strip diagrams, number lines, and numbers.
6P09	identify equivalent fractions, decimals, and percents.
6P10	determine solutions to real-world problems, finding the whole, given a part and the percent; finding the part, given the whole and the percent; and finding the percent, given the part and the whole**.**

Grade 6

Expressions, Equations and Relationships. The student is expected to: **6A**

Apply and Extend Arithmetic to Expressions and Equations

6A01	represent independent and dependent quantities within a mathematical or real-world problem using variables.
6A02	write an equation that represents the relationship between independent and dependent quantities within a mathematical or real-world problem.
6A03	represent mathematical and real-world problems with a rule equivalent to the form $y=kx$ or $y=x+b$.
6A04	represent key aspects of mathematical and real-world problems using verbal descriptions, tables, graphs, or equations.
6A05	transform numerical expressions using the order of operations, including positive exponents.
6A06	represent expressions verbally, numerically, and algebraically.
6A07	distinguish between expressions and equations.
6A08	generate equivalent expressions using the properties of operations.
6A09	determine if two expressions are equivalent using concrete models, pictorial models, and algebraic representations.

Apply and Extend Arithmetic to Geometric Formulas

6A10	illustrate and explain the relationships for areas of parallelograms, trapezoids, and triangles by decomposing and rearranging parts of these figures For example, parallelogram can be decomposed into a trapezoid and a right triangle with congruent heights; the triangle can be moved so that a rectangle is created having the same base length and height as the original parallelogram. (Figures include oblique triangles and parallelograms.)
6A11	write equations that represent mathematical and real-world problems related to area (rectangles, parallelograms, trapezoids, and triangles) and volume of right rectangular prisms. (Dimensions should include positive rational numbers.)

6A12	determine solutions for mathematical and real-world problems involving area (rectangles, parallelograms, trapezoids, and triangles) and volume of rectangular prisms. (Dimensions should include positive rational numbers.)
Represent and Solve Equations and Inequalities	
6A13	write a one-variable (one-step) equation or inequality to represent constraints or conditions within a mathematical (including number lines) or real-world problem.
6A14	represent solutions for a one-variable (one-step) inequality on a number line.
6A15	write a corresponding real-world problem given a one-variable (one-step) equation or inequality.
6A16	solve one-variable (one-step) equations and inequalities that represent real-world and mathematical problems.
6A17	determine the value(s) that make(s) a one-variable (one-step) equation or inequality true.

Grade 6

Measurement and Data. The student is expected to:		**6M**
6M01	graph points in all four quadrants, including points such as (½ , -½).	
6M02	summarize numeric data with graphical representations, including dot plots, stem-and-leaf plots, histograms and boxplots.	
6M03	solve multi-step mathematical and real-world problems using numeric data summarized in dot plots, stem-and-leaf plots, histograms, or boxplots.	
6M04	use the graphical representation of numeric data to describe the center, spread, and shape of the data distribution.	
6M05	distinguish between questions that yield data with and without variability. For example, the question "How tall am I?" will be answered with a single height versus the question "How tall are the students in my class?" which would be answered based on heights that vary.	
6M06	summarize numeric data with numerical summaries including the mean and median (measures of center) and the range and IQR (measures of spread), and use these summaries to describe the center, spread, and shape of the data distribution.	
6M07	summarize categorical data with numerical and graphical summaries including the mode (most frequent), and the percent of values in each category (relative frequency table) and the percent bar graph, and use these summaries to describe the data distribution.	

Grade 7

Mathematical Process Standards Grade 7	
I.	Apply mathematics to problems arising in everyday life, society and the workplace.
II.	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.
III.	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
IV.	Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.
V.	Create and use representations to organize, record, and communicate mathematical ideas.
VI.	Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

Grade 7 Focal Areas		
Number and Operations	▲	Solving problems with addition, subtraction, multiplication, and division of rational numbers
Proportionality	●	Rates and their multiple representations
Proportionality	●	Understanding and applying the critical attributes of similarity
Expressions, Equations, and Relationships	■	Extending measurement to area of triangles and quadrilaterals

Supporting Topics for the Focal Areas in Grade 7 and Grade 8		
Proportionality	+	Using measures (pi and conversions)
	▲	Determining probabilities
Expressions, Equations, and Relationships	▲●	Using multiple representations of independent and dependent quantities
	▲●■	Writing and evaluating expressions and solving equations
Measurement and Data	▲	Summarizing data using appropriate graphical representations

Color and symbol shows the connection between Focal Areas and Supporting Topics.
+ Indicates topic supports Focal Area in Grade 8

Grade 7	
Number and Operations. The student is expected to:	**7N**
7N01	apply and extend previous understandings of operations to add, subtract, multiply, and divide rational numbers.
7N02	determine solutions to mathematical and real-world problems containing rational numbers.

Grade 7

Proportionality. The student is expected to: **7P**

Number	
7P01	represent constant rates in mathematical and real-world problems given a pictorial, tabular, verbal, numeric, graphical or algebraic representations, including *d*=*rt*.
7P02	calculate unit rates from rates in mathematical and real-world problems, including rates such as 1 ½ miles in each ¾ hour is the same as 6 miles in each 3 hours or 2 miles per hour.
7P03	determine the constant of proportionality (*k*=*y*/*x*) to identify the invariant rate within mathematical and real-world problems.
7P04	determine solutions to real-world and mathematical problems involving ratios, rates, and percents, including multi-step problems involving percent increase and percent decrease.
Geometry and Measurement	
7P05	generalize the critical attributes of similarity, including invariant and covariant relationships. (If a, a' and b, b' are side lengths of two pairs of corresponding sides, then a/a' = b/b' and a/b = a'/b'. Corresponding angles of similar figures are congruent.)
7P06	represent π as the ratio of the circumference of a circle to its diameter and the area of a circle to the square of its radius.
7P07	determine solutions to mathematical and real-world problems involving similar figures and/or scale drawings.
7P08	determine conversions between measurement systems mathematical and real-world problems, including the use of proportions and the use of unit rates.
Statistics and Probability	
7P09	represent sample spaces for simple and compound events using lists and tree diagrams.
7P10	determine solutions to mathematical and real-world problems involving random sampling and application to the full population.
7P11	determine solutions to mathematical and real-world problems involving experimental data for probabilistic events and make predictions with this data.

7P12	determine solutions to mathematical and real-world problems using data represented in bar graphs, dot plots, and circle graphs, including part-to-whole and part-to-part comparisons and equivalents.
7P13	determine solutions to mathematical and real-world problems involving qualitative and quantitative data from simple experiments. [Include qualitative prediction and qualitative comparison problems in addition to quantitative prediction and quantitative comparison problems based on the probabilities of simple events. P(E)=0 vs. P(E)=1/2 vs. P(E)=1.]
7P14	determine experimental and theoretical probabilities related to simple and compound events using data and sample spaces.
7P15	determine solutions to mathematical and real-world problems based on prediction using probabilities of simple events**.**

Grade 7

Expressions, Equations and Relationships. The student is expected to: **7A**

Apply and Extend Arithmetic to Expressions and Equations

7A01	represent mathematical and real-world problems using verbal descriptions, tables, graphs, and equations that simplify to the form $y=mx+b$

Apply and Extend Measurement to Geometric Formulas

7A02	illustrate and explain the relationship between the volume of a rectangular prism and a rectangular pyramid having congruent bases and heights. For example, the volume of a pyramid is 1/3 the volume of the prism that has the same base area and height.
7A03	illustrate and explain the relationship between the volume of a triangular prism and a triangular pyramid having congruent bases and heights.
7A04	determine solutions to mathematical and real-world problems involving the volume of rectangular prisms, triangular prisms, rectangular pyramids, or triangular pyramids.
7A05	determine the circumference and area of circles in mathematical and real-world problems.
7A06	determine the area of composite figures comprised of rectangles, squares, parallelograms, trapezoids, triangles, semi-circles and quarter-circles.

Represent and Solve Equations and Inequalities

7A07	write a one-variable (two-step) equation or inequality to represent constraints or conditions within a mathematical or real-world problem.
7A08	represent the solutions for a one-variable (two-step) inequality on a number line.
7A09	write a corresponding real-world problem given a one-variable (two-step) equation or inequality.
7A10	solve one-variable (two-step) equations and inequalities that represent mathematical and real-world problems.
7A11	determine the value(s) that make(s) a one-variable (two-step) equation or inequality true.

Grade 7	
Two-Dimensional and Three-Dimensional Figures. The student is expected to:	**7G**
7G01	solve mathematical and real-world problems involving the surface area of a rectangular prism, rectangular pyramid, triangular prism, or triangular pyramid by determining the area of the figure's net.

Grade 7	
Measurement and Data. The student is expected to:	**7M**
7M01	compare two groups of numeric data using comparative dot plots (in the case of an equal number of data points in each group) or boxplots (in the case of an unequal number of data points in each group) by comparing their shapes, centers, and spreads
7M02	use data from a random sample to make informal inferences about a population.
7M03	compare two populations based on data in random samples from these populations, including informal comparative inferences about differences between the two populations.

Grade 8

Mathematical Process Standards Grade 8	
I.	Apply mathematics to problems arising in everyday life, society and the workplace.
II.	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.
III.	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
IV.	Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.
V.	Create and use representations to organize, record, and communicate mathematical ideas.
VI.	Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

Grade 8 Focal Areas		
Proportionality	▲	Understanding and applying rate of change and slope
Proportionality	▲	Understanding and applying foundations for linear functions
Expressions, Equations, and Relationships	●	Writing and solving equations

Supporting Topics for the Focal Areas in Grade 8 and Algebra I		
Number and Operation	▲●	Using irrational numbers
	●	Using scientific notation
Proportionality	▲●	Representing similarity and dilations on a coordinate plane
Foundations for Functions	▲●	Determining linear association within data
	▲●	Using multiple representations of linear functions ($y=kx$ and $y=mx+b$)
Expressions, Equations, and Relationships	●	Using integer exponents
	▲●	Extending measurement to volumes of pyramids, cylinders, cones and spheres
	▲	Understanding and applying Pythagorean theorem
Two-Dimensional and Three-Dimensional Figures	●	Understanding and representing transformations
Measurement and Data	▲●	Identifying trends in data to determining deviations in data
	▲●	Identifying trends in data to determine sample to sample variation
Color and symbol shows the connection between Focal Areas and Supporting Topics. + Indicates topic supports Focal Area in Algebra I		

Grade 8

Number and Operation. The student is expected to:	8N
8N01	illustrate the rational approximation of an irrational number and locate the rational number approximation a number line. Numbers include square roots of numbers less than 225, cube roots for numbers less than 225 with a whole number cube root, and π.
8N02	convert between base 10 notation and scientific notation.
8N03	extend previous understandings of number line to include ordering rational and irrational numbers.
8N04	order a set of rational and irrational numbers arising from mathematical or real-world contexts.
8N05	approximate multiplicative comparisons between two numbers expressed in scientific notations based on real-world problems. (The average distance from the earth to the moon is 3.84×10^5 km. The average distance from the earth to Mars is 2.28×10^8 km. How many times greater is the distance from the earth to Mars than the distance from the earth to the moon?)

Grade 8

Proportionality. The student is expected to: 8P

Slope

8P01	generalize that the ratio of the distance between any two points of a geometric figure G and the distance between two corresponding points of its dilation, D(G) are the same for all pairs of distinct points in G and that if two line segments AB and CD are given in G, then the ratio of the length of AB to the length of CD is equal to the ratio of the length of their dilations D(AB) and D(CD).
8P02	explain the similarities and differences between a given figure and its dilation(s) on a coordinate plane.
8P03	explain the effect of given scale factors applied to two-dimensional figures on a coordinate plane using an algebraic representation [For example, when a scale factor of 0.5 is applied to a graphed figure, the transformation can be described with $(x,y)\rightarrow(0.5x, 0.5y)$].
8P04	explain using similar triangles why the slope, m, given as the change in y-values divided by the change in x-values, $(y_2-y_1)/(x_2-x_1)$, is the same for any two points (x_1, y_1) and (x_2, y_2) on the same line.
8P05	graph proportional relationships, interpreting the unit rate as the slope of the line modeling the relationship.

Foundations for Functions

8P06	represent key aspects of proportional situations and direct variation with tables, graphs, and equations ($y=kx$).
8P07	represent key aspects of non-proportional situations with tables, graphs, and equations ($y=mx+b, b\neq0$).
8P08	contrast bivariate sets of data that suggest a linear association with bivariate sets of data that do not suggest a linear association.
8P09	graph a straight line that approximates the relationship between bivariate sets of data that suggest a linear association.
8P10	determine solutions to mathematical and real-world and real-world problems involving direct variation.
8P11	distinguish between proportional and non-proportional situations using tables, graphs, or equations of form $y=kx$ and $y=mx+b$, where $b\neq0$.

8P12	give examples and non-examples of proportional functions that arise from mathematical and real-world problems. Examples should reveal the understanding that each input has exactly one output.
8P13	generate a linear function, written as $y=mx+b$, to model a linear relationship between two quantities represented verbally, numerically (two (x,y) values), tabularly or graphically.
8P14	determine the slope (rate of change) and the *y*-intercept (initial value) of the data in a table or graph that models a given context.

Grade 8

Expressions, Equations and Relationships. The student is expected to: 8A

Apply and Extend Arithmetic to Expressions and Equations

8A01	apply the properties of integer exponents to generate equivalent numerical expressions.

Apply and Extend Measurement to Geometric Formulas

8A02	illustrate and explain the relationship between the base area, height, and volume of a cylinder verbally and symbolically.
8A03	illustrate and explain the relationship between the volume of a cylinder and a cone having congruent bases and heights. For example, the volume of a cone is 1/3 the volume of the cylinder that has the same base area and height.
8A04	illustrate and explain the relationship between the formula for the volume of a sphere as it relates to the volume of a cone whose base radius and height are equal and are congruent to the radius of the sphere.
8A05	determine solutions to mathematical and real-world problems involving the volume of cylinders, cones, and spheres.
8A06	represent, verify, and explain the Pythagorean theorem and its converse using models and diagrams.
8A07	use the Pythagorean theorem and its converse to solve mathematical and real-world problems.
8A08	determine the distance between two points on a coordinate plane using the Pythagorean theorem

Represent and Solve Equations and Inequalities

8A09	write a one-variable equation with variables on both sides that represents a or mathematical or real-world problem. (The equations should include rational number coefficients and constants.)

8A10	match the values of *x* and *y* that simultaneously satisfy two linear equations (in the form $y = mx + b$) with the intersections of the graphed equations.
8A11	write a corresponding real-world problem when given a one-variable equation with variables on both sides of the equal sign, including rational number coefficients and constants.
8A12	solve one-variable equations with variables on both sides of the equal signthat represent mathematical and real-world problems. (The equations should include rational number coefficients and constants.)
8A13	determine the solutions to mathematical and real-world problems involving pairs of simultaneous linear equations (in form $y=mx+b$) using tables, graphs, and algebraic methods.

Grade 8	
Two-Dimensional and Three-Dimensional Figures. The student is expected to:	**8G**
8G01	generalize the properties of orientation and congruence of rotations, reflections, and translations of two-dimensional figures on a coordinate plane. For example, rotations, reflections, and translations preserve congruence of two-dimensional figures.
8G02	differentiate between transformations that preserve congruence and those that do not.
8G03	explain the effect of given transformations (translations, reflections, and rotations) applied to two-dimensional figures on a coordinate plane using an algebraic representation For example, $(x, y) \rightarrow (x+2, y+2)$ describes a translation of the point two units up and two units to the right.

Grade 8		
Measurement and Data. The student is expected to:		**8M**
8M01	construct a scatterplot and describe the observed trend in the scatterplot (positive trend, negative trend, no trend, linear association, non-linear association, and/or no association.) to address questions of association between bivariate data.	
8M02	determine deviations from the mean in order to describe the mean as a "balance point" (the sum of the deviations is 0).	
8M03	determine the mean absolute deviation and use this quantity as a measure of the average distance data are from the mean.	
8M04	simulate generating random samples of the same size from a population with known characteristics to gauge the sample to sample variation in sample statistics and to develop the notion of a random sample being representative of the population from which it was selected.	

Algebra I

Mathematical Process Standards Algebra I	
I.	Apply mathematics to problems arising in everyday life, society and the workplace.
II.	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.
III.	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
IV.	Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.
V.	Create and use representations to organize, record, and communicate mathematical ideas.
VI.	Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

Algebra I Focal Areas
Linear functions, equations and inequalities
Quadratic functions, equations and inequalities
Exponential functions, equations and inequalities
Number and algebraic methods

Algebra I

Linear Functions, Equations, and Inequalities. The student is expected to: **A1L**

Representation	
A1L01	determine the domain and range of a linear function in mathematical and real-world problems.
A1L02	generate linear equations in two variables for mathematical and real-world problems.
A1L03	generate linear inequalities in two variables for mathematical and real-world problems.
A1L04	generate systems of two linear equations for mathematical and real-world problems.
A1L05	write an equation of a line in various forms including $y = mx + b$, $ax + by = c$ and $y - y_1 = m(x - x_1)$.
A1L06	write an equation of a line that is parallel or perpendicular to the *x* or *y* axis, including determining whether its slope is 0 or undefined.
Coordinate Geometry	
A1L07	determine the effect on the graph of the linear function $f(x) = x$ when *f(x)* is replaced by *a·f(x)*, *f(x)* $+d$, $f(x - c)$, $f(b \cdot x)$ for specific values of *a*, *b*, *c* and *d.*
A1L08	graph a linear function on the coordinate plane and determine key features including *x*-intercept, *y*-intercept, and slope in mathematical and real-world problems.
A1L09	approximate the solution graphically to a system of two linear equations with two variables in mathematical and real-world problems.
A1L10	graph the solution to a linear inequality in two variables on the coordinate plane.
A1L11	graph the solution to a system of two linear inequalities in two variables on the coordinate plane.

Linear Functions and Data	
A1L12	determine the correlation coefficient using technology and interpret this quantity as a measure of the strength of linear association between two quantitative variables.
A1L13	differentiate between association and causation in real-world problems.
A1L14	determine, when appropriate, a linear equation that provides a reasonable fit to bivariate data in a scatter plot to approximate solutions to real-world problems and make predictions.
Solving Linear Equations, Inequalities, and Systems of Equations	
A1L15	solve linear equations for mathematical and real-world problems.
A1L16	determine the reasonableness, including using the appropriate units, of a solution to a linear equation as applied to mathematical and real-world problems.
A1L17	solve linear inequalities in two variables, including solving inequalities for which the application of the distributive property is necessary and/or involves variables on both sides of the inequality.
A1L18	determine the reasonableness, including using the appropriate units, of a solution to linear inequalities as applied to mathematical and real-world problems.
A1L19	solve algebraically systems of two linear equations with two variables for mathematical and real-world problems.
A1L20	determine the reasonableness, including using the appropriate units, of a solution to a system of linear equations as applied to mathematical and real-world problems.

Algebra I

Quadratic Functions, Equations, and Inequalities. The student is expected to: **A1Q**

Representation	
A1Q01	determine the domain and range of a quadratic function in mathematical and real-world problems.
A1Q02	apply the Remainder Theorem to a quadratic function. [For a quadratic polynomial $q(x)$ and a number a, the remainder on division of $q(x)$ by $x - a$ is $q(a)$, so $q(a) = 0$ if and only if $(x - a)$ is a factor of $q(x)$.]
Coordinate Geometry	
A1Q03	write the equation of a quadratic function in standard form and vertex form.
A1Q04	determine the effect on the graph of the quadratic function $f(x) = x^2$ when *f(x)* is replaced by $a \cdot fx,\ fx + d, fx - c, fb \cdot x$ for specific values of a, b, c and d.
A1Q05	relate the linear factors of a quadratic expression to the zeros of the associated quadratic function.
A1Q06	graph a quadratic function on the coordinate plane and determine key features, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and axis of symmetry in mathematical and real-world problems.
Solving Equations	
A1Q07	solve quadratic equations having real roots in mathematical and real-world problems by inspection e.g., $x^2=81$), factoring, taking square roots, completing the square, and applying the quadratic formula.
A1Q08	determine the reasonableness, including using the appropriate units, of a solution to a quadratic equation applied to mathematical and real-world problems.

Algebra I

Other Functions, Equations, and Inequalities. The student is expected to: **A1O**

Representation

A1O01	determine the domain and range of an exponential function of the form $f(x) = ab^x$ in mathematical and real-world problems.
A1O02	determine the meaning of the values of a and b in an exponential function of the form $f(x) = ab^x$ in mathematical and real-world problems.
A1O03	generate an exponential function in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical and real-world situations including growth and decay.
A1O04	graph an exponential function that models growth or decay and determine key features including x-intercept, y-intercept, and asymptotes in mathematical and real-world problems.
A1O05	identify an exponential function that approximately fits data graphed on a scatter plot to approximate solutions for real-world problems.

Algebra I

Number and Algebraic Methods. The student is expected to: **A1A**

A1A01	transform radical expressions involving square roots to solve mathematical and real-world problems.
A1A02	determine whether a relation represented with words, a table, graph, or symbols defines a function.
A1A03	calculate the rate of change of a linear function, given as a table, as a graph, or as an equation, over a specified interval within a mathematical or real-world problem.
A1A04	express slope as a rate of change for a linear function represented with a table, a graph, and an equation.
A1A05	determine the slope of a line given the standard form of a line.
A1A06	determine the value of a linear, quadratic, or exponential function expressed in function notation, given an element in its domain such as finding $f(2)$ if $f(x) = x + 4$.
A1A07	identify terms of an arithmetic or geometric sequence when the sequence is given in function or recursive form.
A1A08	find a formula for the general term of an arithmetic or geometric sequence given several of its terms.
A1A09	transform algebraic expressions using the laws of integer exponents.
A1A10	extend previous understandings of the laws of integral exponents to the corresponding laws for rational exponents.
A1A11	determine the sum, difference, and product of polynomials of degree one or two.
A1A12	determine the quotient of a polynomial of degree of one or two when divided by a polynomial of degree one or two.
A1A13	determine the factors of a polynomial of degree one or two and write the polynomial in factored form.

A1A14	determine the factors of simple trinomials of degree one or two.
A1A15	determine a quadratic function when given the roots or graph of its related equation.
A1A16	determine if a binomial can be written as the difference of two squares and if possible transform it to illustrate this structure, such as an expression $49x^4 - y^4 = (7x^2)^2 - (y^2)^2 = (7x^2 + y^2)(7x^2 - y^2)$.
A1A17	transform polynomial expressions with degree of one or two to equivalent forms using the distributive property.
A1A18	solve a literal equation for a specified variable.

Geometry

Mathematical Process Standards Geometry	
I.	Apply mathematics to problems arising in everyday life, society and the workplace.
II.	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.
III.	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
IV.	Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.
V.	Create and use representations to organize, record, and communicate mathematical ideas.
VI.	Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

Geometry Focal Areas
Representations connecting algebra and geometry
Logical argument, proof, congruence and constructions
Similarity and Trigonometry
Problem solving with surface area and volume
Basic theorems about circles
Probability

Geometry

Representations: Connecting Algebra and Geometry. The student is expected to: **GA**

Representation	
GA01	determine the coordinates of a point that is a given fractional distance from one end of a line segment to the other in the coordinate plane, including finding the midpoint.
GA02	determine an equation with graph of a line parallel or perpendicular to a given line and that passes through a given point.
GA03	prove geometric relationships including congruence of segments and parallelism or perpendicularity of pairs of lines, using coordinates and algebraic methods.
GA04	determine the equation of a parabola given its focus and directrix.
GA05	solve problems with geometric contexts arising from mathematical and real-world situations that include symbolic representations.

Geometry

Logical Argument, Proof, Congruence and Constructions. The student is expected to: **GG**

GG01	distinguish between undefined terms, definitions, postulates and theorems using mathematical induction and deductive reasoning.
GG02	identify the converse, inverse, and contrapositive of a conditional statement.
GG03	verify that a conjecture is false using counterexamples.
GG04	represent formal geometric constructions choosing from a variety of tools such as compass and straightedge, paper folding and dynamic geometric software. (Constructions include duplicating a line segment; duplicating an angle; constructing an angle bisector; finding the midpoint of a line segment; finding a line parallel or perpendicular to a given line through a point not on the line; and constructing the perpendicular bisector of a line segment.)
GG05	represent the construction of an equilateral triangle, a square or a regular hexagon inscribed in a circle choosing from a variety of tools such as compass and straightedge, paper folding and dynamic geometric software.
GG06	identify key differences between geometric relationship within Euclidean and spherical geometries. (Include parallel lines and the sum of the angles in a triangle.)
GG07	identify transformations of figures in a plane using function notation, taking points in the plane as inputs and giving other points as outputs.
GG08	determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations (translation, reflection, rotation) or non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). (Sequences include rotations and dilations where the center can be any point in the plane.)
GG09	identify the sequence of Euclidean transformations including rotations and reflections that will carry the image of a given figure onto itself in a given number of steps.
GG10	identify congruent figures and their corresponding sides and angles using the definition of congruence in terms of rigid motions.
GG11	prove whether two triangles are congruent by applying the SAS, ASA, AAS or SSS triangle congruence conditions.

GG12	use the fact that the sum of the measures of the lengths of any two sides of a triangle is greater than the measure of the length of the third side (Triangle Inequality theorem) in mathematical and real-world problems.
GG13	prove theorems about the relationships between line segments, lines, and angles that are formed choosing from various formats of proof such as paragraph, flow, two-column, coordinate or transformational, and use these relationships to solve problems. (Theorems include vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; and points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.)
GG14	prove theorems about the angle relationships in triangles choosing from various formats of proof such as paragraph, flow, two-column, coordinate or transformational, and use these relationships to solve problems. (Theorems include measures of interior angles of a triangle sum to 180°; base angles of an isosceles triangle are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.)
GG15	prove theorems about parallelograms choosing from various formats of proof such as paragraph, flow, two-column, coordinate or transformational, and use these theorems to solve problems. (Theorems include opposite sides are congruent; opposite angles are congruent; the diagonals of a parallelogram bisect each other; and rectangles are parallelograms with congruent diagonals.)

Geometry	
Similarity and Trigonometry. The student is expected to:	**GS**
GS01	apply the definition of similarity in terms of a similarity transformation to determine whether two figures are similar including identifying proportional sides and the congruent corresponding angles.
GS02	apply the AA criterion to verify similar triangles and apply the proportionality of the sides to solve problems in real-world and mathematical problems.
GS03	prove theorems about triangles choosing from various formats of proof such as paragraph, flow, two-column, coordinate or transformational, and use these theorems to solve problems. (Theorems include a line parallel to one side of a triangle divides the other two proportionally and conversely and the Pythagorean theorem proved using triangle similarity.)
GS04	prove the theorem that the length of the altitude drawn to the hypotenuse of a right triangle is the geometric mean between the lengths of the segments on the hypotenuse, choosing from various formats of proof such as paragraph, flow, two-column, coordinate, or transformational, and use this theorem to solve problems.
GS05	determine the lengths and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine and tangent in mathematical and real-world problems.
GS06	apply the relationships in special right triangles ($30° - 60° - 90°$ and $45° - 45° - 90°$) and the Pythagorean theorem in mathematical and real-world problems.

Geometry

Measurement. The student is expected to:	**GM**
GM01	use appropriate units of measure to solve real-world problems, including conversions between measurement systems.
GM02	identify the shapes of two-dimensional cross-sections of three-dimensional object and identify three-dimensional objects generated by rotations of two-dimensional objects.
GM03	determine how changes in the linear dimensions of a shape affect its perimeter, area, surface area or volume.
GM04	determine the area of regular polygons and the area of composite two-dimensional figures in mathematical and real-world problems.
GM05	determine the total and lateral surface area (where applicable) of three-dimensional figures in mathematical and real-world problems. (These figures include prisms, pyramids, cones, cylinders, spheres and composite figures. Dimensions may be labeled with single variables.)
GM06	determine the volume of three-dimensional figures in mathematical and real-world problems. (These figures include prisms, pyramids, cylinders, cones, spheres, and composite figures. Dimensions may be labeled with single variables.)

Geometry	
Circles. The student is expected to:	**GC**
GC01	prove theorems about circles, including relationships among inscribed angles, radii, chords, lines, and line segments, and use these relationships to solve problems.
GC02	apply the proportional relationship between the measure of an arc length of a circle and the circumference of the circle in mathematical and real-world problems. This includes the ratio of the length of an arc intercepted by a central angle and the radius of the circle and the radian measure of an angle.
GC03	apply the proportional relationship between the measure of the area of a sector of a circle and the area of the circle in mathematical and real-world problems.
GC04	determine the equation for the graph of a circle with radius r and center (h, k), $(x-h)^2 + (y-k)^2 = r^2$, and justify the derivation of this equation using the Pythagorean theorem and properties of translations.

Geometry

Probability. The student is expected to: **GD**

GD01	determine probabilities based on area in mathematical and real-world problems. [Obtain the probability measure by taking the measure (area) of a subset and dividing it by the measure (area) of the entire set].
GD02	represent events as subsets of a sample space using the characteristics of the outcomes or as unions, intersections or complements of other events in mathematical and real-world problems.
GD03	identify whether two events are independent and give an example of how the probability of the two events occurring together is the product of their probabilities.
GD04	interpret results in a two-way frequency table of data when the two variables are related.
GD05	treating a two-way frequency table as a sample space, identify whether two events are independent and determine conditional probabilities.
GD06	apply conditional probability of A given B and independence in real-world problems.
GD07	use the Addition rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ in mathematical and real-world problems.

Algebra II

Mathematical Process Standards Algebra II	
I.	Apply mathematics to problems arising in everyday life, society and the workplace.
II.	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.
III.	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
IV.	Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.
V.	Create and use representations to organize, record, and communicate mathematical ideas.
VI.	Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

Algebra II Focal Areas
Features of functions
Systems of linear functions, equations and inequalities
Quadratic, square root, cubic and cube root functions, equations and inequalities
Other functions, equations and inequalities
Number and algebraic methods
Data analysis

Algebra II

Features of Functions. The student is expected to: **A2F**

A2F01	graph the functions $f(x) = b^x, f(x) = \log(x), f(x) = \lvert x \rvert, f(x) = \sqrt{x}, \ f(x) = \frac{a}{x}, f(x) = x^3$ and when applicable determine the key features such as domain, range, intercepts, relative maximums and minimums given an interval, symmetries, and asymptotic behavior. For the functions $f(x) = b^x$ and *f(x)*=log(*x*), *b* is 2, 10 or *e*.
A2F02	determine the composition of two functions, including the necessary restrictions on the domain.
A2F03	explain the relationship between a function and its inverse, if it exists, including the restrictions on domains and ranges. (Include quadratic, square root, logarithmic and exponential functions.)
A2F04	graph the inverse of a function, if it exists, by reflection across the line $y = x$.
A2F05	graph step and other piecewise-defined functions, including the greatest integer function, and when applicable determine key features such as domain, range and symmetry in mathematical and real-world problems.

Algebra II

Linear Functions, Equations, and Inequalities. The student is expected to:	**A2L**
A2L01	generate systems of equations for mathematical and real-world problems, including systems consisting of three linear equations in three unknowns and systems consisting of two equations, the first linear and the second quadratic.
A2L02	generate systems of at least two linear inequalities in two variables to solve mathematical and real-world problems.
A2L03	represent a system of linear equations using a matrix in mathematical and real-world problems ,and explain why it might be an advantage to replace the system by the matrix.
A2L04	solve systems of three linear equations with three variables algebraically in mathematical and real-world problem. (Include the use of algebraic methods and matrices.)
A2L05	determine the reasonableness, including using the appropriate units, of solutions to systems of three linear equations in three variables in mathematical and real-world problems.
A2L06	solve systems of two or more linear inequalities with two variables in mathematical and real-world problems both algebraically and using matrices.
A2L07	determine the reasonableness, including using the appropriate units, of solutions to systems of two or more linear inequalities in two variables in mathematical and real-world problems.
A2L08	solve algebraically systems of two equations in two variables made up of a linear equation and a quadratic equation in mathematical and real-world problems.
A2L09	determine the reasonableness, including using the appropriate units, of solutions to systems of a linear equation and a quadratic equation in two variables in mathematical and real-world problems.

Algebra II

Quadratic, Square Root, Cubic and Cube Root Functions, Equations, and Inequalities. The student is expected to:	**A2Q**
A2Q01	generate a quadratic function with graph having a given vertex and axis of symmetry, and generate a quadratic function with a graph that contains two or more specified points in the plane.
A2Q02	generate square root functions for mathematical and real-world problems.
A2Q03	generate quadratic inequalities for mathematical and real-world problems.
A2Q04	generate quadratic, square root, cubic, and cube root equations for real-world problems.
A2Q05	rewrite a quadratic function $f(x) = ax^2 + bx + c$ in the form $f(x) = a(x-h)^2 + k$ to reveal different properties of *f(x)* in mathematical and real-world problems.
A2Q06	determine the effect on the graphs of $f(x) = \sqrt{x}, f(x) = x^3,$ and $f(x) = \sqrt[3]{x}$ when *f(x)* is replaced by *a f(x), f(x) + d, f(bx)*, or *f(x+c)* for specific positive and negative values of *a,b,c,* and *d.*
A2Q07	generate the equation of a parabola using given features that may include vertex, focus, axis of symmetry, direction of opening and focal width in mathematical and real-world problems.
A2Q08	solve quadratic and square root equations that may have real or complex roots in mathematical and real-world problems.
A2Q09	give examples showing how extraneous solutions may arise with quadratic equations in real-world problems.
A2Q10	solve cube root equations that have real or complex roots in mathematical and real-world problems.
A2Q11	determine the reasonableness, including using the appropriate units, of a solution to a square root or cube root equation in mathematical and real-world problems.

Algebra II

Other Functions, Equations, and Inequalities. The student is expected to: **A2E**

Exponential and Logarithmic Functions	
A2E01	generate exponential and logarithmic equations that model real-world situations.
A2E02	determine the effect on the graphs of $f(x) = b^x$, and $f(x) = \log(x)$ when *f(x)* is replaced by *a f(x), f(x) + d, f(bx)*, or *f(x+c)* for specific positive and negative values of *a,b,c* and *d.*
A2E03	solve exponential and logarithmic equations that have real roots in mathematical and real-world problems.
A2E04	determine the reasonableness, including using the appropriate units, of a solution to an exponential equation in mathematical and real-world problems.
A2E05	determine the reasonableness, including using the appropriate units, of a solution to a logarithmic equation in mathematical and real-world problems.
Absolute Value Functions, Equations, and Inequalities	
A2E06	generate absolute value equations that model mathematical and real-world situations.
A2E07	determine the effect on the graphs of $f(x) = \|x\|$ when *f(x)* is replaced by *a f(x), f(x) + d, f(bx)*, or *f(x+c)* for specific positive and negative values of *a,b,c* and *d.*
A2E08	solve absolute value equations that have real or complex roots in mathematical and real-world problems.
A2E09	solve absolute value inequalities in mathematical and real-world problems.
Rational Functions and Equations	
A2E10	generate rational equations that model mathematical and real-world situations.
A2E11	determine the effect on the graphs of $f(x) = \frac{a}{x}$, when *f(x)* is replaced by *a f(x), f(x) + d, f(bx)*, or *f(x+c)* for specific positive and negative values of *a,b,c,* and *d.*

A2E12	solve rational equations that have real or complex roots in mathematical and real-world problems.
A2E13	determine the reasonableness, including using the appropriate units, of a solution to a rational equation in mathematical and real-world problems.
A2E14	generate examples showing how extraneous solutions may arise with rational equations in real-world problems.

Algebra II

Number and Algebraic Methods. The student is expected to: **A2A**

A2A01	use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
A2A02	apply the properties of matrix addition, matrix subtraction, scalar multiplication of a matrix and matrix multiplication in mathematical and real-world problems.
A2A03	transform algebraic expressions involving rational exponents.
A2A04	determine the sum, difference, and product of polynomials in mathematical and real-world problems.
A2A05	determine the quotient of a polynomial divided by a binomial in mathematical and real-world problems, including quotients with remainders
A2A06	apply the Remainder Theorem to determine the linear factors of a polynomial.
A2A07	transform radical expressions that contain variables to equivalent forms in mathematical and real-world and problems.
A2A08	transform exponential expressions to their corresponding logarithmic expressions and logarithmic expressions to their corresponding exponential expressions in mathematical and real-world problems.
A2A09	determine the sum, difference, product and quotient of simple rational expressions including determining the restrictions on the domain in mathematical and real-world problems.
A2A10	determine linear and quadratic factors of a polynomial, including factoring the sum and difference of two cubes, when suitable factorizations are available.

Algebra II

Data. The student is expected to: **A2D**

A2D01	when appropriate, use the mean and standard deviation of a data set to fit a normal distribution and to approximate normal population percentages using tools such as calculators, spreadsheets and tables.
A2D02	recognize that there are data sets for which it is not appropriate to model with a normal distribution.
A2D03	determine whether data from generating process such as simulation are consistent with a specified model.
A2D04	distinguish the purposes and differences among sample surveys, experiments and observation studies including explaining the role of randomization in each type of study and the scope of inference from each type of study.
A2D05	use data from a sample survey to estimate population mean or population proportion including developing the margin of error through the use of simulation models for random sampling.
A2D06	use data from a randomized experiment to compare two treatments and use simulation to decide if the observed differences are statistically significant.
A2D07	determine the strengths and weaknesses of reports based on data when solving problems in real-world situations.

Pre-Calculus

Mathematical Process Standards Pre-Calculus	
I.	Apply mathematics to problems arising in everyday life, society and the workplace.
II.	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.
III.	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
IV.	Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.
V.	Create and use representations to organize, record, and communicate mathematical ideas.
VI.	Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

Pre-Calculus Focal Areas
Functions
Geometric reasoning
Measurement
Number and algebraic methods

Pre-Calculus

Functions. The student is expected to: **PF**

PF01	use the composition of two functions to model and solve real-world problems.
PF02	give an example that function composition is not always commutative.
PF03	represent a given function as a composite function of two or more functions. For example, $f(x)=\sqrt{x^2+3}$ can be represented as $f(x)=(g\circ h)(x)$ where $g(x)=\sqrt{x}$ and $h(x)=x^2+3$, or *f* can be represented as $f(x)=(g\circ w\circ v)(x)$ where $g(x)=\sqrt{x}$, $w(x)=x+3$, and $v(x)=x^2$.
PF04	describe symmetry of graphs of even and odd functions in mathematical and real-world problems.
PF05	determine an inverse function, when it exists, for a given function over its domain or a subset of its domain and represent the inverse graphically and/or algebraically.
PF06	graph exponential functions and logarithmic functions (including base *e*), trigonometric functions, piece-wise defined functions, rational functions and inverse trigonometric functions.
PF07	analyze the key features of exponential functions and logarithmic functions (including base *e*), trigonometric functions, piece-wise defined functions, rational functions and inverse trigonometric functions such as domain, range, symmetry, relative maximum, relative minimum, zeros , asymptotes, and intervals over which the function is increasing or decreasing.

Pre-Calculus

Geometric Reasoning. The student is expected to: **PG**

PG01	graph a set of parametric equations.
PG02	convert parametric equations into rectangular relations and convert rectangular relations into parametric equations to solve mathematical and real-world problems.
PG03	graph logarithmic functions with various bases, including the natural log function, and their transformations, including $a \cdot f(x)$, $f(x) + d, f(x - c), f(b \cdot x)$ for specific values of *a*, *b*, *c*, and *d*, in mathematical and real-world problems.
PG04	graph power functions (including radical) and their transformations including the concept of end behavior using infinity notation to communicate this characteristic in mathematical and real-world problems.
PG05	graph rational functions and determine characteristics such as domain, asymptotes (horizontal, vertical, slant) and describe the differences between the domains of the rational functions ($p(x)$ $s(x)$) / ($q(x)$ $s(x)$) and $p(x)$ / $q(x)$ for *p*, *q* and *s* polynomial functions in mathematical and real-world problems.
PG06	determine various types of discontinuities in the interval $(-\infty, \infty)$ as they relate to rational functions and explore the limitations of the graphing calculator in mathematical and real-world problems.
PG07	graph exponential functions and their transformations, including $a \cdot f(x)$, $f(x) + d, f(x - c), f(b \cdot x)$ for specific values of *a*, *b*, *c*, and *d*, to solve problems in mathematical and real-world problems.
PG08	graph points in the polar coordinate system and convert between the rectangular and polar coordinate systems in mathematical and real-world problems.
PG09	graph polar equations by plotting points, using symmetry, using zeros and maximum values including recognizing special polar graphs.
PG10	derive, in rectangular coordinates, the equation of a circle, parabola, ellipse, and hyperbola from their locus definitions.
PG11	write the equation of an ellipse with center (*h*,*k*) and determine the foci and eccentricity in mathematical and real-world problems.
PG12	write the equation of a hyperbola with center (*h*,*k*) and determine the foci, eccentricity and the equations of the asymptotes in mathematical and real-world problems.
PG13	determine the conic section formed when a plane intersects a double napped cone.

PG14	use trigonometric identities such as reciprocal, quotient, Pythagorean, cofunctions, even/odd, and sum and difference identities for cosine and sine to simplify trigonometric expressions in mathematical and real-world problems.
PG15	determine whether a situation can be modeled by a sinusoidal function, develop a mathematical model to describe the situation, and use the model to solve mathematical and real-world problems.
PG16	graph the sine and cosine functions and apply one or more transformations to these functions, including $a \cdot f(x),\ f(x)+d, f(x-c), f(b \cdot x)$ for specific values of *a*, *b*, *c* and *d* in mathematical and real-world problems.
PG17	graph inverse trigonometric functions (arcsin *x*, arccos *x*) with and without technology including explaining why there is a need for restricted domains and ranges in mathematical and real-world problems.
PG18	estimate the limit of a function at a point, including one-sided limits, using graphs and tables.
PG19	illustrate cases in which a limit of a function fails to exist at a point or as *x* grows without bound, including unequal left-hand and right-hand limits at a point, unbounded behavior, and oscillating behavior.
PG20	use knowledge of the limiting process to describe the behavior of a function including end-behavior.
PG21	explain, informally, why a limit fails to exist at a point or as *x* grows without bound, including unequal left-hand and right-hand limits at a point, unbounded behavior, and oscillating behavior.
PG22	solve problems requiring an understanding of the limiting process in mathematical and real-world problems.
PG23	use vectors to model situations involving magnitude and direction.
PG24	represent the addition of vectors and the multiplication of a vector by a scalar geometrically and symbolically.
PG25	apply vector addition and multiplication of a vector by a scalar in mathematical and real-world problems.

Pre-Calculus

Measurement. The student is expected to:		PM
PM01	determine the relationship between the unit circle, the wrapping function (W(x)=(cos x, sin x)), and the definition of a periodic function to evaluate trigonometric functions in mathematical and real world problems.	
PM02	determine the values of the trigonometric functions at the special angles (30°, 45°, 60°) and the angles, such as the half-angles, related to them in mathematical and real-world problems.	
PM03	determine, using reference angles, the value of trigonometric ratios of any angle, including solving problems involving points on the terminal side of an angle in mathematical and real world problems.	
PM04	use the Law of Sines in mathematical and real-world problems.	
PM05	use the Law of Cosines in mathematical and real-world problems.	
PM06	identify radian measure of a central angle of a unit circle as the length of the arc subtended by that angle.	
PM07	represent angles in radians and degrees based on the concept of rotation and find the measure of reference angles and angles in standard position with a common terminal side in mathematical and real-world problems involving arc length, linear and angular speeds and area of the sector of a circle.	
PM08	use trigonometry to determine directional bearing and harmonic motion in mathematical and real world problems .	

Pre-Calculus

Number and Algebraic Methods. The student is expected to:	**PA**
PA01	represent finite sums and infinite series using sigma notation.
PA02	calculate the value, when it exists, of an expression written in sigma notation.
PA03	represent arithmetic sequences and series using a recursion formula and sigma notation.
PA04	determine the n^{th} terms and the sum of a finite arithmetic series in mathematical and real-world problems.
PA05	represent geometric sequences and series using a recursion formula and sigma notation.
PA06	calculate the n^{th} term, n^{th} partial sum, and sum of a geometric series when this sum exists.
PA07	determine the trigonometric form of a complex number and relate to polar coordinates.
PA08	determine the product and quotient of complex numbers in trigonometric form.
PA09	determine powers and all the n^{th} roots of complex numbers.
PA10	use the properties of logarithms to evaluate or transform logarithmic expressions requiring the change of base formula in both mathematical and real-world problems.
PA11	use the Binomial Theorem to write the expression $(a + b)^n$ (n a positive integer) in expanded form.
PA12	use Pascal's Relation (triangle) to give a recursive definition of the coefficient $a^p b^{n-p}$ in the expansion of $(a+b)n$.
PA13	generate and solve logarithmic equations including those requiring change of base in mathematical and real-world problems.

PA14	generate and solve exponential equations in mathematical and real-world problems.
PA15	use parametric equations to model problems involving motion in mathematical and real-world problems.
PA16	solve trigonometric equations in mathematical and real-world problems.
PA17	solve polynomial equations with real coefficients by applying a variety of techniques including the Fundamental Theorem of Algebra, factoring, Descartes Rule of Signs, and knowing that complex zeros occur in conjugate pairs in mathematical and real-world problems.
PA18	solve polynomial and rational inequalities with real coefficients using critical numbers, by testing intervals and writing the solution set in interval notation in mathematical and real-world problems.

Advanced Quantitative Reasoning (AQR)

Mathematical Process Standards Advanced Quantitative Reasoning (AQR)	
I.	Apply mathematics to problems arising in everyday life, society and the workplace.
II.	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.
III.	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
IV.	Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.
V.	Create and use representations to organize, record, and communicate mathematical ideas.
VI.	Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

AQR Focal Areas
Expressions, equations and generalized relationships
Geometric reasoning
Probabilistic and statistical reasoning

Advanced Quantitative Reasoning

Numeric Reasoning. The student is expected to:	**AQRN**
Develop and Apply Skills Used in College and Careers	
AQRN01	gather data, conduct investigations, and apply mathematical concepts and models to solve problems in mathematics and other disciplines.
AQRN02	demonstrate reasoning skills in developing, explaining, and justifying sound mathematical arguments, and analyze the soundness of mathematical arguments of others.
AQRN03	communicate with and about mathematics orally and in writing as part of independent and collaborative work, including making accurate and clear presentations of solutions to problems.
Analyze Numerical Data	
AQRN04	apply, compare, and contrast published ratios, rates, ratings, averages, weighted averages, and indices to make informed decisions.
AQRN05	solve problems involving large quantities that are not easily measured.
AQRN06	use arrays to efficiently manage large collections of data and add, subtract, and multiply matrices to solve applied problems.
AQRN07	apply algorithms and identify errors in recording and transmitting identification numbers.
Use Ranking and Selection	
AQRN08	apply and analyze various ranking algorithms to determine an appropriate method for a given situation.
AQRN09	analyze various voting and selection processes to determine an appropriate method for a given situation.
Use Network Models	
AQRN10	solve problems involving scheduling or routing situations that can be represented by a vertex-edge graph, and find critical paths, Euler paths, or minimal spanning trees.
AQRN11	construct, analyze, and interpret flow charts in order to develop and describe problem-solving procedures.

Advanced Quantitative Reasoning

Algebraic Reasoning (Expressions, Equations, and Generalized Relationships). The student is expected to: **AQRA**

Model Data	
AQRA01	determine whether or not there is a linear relationship in a set of bivariate data by finding and interpreting the correlation coefficient for the data.
AQRA02	collect numerical bivariate data; use the data to create a scatterplot; select a function to model the data, justify the selection, and use the model to make predictions.
Model Change and Relationships	
AQRA03	determine or analyze an appropriate growth or decay model for problem situations, including linear, exponential, and logistic functions.
AQRA04	determine or analyze an appropriate cyclical model for problem situations that can be modeled with trigonometric functions.
AQRA05	determine or analyze an appropriate piecewise model for problem situations.
AQRA06	solve problems using recursion or iteration, including those involving population growth or decline and compound interest.
Model Financial Situations	
AQRA07	determine, represent, and analyze mathematical models for various types of income calculations to determine the best option for a given situation.
AQRA08	determine, represent, and analyze mathematical models for expenditures, including those involving credit, to determine the best option for a given situation.
AQRA09	determine, represent, and analyze mathematical models and appropriate representations for various types of loans and investments to determine the best loan or investment plan for a given situation.

Advanced Quantitative Reasoning

Geometric Reasoning. The student is expected to:	**AQRG**
Model with Geometric Tools	
AQRG01	create and use two- and three-dimensional representations of authentic situations using geometric models or dynamic geometric environments for computer-aided design and other applications**.**
AQRG02	use vectors to represent and solve applied problems.
AQRG03	use matrices to represent geometric transformations and solve applied problems.
AQRG04	solve geometric problems involving inaccessible distances.

Advanced Quantitative Reasoning

Probabilistic and Statistical Reasoning. The student is expected to:		**AQRD**
Analyze and Evaluate Risk and Return in the Context of Everyday Situations		
AQRD01	determine and interpret conditional probabilities and probabilities of compound events by constructing and analyzing representations, including tree diagrams, Venn diagrams, and area models, to make decisions in problem situations.	
AQRD02	use probabilities to make and justify decisions about risks in everyday life.	
AQRD03	calculate expected value to analyze mathematical fairness, payoff, and risk.	
Critique Applications of Statistics		
AQRD04	identify limitations or lack of information in studies reporting statistical information, especially when studies are reported in condensed form.	
AQRD05	interpret and compare the results of polls, given a margin of error.	
AQRD06	identify uses and misuses of statistical analyses in studies reporting statistics or using statistics to justify particular conclusions, including assertions of a cause and effect relationship rather than an association.	
AQRD07	describe strengths and weaknesses of sampling techniques, data and graphical displays, and interpretations of summary statistics and other results appearing in a study, including reports published in the media.	
Conduct Statistical Analyses		
AQRD08	determine the need for and purpose of a statistical investigation and what type of statistical analysis can be used to answer a specific question or set of questions.	
AQRD09	identify the population of interest, select an appropriate sampling technique, and collect data**.**	
AQRD10	identify the variables to be used in a study.	
AQRD11	determine possible sources of statistical bias in a study and how such bias may affect the ability to generalize the results.	

AQRD12	create data displays for given data sets to investigate, compare, and estimate center, shape, spread, and unusual features of the data**.**
AQRD13	determine possible sources of variability of data, both those that can be controlled and those that cannot be controlled.
Communicate Statistical Information	
AQRD14	report results of statistical studies to a particular audience, including selecting an appropriate presentation format, creating graphical data displays, and interpreting results in terms of the question studied.
AQRD15	justify the design and the conclusion(s) of statistical studies, including the methods used for each**.**
AQRD16	communicate statistical results in both oral and written formats using appropriate statistical and nontechnical language.

Mathematical Models with Applications (MMA)

Mathematical Process Standards Mathematical Models with Applications (MMA)	
I.	Apply mathematics to problems arising in everyday life, society and the workplace.
II.	Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process.
III.	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
IV.	Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and language.
V.	Create and use representations to organize, record, and communicate mathematical ideas.
VI.	Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.

MMA Focal Areas
Numeric reasoning
Expressions, equations and generalized relationships
Geometric reasoning
Probabilistic and statistical reasoning

Mathematical Models with Applications

Numeric Reasoning. The student is expected to:		**MMAN**
MMAN01	compare and analyze various methods for solving a real-life problem.	
MMAN02	use multiple approaches (algebraic, graphical, and geometric methods) to solve problems from a variety of disciplines.	
MMAN03	select a method to solve a problem, defend the method, and justify the reasonableness of the results.	

Mathematical Models with Applications

Algebraic Reasoning (Expressions, Equations, and Generalized Relationships). The student is expected to: **MMAA**

MMAA01	use rates, linear functions, and direct variation to solve problems involving personal finance and budgeting, including compensations and deductions.
MMAA02	solve problems involving personal taxes.
MMAA03	analyze data to make decisions about banking.
MMAA04	analyze methods of payment available in retail purchasing and compare relative advantages and disadvantages of each option.
MMAA05	use amortization models to investigate home financing and compare buying and renting a home.
MMAA06	use amortization models to investigate automobile financing and compare buying and leasing a vehicle.
MMAA07	analyze types of savings options involving simple and compound interest and compare relative advantages of these options.
MMAA08	analyze and compare coverage options and rates in insurance.
MMAA09	investigate and compare investment options including stocks, bonds, annuities, and retirement plans.
MMAA10	use direct and inverse variation to describe physical laws such as Hook's, Newton's, and Boyle's laws.

Mathematical Models with Applications

Geometric Reasoning. The student is expected to: **MMAG**

MMAG01	use geometric models available through technology to model growth and decay in areas such as population, biology, and ecology.
MMAG02	use trigonometric ratios and functions available through technology to calculate distances and model periodic motion.
MMAG03	use geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and architecture.
MMAG04	use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music.

Mathematical Models with Applications

Probabilistic and Statistical Reasoning. The student is expected to:	**MMAD**
MMAD01	interpret information from various graphs, including line graphs, bar graphs, circle graphs, histograms, scatterplots, dot plots, stem and leaf plots, and box and whisker plots to draw conclusions from the data.
MMAD02	analyze numerical data using measures of central tendency, variability, and correlation in order to make inferences.
MMAD03	analyze graphs from journals, newspapers, and other sources to determine the validity of stated arguments.
MMAD04	use regression methods available through technology to describe various models for data such as linear, quadratic, exponential, etc., select the most appropriate model, and use the model to interpret information.
MMAD05	formulate a meaningful question, determine the data needed to answer the question, gather the appropriate data, analyze the data, and draw reasonable conclusions.
MMAD06	communicate methods used, analyses conducted, and conclusions drawn for a data-analysis project by written report, visual display, oral report, or multi-media presentation.
MMAD07	determine the appropriateness of a model for making predictions from a given set of data.
MMAD08	compare theoretical and empirical probability.
MMAD09	use experiments to determine the reasonableness of a theoretical model such as binomial, geometric, etc.

Frequently Asked Questions

1. **Are the Commissioner's Standards for Mathematics a "total overhaul" from our current mathematics TEKS?**

   Yes.

2. **How are the Commissioner's Standards for Mathematics different from our current TEKS?**

   They are more rigorous and specific. The verbs add clarity to the intent.

   The placement of content in these standards is aligned to recent research, the report of the National Mathematics Advisory Panel and to the practices of countries that are high performing in mathematics.

   Vertical development is clear as is the long-term intent of the vertical development.

   - For example, we can trace number concepts, fractions and decimals into rational numbers and irrational numbers.

   A big part of grade 8 is the algebraic underpinnings of foundations of functions. The development of equation solving skills is much more explicit – helpful for Algebra I.

   These standards have more of a focal point flavor than a strand flavor.

   - For example, proportionality is named as a focal point and the relevant math content is listed under proportionality as a unifying theme for the learning related to these standards.

   Under the TEKS, the mathematical connections to proportionality are not emphasized and teachers often miss these connections.

   Proportionality is a critical concept for success in Algebra. It lays the foundation for linear functions and equations.

3. **Are there other differences between the Commissioner's Standards for Mathematics and the current mathematics TEKS?**

   Yes.

   These standards are the target for learning, the mathematics that students should be able to do at the end of each school year or course.

   Some of the current math TEKS have instructional strategies embedded. These standards remove instructional strategies so that teachers may determine how to best meet the needs of their students.

A great deal of research has been done since the mathematics TEKS were first written. Students around the country are experiencing success with concepts at younger grades when the content is addressed appropriately. In addition, teachers will appreciate the clarity.

These standards meet or exceed the difficulty level of the Common Core State Standards in Mathematics.

4. How are the Commissioner's Standards for Algebra I, Geometry, and Algebra II different from the TEKS?

Texas now has College and Career Readiness Standards that were not in place when the TEKS were written. The CCRS underscore that mathematics of everyday life is different in this century than it was in the last. The increased use of data and statistics in everyday life makes it more important than ever for all students to know how to interpret data and make informed decisions using data.

The Commissioner's Standards for these high school courses also include carefully chosen data and statistics topics that support the course content. These same topics are selectively included in grades K-8 to support the focal concepts for the grade level.

The Commissioner's Standards for Mathematics have greater specificity regarding what students should know and be able to do. This provides clarity for teachers and for assessment development.