Chapter 111, Texas Essential Knowledge and Skills for Mathematics, Subchapter C, High School

§111.46. Algebraic Reasoning, Adopted 2015 (One Credit).

(a) General requirements. Students shall be awarded one credit for successful completion of this course. Prerequisite: Algebra I.

(b) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) In Algebraic Reasoning, students will build on the knowledge and skills for mathematics in Kindergarten-Grade 8 and Algebra I. Students will broaden their knowledge of multiple representations and linear, quadratic, square root, rational, cubic, cube root, exponential, absolute value, and logarithmic functions. Students will study these functions through explorations of patterns and structure, composition of functions, number and algebraic methods, and modeling from data.

(4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Patterns and structure. The student applies mathematical processes to connect finite differences to attributes of functions and related rates of change. The student is expected to:

(A) classify a polynomial function represented tabularly or graphically using finite differences;

(B) describe and analyze the relationship between a function, including linear, quadratic, cubic, and exponential functions, its rate of change, and related finite differences numerically, tabularly, and graphically; and

(C) write a function and its restricted domain and range for a given numeric pattern or numeric sequence using finite differences.

(3) Patterns and structure. The student applies mathematical processes to understand the connections among representations of combinations of functions, functions and their inverses, and key attributes of these functions. The student is expected to:

(A) represent a resulting function tabularly, graphically, and symbolically when functions, including the constant function, \( f(x) = x, f(x) = x^2, f(x) = |x|, f(x) = 1/x, f(x) = x^3, f(x) = \sqrt[3]{x}, f(x) = b^x, f(x) = x^3, f(x) = \log_b(x) \) where \( b \) is 10 and \( e \), are combined or separated using addition, subtraction, multiplication, division, and a sequence of these operations;

(B) compare and contrast the domain and range of the resulting function when functions, including the constant function, \( f(x) = x, f(x) = x^2, f(x) = |x|, f(x) = 1/x, f(x) = x^3, f(x) = \sqrt[3]{x}, f(x) = b^x, f(x) = x^3, f(x) = \log_b(x) \) where \( b \) is 10 and \( e \), are combined or separated using addition, subtraction, multiplication, division, and a sequence of these operations, with the domains and ranges of the original functions;

(C) generalize the key attributes of the resulting function, including zeros, maxima, minima, and \( y \)-intercepts of functions when functions, including the constant function, \( f(x) = x, f(x) = x^2, f(x) = |x|, f(x) = 1/x, f(x) = x^3, f(x) = \sqrt[3]{x}, f(x) = b^x, f(x) = x^3, f(x) = \log_b(x) \) where \( b \) is 10 and \( e \), are combined or separated using addition, subtraction, multiplication, division, and a sequence of these operations;

(D) classify a resulting function when functions including the constant function, \( f(x) = x, f(x) = x^2, f(x) = |x|, f(x) = 1/x, f(x) = x^3, f(x) = \sqrt[3]{x}, f(x) = b^x, f(x) = x^3, f(x) = \log_b(x) \) where \( b \) is 10 and \( e \), are combined or separated using addition, subtraction, multiplication, division, and a sequence of these operations; and

(E) compare and contrast the key attributes, including domain, range, zeros, maxima, minima, and \( y \)-intercepts, of a function and its inverse numerically, tabularly, graphically, and symbolically.

(4) Composition of functions. The student applies mathematical processes to understand composition of functions numerically and symbolically. The student is expected to:

(A) write a function resulting from a composition of functions when the composition is represented using symbolic notation, including \( f \circ g \) and \( f(g(x)) \);

(B) represent the changes numerically and symbolically to \( g \) when \( g \) is defined in terms of \( f \) such as \( g(x) = f(x) + 4 \);

(C) represent the changes numerically and symbolically to \( h \) when \( h = g \circ f \) such as \( h(x) = 2g(f(x)) \) and \( h(x) = g^{-1}(f(x)) \);
(D) verify that two functions are inverses of each other numerically and symbolically through function composition; and

(E) extend the order of operations to function composition such as determining \( f, g, \) and \( h \) for \( r(x) = f \circ g \circ h \) when \( r(x) = \sqrt{x^2 - 4} \).

(5) Composition of functions. The student applies mathematical processes to understand composition of functions with connections to transformational geometry. The student is expected to:

(A) represent a function resulting from a composition of functions tabularly and graphically when the composition is represented using symbolic notation, including \( f \circ g \) and \( f(g(x)) \);

(B) represent the changes tabularly and graphically to \( g \) when \( g \) is defined in terms of \( f \) such as \( g(x) = f(x) + 4 \);

(C) represent the changes tabularly and graphically to \( h \) when \( h = g \circ f \) such as \( h(x) = 2g(f(x)) \) and \( h(x) = g^{-1}(f(x)) \); and

(D) represent a sequence of transformations of a line segment as a composition of functions.

(6) Number and algebraic methods. The student applies mathematical processes to understand number sets and define purely imaginary numbers and complex numbers, describe characteristics of complex numbers, and perform operations using complex numbers. The student is expected to:

(A) represent purely imaginary numbers using \( bi \), where \( b \) is a real number;

(B) determine real and purely imaginary numbers derived from square root expressions;

(C) extend previous knowledge of sets and subsets of numbers using a visual representation to describe relationships between sets of real numbers and complex numbers; and

(D) define complex numbers such as \( a + bi \) and operations on them, including addition, subtraction, and multiplication.

(7) Number and algebraic methods. The student applies mathematical processes to simplify and perform operations on functions represented in a variety of ways. The student is expected to:

(A) add, subtract, and multiply polynomial functions that are represented tabularly and symbolically;

(B) extend the definition of additive and multiplicative relationships to include the addition and multiplication of pairs of linear functions;

(C) compare and contrast the results when adding two linear functions and multiplying two linear functions that are represented tabularly, graphically, and symbolically;

(D) decompose a quadratic function into the product of two linear functions;

(E) determine the quotient of a polynomial function of degree three and of degree four when divided by a polynomial function of degree one and of degree two when represented tabularly and symbolically; and

(F) determine the linear factors of a polynomial function of degree two and of degree three when represented tabularly, graphically (where appropriate), and symbolically.

(8) Number and algebraic methods. The student applies mathematical processes to represent, simplify and perform operations on matrices and to solve systems of equations using matrices. The student is expected to:

(A) add and subtract matrices;

(B) multiply matrices;

(C) multiply matrices by a scalar;

(D) represent systems of two linear equations arising from mathematical and real-world situations using matrices;
(E) solve systems of two linear equations arising from mathematical and real-world situations using matrices;

(F) represent systems of three linear equations arising from mathematical and real-world situations using matrices and technology; and

(G) solve systems of three linear equations arising from mathematical and real-world situations using matrices and technology.

(9) Number and algebraic methods. The student applies mathematical processes to estimate and determine solutions to rational and exponential equations. The student is expected to:

(A) estimate a reasonable input value that results in a given output value for a rational function;

(B) estimate a reasonable input value that results in a given output value for an exponential function; and

(C) solve rational and exponential equations tabularly, graphically, and symbolically.

(10) Modeling from data. The student applies mathematical processes to analyze data and model data with corresponding functions. The student is expected to:

(A) represent the domain and range of a given linear, quadratic, and exponential function using interval notation, inequalities, and set notation;

(B) determine the accuracy of a prediction from a function that models a set of data compared to the actual data using comparisons between average rates of change and finite differences;

(C) determine a line of best fit or a curve of best fit for a set of data for situations that may be modeled with a linear, quadratic, or exponential function using finite differences; and

(D) determine the reasonableness of a line of best fit for a set of data.

§111.47. Statistics, Adopted 2015 (One Credit).

(a) General requirements. Students shall be awarded one credit for successful completion of this course. Prerequisite: Algebra I.

(b) Introduction.

(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display.
explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) In Statistics, students will build on the knowledge and skills for mathematics in Kindergarten-Grade 8 and Algebra I. Students will broaden their knowledge of variability and statistical processes. Students will study sampling and experimentation, categorical and quantitative data, probability and random variables, inference, and bivariate data. Students will connect data and statistical processes to real-world situations. In addition, students will extend their knowledge of data analysis.

(4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;
(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
(E) create and use representations to organize, record, and communicate mathematical ideas;
(F) analyze mathematical relationships to connect and communicate mathematical ideas; and
(G) display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Statistical process sampling and experimentation. The student applies mathematical processes to apply understandings about statistical studies, surveys and experiments to design and conduct a study and use graphical, numerical, and analytical techniques to communicate the results of the study. The student is expected to:

(A) compare and contrast the benefits of different sampling techniques, which include random sampling and convenience sampling methods;
(B) distinguish among observational studies, surveys, and experiments;
(C) analyze generalizations made from observational studies, surveys, and experiments;
(D) distinguish between sample statistics and population parameters;
(E) formulate a meaningful question, determine the data needed to answer the question, gather the appropriate data, analyze the data, and draw reasonable conclusions;
(F) communicate methods used, analyses conducted, and conclusions drawn for a data-analysis project through the use of one or more of the following: a written report, a visual display, an oral report, or a multi-media presentation; and
(G) critically analyze published findings for appropriateness of study design implemented, sampling methods used, or the statistics applied.

(3) Variability. The student applies the mathematical process standards when describing and modeling variability. The student is expected to:

(A) distinguish between mathematical models and statistical models;
(B) construct a statistical model to describe variability around the structure of a mathematical model for a given situation;
(C) distinguish among different sources of variability, including measurement, natural, induced, and sampling variability; and
(D) describe and model variability using population and sampling distributions.

(4) Categorical and quantitative data. The student applies the mathematical process standards to represent and analyze both categorical and quantitative data. The student is expected to:
(A) distinguish between categorical and quantitative data;
(B) represent and summarize data and justify the representation;
(C) analyze the distribution characteristics of quantitative data, including determining the possible existence and impact of outliers;
(D) compare and contrast different graphical or visual representations given the same data set;
(E) compare and contrast meaningful information derived from summary statistics given a data set; and
(F) analyze categorical data using two-way frequency tables.

(5) Probability and random variables. The student applies the mathematical process standards to connect probability and statistics. The student is expected to:
(A) determine probabilities including the use of a two-way table;
(B) describe the relationship between theoretical and empirical probabilities using the Law of Large Numbers;
(C) construct a distribution based on a technology-generated simulation or collected samples for a discrete random variable; and
(D) compare statistical measures such as sample mean and standard deviation from a technology-simulated sampling distribution to the theoretical sampling distribution.

(6) Inference. The student applies the mathematical process standards to make inferences and justify conclusions from statistical studies. The student is expected to:
(A) explain how a sample statistic provides evidence against a claim about a population parameter when using a hypothesis test;
(B) construct null and alternative hypothesis statements about a population parameter;
(C) explain the meaning of the $p$-value in relation to the significance level in providing evidence to reject or fail to reject the null hypothesis in the context of the situation;
(D) interpret the results of a hypothesis test using technology-generated results such as large sample tests for proportion, mean, difference between two proportions, and difference between two independent means; and
(E) describe the potential impact of Type I and Type II Errors.

(7) Bivariate data. The student applies the mathematical process standards to analyze relationships among bivariate quantitative data. The student is expected to:
(A) analyze scatterplots for patterns, linearity, outliers, and influential points;
(B) transform a linear parent function to determine a line of best fit;
(C) compare different linear models for the same set of data to determine best fit, including discussions about error;
(D) compare different methods for determining best fit, including median-median and absolute value;

(E) describe the relationship between influential points and lines of best fit using dynamic graphing technology; and

(F) identify and interpret the reasonableness of attributes of lines of best fit within the context, including slope and y-intercept.