Master Mathematics Teacher Standards

FINAL

Approved on January 4, 2002[*]

Texas State Board for Educator Certification

MASTER MATHEMATICS TEACHER
STANDARDS

Standard I.   Number Concepts: The Master Mathematics Teacher understands and applies knowledge of numbers, number systems and their structure, operations and algorithms, quantitative reasoning, and the vertical alignment of number concepts to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Standard II.   Patterns and Algebra: The Master Mathematics Teacher understands and applies knowledge of patterns, relations, functions, algebraic reasoning, analysis, and the vertical alignment of patterns and algebra to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Standard III.   Geometry and Measurement: The Master Mathematics Teacher understands geometry, spatial reasoning, measurement concepts and principles, and the vertical alignment of geometry and measurement to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Standard IV.   Probability and Statistics: The Master Mathematics Teacher understands probability and statistics, their applications, and the vertical alignment of probability and statistics to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Standard V.   Mathematical Processes: The Master Mathematics Teacher understands and uses mathematical processes to reason mathematically, to solve mathematical problems, to make mathematical connections within and outside of mathematics, and to communicate mathematically.

Standard VI.   Instruction: The Master Mathematics Teacher applies knowledge of mathematical content, uses appropriate theories for learning mathematics, implements effective instructional approaches for teaching mathematics, including teaching students who are at-risk, and demonstrates effective classroom management techniques.

Standard VII.   Creating and Promoting a Positive Learning Environment: The Master Mathematics Teacher demonstrates behavior that reflects high expectations for every student, promotes positive student attitudes towards mathematics, and provides equitable opportunities for all students to achieve at a high level.

Standard VIII.   Assessment: The Master Mathematics Teacher selects, constructs, and administers appropriate assessments to guide, monitor, evaluate, and report student progress to students, administrators, and parents, and develops these skills in other teachers.

Standard IX.   Mentoring and Leadership: The Master Mathematics Teacher facilitates appropriate standards-based mathematics instruction by communicating and collaborating with educational stake-holders; mentoring, coaching, exhibiting leadership, and consulting with colleagues; providing professional development opportunities for faculty; and making instructional decisions based on data and supported by evidence from research.

Standard X.   Mathematical Perspectives: The Master Mathematics Teacher understands the historical development of mathematical ideas, the interrelationship between society and mathematics, the structure of mathematics, and the evolving nature of mathematics and mathematical knowledge.

Standard I. **Number Concepts: The Master Mathematics Teacher understands and applies knowledge of numbers, number systems and their structure, operations and algorithms, quantitative reasoning, and the vertical alignment of number concepts to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).**

Teacher Knowledge: What Master Mathematics Teachers Know	Application: What Master Mathematics Teachers Can Do
EC–12 Number Concepts	*EC–4 Number Concepts*
The EC–12 Master Mathematics Teacher knows and understands:	The EC–4 Master Mathematics Teacher is able to:
1.1k the structure of number systems, the development of a sense of quantity, and the relationship between quantity and symbolic representation;	1.1s compare and contrast numeration systems;
1.2k the connections of operations, algorithms, and relations with their associated concrete and visual representations;	1.2s analyze, explain, and model the structure of numeration systems, and in particular, the role of place value and zero in the base ten system;
1.3k the relationship among number concepts, the number line, operations and algorithms, and the properties of numbers, including ideas of number theory;	1.3s demonstrate a sense of quantity and estimation for the real numbers (i.e., whole numbers, integers, rational and irrational numbers);
1.4k how to model, construct, and solve number concept problems within and outside of mathematics; and	1.4s analyze, explain, and model the four basic operations with real numbers (i.e., whole numbers, integers, rational and irrational numbers);
1.5k how number concepts, operations, and algorithms are developmental and connected between and across all grade levels.	1.5s recognize, model, and describe different ways to interpret the four basic operations involving real numbers (e.g., whole numbers, integers, fractions, decimals);
	1.6s select appropriate representations of real numbers (e.g., fractions, decimals, percents, roots, exponents, scientific notation) for particular situations and justify that selection;
	1.7s communicate the vertical alignment of number concepts across the grade levels;
	1.8s analyze, describe, and connect relationships among number properties, operations, and algorithms involving the four basic operations with real numbers (e.g., whole numbers, integers, fractions, decimals); translate among concrete, pictorial, and symbolic representations; and recognize these concepts and relationships in real life situations;

Standard I. **Number Concepts: The Master Mathematics Teacher understands and applies knowledge of numbers, number systems and their structure, operations and algorithms, quantitative reasoning, and the vertical alignment of number concepts to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).**

Application: What Master Mathematics Teachers Can Do

EC–4 Number Concepts (continued)

1.9s   work with real numbers and demonstrate, explain, and model how some situations that have no solution in the whole, integer, or rational number systems have solutions in the real number systems;

1.10s   analyze error patterns that often occur when students use algorithms to perform operations;

1.11s   recognize and analyze appropriate alternative algorithms for the four basic operations with whole numbers, fractions, and decimals;

1.12s   describe ideas from number theory (e.g., prime numbers, composite numbers, greatest common factors) as they apply to whole numbers, integers, and fractions, and use these ideas in problem situations;

1.13s   apply place value and other number properties to develop techniques of mental mathematics and computational estimation; and

1.14s   demonstrate a sense of equivalency among different representations of rational numbers.

Standard I. **Number Concepts: The Master Mathematics Teacher understands and applies knowledge of numbers, number systems and their structure, operations and algorithms, quantitative reasoning, and the vertical alignment of number concepts to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).**

Application: What Master Mathematics Teachers Can Do

4–8 Number Concepts

The 4–8 Master Mathematics Teacher is able to:

1.15s   apply all skills specified for teachers in grades EC–4, using content and contexts appropriate for grades 4–8;

1.16s   work with complex numbers and demonstrate, explain, and model how some problems have no solutions in various subsets of the real number system but may have solutions in the complex number system;

1.17s   explain and justify the traditional algorithms for the four basic operations with real numbers and analyze common error patterns that may occur in their application;

1.18s   use real numbers to describe and quantify phenomena such as money, length, area, volume, and density; and

1.19s   extend and generalize the operations of rational numbers to include exponents, their operations, their properties, and their application to the real numbers.

Standard I. **Number Concepts: The Master Mathematics Teacher understands and applies knowledge of numbers, number systems and their structure, operations and algorithms, quantitative reasoning, and the vertical alignment of number concepts to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).**

Application: What Master Mathematics Teachers Can Do

8–12 Number Concepts

The 8–12 Master Mathematics Teacher is able to:

1.20s   apply all skills specified for teachers in grades EC–8, using content and contexts appropriate for grades 8–12;

1.21s   demonstrate an understanding of the real and complex number systems as algebraic fields;

1.22s   describe and analyze properties of subsets of the real numbers (e.g., rational, irrational, algebraic, transcendental) and the complex numbers (e.g., real numbers, imaginary numbers);

1.23s   select appropriate representations of complex numbers (e.g., vector, ordered pair, polar, exponential) for particular situations and justify that selection;

1.24s   describe real and complex number operations and their interrelationships using geometric and symbolic representations;

1.25s   apply properties of real and complex numbers to explain and justify algebraic algorithms; and

1.26s   investigate and apply fundamental number theory concepts and principles (e.g., divisibility, Euclidean algorithm, congruence classes, modular arithmetic, the fundamental theorem of arithmetic) in a variety of situations.

Standard II. Patterns and Algebra: The Master Mathematics Teacher understands and applies knowledge of patterns, relations, functions, algebraic reasoning, analysis, and the vertical alignment of patterns and algebra to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Teacher Knowledge: What Master Mathematics Teachers Know	Application: What Master Mathematics Teachers Can Do
EC–12 Patterns and Algebra	*EC–4 Patterns and Algebra*
The EC–12 Master Mathematics Teacher knows and understands:	The EC–4 Master Mathematics Teacher is able to:
2.1k how to use algebraic concepts and reasoning to investigate patterns, make generalizations, formulate mathematical models, make predictions, and validate results;	2.1s use inductive reasoning to identify and generalize patterns using concrete models, geometric figures, tables, graphs, and algebraic expressions;
2.2k how to use properties, graphs, and applications of relations and functions to analyze, model, and solve problems;	2.2s formulate implicit and explicit rules to describe and construct sequences using concrete models, geometric figures, tables, graphs, and algebraic expressions;
2.3k the concept of and relationships among variables, expressions, equations, inequalities, and systems in order to analyze, model, and solve problems;	2.3s illustrate concepts of relations and functions (i.e., linear and nonlinear) using concrete models, geometric figures, tables, graphs, and algebraic expressions;
2.4k the connections among geometric, graphic, numeric, and symbolic representations of functions and relations;	2.4s apply relations and functions to represent mathematical and real-world situations;
2.5k how to model, construct, and solve algebraic problems within and outside of mathematics;	2.5s translate problem-solving situations into expressions and equations;
2.6k that in many situations, a pattern is only a trend and is accompanied by random variation from the trend; and	2.6s model and solve problems, including proportion problems, using concrete, geometric, tabular, graphic, and algebraic methods;
2.7k how patterns, relations, functions, algebraic reasoning, and analysis are developmental and connected between and across all grade levels.	2.7s make, test, validate, and use conjectures about patterns and relationships in data presented in tables, sequences, or graphs;
	2.8s use linear functions and relations to model problems;
	2.9s use tables, graphs, and algebraic techniques to solve linear equations;
	2.10s give appropriate justification of the manipulation of algebraic expressions and equations in one variable; and
	2.11s communicate the vertical alignment within Patterns and Algebra across the grade levels.

Standard II. Patterns and Algebra: The Master Mathematics Teacher understands and applies knowledge of patterns, relations, functions, algebraic reasoning, analysis, and the vertical alignment of patterns and algebra to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

	Application: What Master Mathematics Teachers Can Do
	4–8 Patterns and Algebra
	The 4–8 Master Mathematics Teacher is able to:
	2.12s apply all skills specified for Master Mathematics Teachers in grades EC–4, using content and contexts appropriate for grades 4–8;
	2.13s use linear and nonlinear functions and relations, including polynomial, absolute value, trigonometric, rational, radical, exponential, logarithmic, and piecewise functions, to model and solve problems using a variety of methods, including technology;
	2.14s use tables, graphs, algebraic techniques, and appropriate technology to solve linear and nonlinear equations, inequalities, and systems;
	2.15s use transformations to illustrate properties of functions and relations and to solve problems;
	2.16s describe and predict the effects of transformations on the shape and location of graphs of functions and relations, and conversely;
	2.17s give appropriate justification of the manipulation of algebraic expressions, equations, and inequalities;
	2.18s relate the concept of limit as a conceptual foundation of calculus to middle school mathematics;
	2.19s relate the rate of change as a conceptual foundation of calculus to middle school mathematics;
	2.20s relate the area under a curve as a conceptual foundation of calculus to middle school mathematics;
	2.21s work with patterns with random variations;

Standard II. Patterns and Algebra: The Master Mathematics Teacher understands and applies knowledge of patterns, relations, functions, algebraic reasoning, analysis, and the vertical alignment of patterns and algebra to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Application: What Master Mathematics Teachers Can Do

4–8 Patterns and Algebra (continued)

2.22s   use methods of recursion and iteration to model and solve problems;

2.23s   analyze the properties of sequences and series and use them to solve problems involving finite and infinite processes including problems related to simple and compound interest rate;

2.24s   use deductive reasoning to simplify and justify algebraic processes; and

2.25s   analyze attributes of functions and relations (e.g., domain, range, one-to-one functions, inverse functions) and their graphs.

Standard II. **Patterns and Algebra: The Master Mathematics Teacher understands and applies knowledge of patterns, relations, functions, algebraic reasoning, analysis, and the vertical alignment of patterns and algebra to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).**

Application: What Master Mathematics Teachers Can Do

8–12 Patterns and Algebra

The 8–12 Master Mathematics Teacher is able to:

2.26s   apply all skills specified for Master Mathematics Teachers in grades EC–8, using content and contexts appropriate for grades 8–12;

2.27s   analyze the properties of sequences and series and use them to solve problems involving finite and infinite processes including problems related to simple, compound, and continuous interest rates, as well as annuities;

2.28s   use mathematical induction to prove theorems;

2.29s   analyze attributes of functions and relations (e.g., domain, range, one-to-one functions, composite functions, inverse functions, odd and even functions, continuous functions) and their graphs;

2.30s   describe linear, quadratic, and other polynomial functions and analyze their algebraic and graphical properties to model and solve problems using a variety of methods, including technology;

2.31s   describe exponential, logarithmic, and logistic functions algebraically and graphically and analyze their algebraic and graphical properties to model and solve problems using a variety of methods, including technology;

2.32s   describe trigonometric and circular functions algebraically and graphically and analyze their algebraic and graphical properties to model and solve problems using a variety of methods, including technology;

2.33s   describe rational, radical, absolute value, and piecewise functions algebraically and graphically and analyze their algebraic and graphical properties to model and solve problems using a variety of methods, including technology;

Standard II. Patterns and Algebra: The Master Mathematics Teacher understands and applies knowledge of patterns, relations, functions, algebraic reasoning, analysis, and the vertical alignment of patterns and algebra to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

	Application: What Master Mathematics Teachers Can Do *8–12 Patterns and Algebra (continued)* 2.34s represent and solve problems using parametric and polar equations; 2.35s investigate and solve problems using techniques of differential and integral calculus along with a variety of other methods, including technology; 2.36s represent and solve problems using techniques of linear and matrix algebra; 2.37s apply the properties of vectors and vector algebra to solve pure and applied problems; 2.38s demonstrate an understanding of algebraic structures (e.g., groups, rings, fields, vector spaces) and their relationship to secondary mathematics; 2.39s demonstrate an understanding of analysis (e.g., analytic geometry and calculus) and its relationship to secondary mathematics; 2.40s integrate concepts from algebra, relations, functions, algebraic reasoning, analysis, and technology in other disciplines (e.g., agriculture, business, science, fine arts, physical education); and 2.41s evaluate the appropriateness of materials, instructional strategies, terminology, and technology for the appropriate developmental level.

Standard III. Geometry and Measurement: The Master Mathematics Teacher understands geometry, spatial reasoning, measurement concepts and principles, and the vertical alignment of geometry and measurement to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Teacher Knowledge: What Master Mathematics Teachers Know	Application: Master Mathematics Teachers Can Do
EC–12 Geometry and Measurement	*EC–4 Geometry and Measurement*
The EC–12 Master Mathematics Teacher knows and understands:	The EC–4 Master Mathematics Teacher is able to:
3.1k how to use spatial reasoning to investigate concepts such as direction, orientation, perspective, shape, and structure;	3.1s describe shape in terms of dimension, direction, orientation, perspective, and relationships among these concepts;
3.2k the use of mathematical reasoning to develop, generalize, justify, and prove geometric relationships;	3.2s develop, explain, and use formulas to find length, perimeter, area, and volume of geometric objects;
3.3k connections among geometric ideas and number concepts, measurement, probability and statistics, algebra, and analysis;	3.3s explain and illustrate the use of numbers and units of measurement for quantities such as length, perimeter, area, volume, temperature, money, percent, speed, and acceleration;
3.4k measurement as a process;	
3.5k methods of approximation and estimation and the effects of error on measurement;	3.4s develop, justify, and use conversions within and between different measurement systems (e.g., standard and nonstandard);
3.6k how to use measurement to collect data, to recognize relationships, and to develop generalizations, including formulas;	3.5s use translations, rotations, reflections, dilations, and contractions to illustrate similarities, congruencies, and symmetries of figures;
3.7k how to model, construct, and solve geometry and measurement problems within and outside of mathematics;	3.6s identify attributes to be measured, quantify the attributes by selecting and using appropriate units, and communicate information about the attributes using the unit measure;
3.8k how to describe geometry from synthetic, coordinate, and transformational approaches;	3.7s use manipulatives and appropriate technology to investigate and illustrate geometric relationships;
3.9k logical reasoning, justification, and proof in relation to an axiomatic structure of geometry; and	3.8s develop, justify, and perform geometric constructions using compass/straightedge, reflection devices, and other appropriate technology;
3.10k how geometry, spatial reasoning, and measurement concepts and principles are developmental and connected between and across grade levels.	3.9s use logical reasoning to investigate, analyze, and prove geometric relationships within the axiomatic structure of Euclidean geometry;

Standard III. Geometry and Measurement: The Master Mathematics Teacher understands geometry, spatial reasoning, measurement concepts and principles, and the vertical alignment of geometry and measurement to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

	Application: Master Mathematics Teachers Can Do
	EC–4 Geometry and Measurement (continued)
	3.10s analyze and solve problems involving one-, two-, and three-dimensional objects (e.g., lines, angles, circles, polygons, cylinders, cones, pyramids, prisms, and spheres);
	3.11s analyze the relationship among three-dimensional objects and related two-dimensional representations (e.g., perspective, projections, cross sections, nets) and use these representations to solve problems;
	3.12s apply measurement concepts and dimensional analysis to derive units and formulas for a variety of situations, including average rates of change of one variable with respect to another;
	3.13s use symmetry to describe tessellations and show how they can be used to illustrate concepts, properties, and relationships;
	3.14s relate geometry to algebra by using the Cartesian coordinate system and use this relationship to solve problems; and
	3.15s communicate the vertical alignment of geometry and measurement across the grade levels.

Standard III. Geometry and Measurement: The Master Mathematics Teacher understands geometry, spatial reasoning, measurement concepts and principles, and the vertical alignment of geometry and measurement to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Application: Master Mathematics Teachers Can Do

4–8 Geometry and Measurement

The 4–8 Master Mathematics Teacher is able to:

3.16s   apply all skills specified for Master Mathematics Teachers in grades EC–4 using content and contexts appropriate for grades 4–8;

3.17s   investigate and prove geometric relationships within the axiomatic structure of Euclidean geometry;

3.18s   demonstrate an understanding of the methods, uses, and results of Euclidean geometry;

3.19s   apply measurement concepts and dimensional analysis to derive units and formulas for a variety of situations, including instantaneous rates of change of one variable with respect to another;

3.20s   relate geometry to algebra and trigonometry by using the Cartesian coordinate system and use this relationship to solve problems;

3.21s   use calculus concepts to answer questions about rates of change, areas, volumes, and properties of functions and their graphs;

3.22s   illustrate axiomatic systems and their components, such as undefined terms, defined terms, theorems, examples, and counter-examples;

3.23s   illustrate geometry from several perspectives, including the use of coordinate systems, transformations, and vectors;

3.24s   investigate and explore geometric concepts and properties using technology; and

3.25s   relate geometry to algebra by representing transformations as matrices and use this relationship to solve problems.

Standard III. Geometry and Measurement: The Master Mathematics Teacher understands geometry, spatial reasoning, measurement concepts and principles, and the vertical alignment of geometry and measurement to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Application: Master Mathematics Teachers Can Do

8–12 Geometry and Measurement

The 8–12 Master Mathematics Teacher is able to:

3.26s   apply all skills specified for Master Mathematics teachers in grades EC–8, using content and contexts appropriate for grades 8–12;

3.27s   illustrate and analyze axiomatic systems and their components, such as undefined terms, defined terms, theorems, examples, and counter-examples, for Euclidean and non-Euclidean geometries;

3.28s   demonstrate an understanding of finite geometries, non-Euclidean geometries, fractal geometry, and networks and graphs;

3.29s   analyze and apply calculus concepts of limit of a function, continuity, differentiability of functions, definite and indefinite integrals, and the fundamental theorem of calculus;

3.30s   illustrate concepts of calculus using slopes, rates of change, areas and volumes, and average value of a function;

3.31s   show how differential calculus is used to answer questions about rates of change and optimization;

3.32s   use integral calculus to compute various measurements associated with curves and regions in the plane, and measurements associated with curves, surfaces, and regions in three-dimensional space;

3.33s   explain and demonstrate applications of calculus in a variety of disciplines other than mathematics;

3.34s   relate trigonometry and algebra using polar coordinates and polar equations;

Standard III. Geometry and Measurement: The Master Mathematics Teacher understands geometry, spatial reasoning, measurement concepts and principles, and the vertical alignment of geometry and measurement to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

	Application: Master Mathematics Teachers Can Do
	8–12 Geometry and Measurement (continued)
	3.35s relate geometry and algebra by representing conic sections using rectangular and polar coordinates; and
	3.36s demonstrate an understanding of conic sections as a locus of points by representing conic sections in rectangular and polar coordinate systems, including the use of appropriate technology, and applying these to real world situations.

Standard IV. Probability and Statistics: The Master Mathematics Teacher understands probability and statistics, their applications, and the vertical alignment of probability and statistics to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).

Teacher Knowledge: What Master Mathematics Teachers Know	Application: What Master Mathematics Teachers Can Do
EC–12 Probability and Statistics	***EC–4 Probability and Statistics***
The EC–12 Master Mathematics Teacher knows and understands:	The EC–4 Master Mathematics Teacher is able to:
4.1k the theory of probability and its relationship to sampling and statistical inference;	4.1s use the concepts and principles of probability to describe the outcome of simple and compound events, including constructing sample spaces to model situations;
4.2k how to design experiments and surveys to answer questions and solve problems;	4.2s explore concepts of probability through data collection, experiments, and simulations;
4.3k how to use graphical and numerical techniques to explore data, characterize patterns, and describe departures from patterns;	4.3s generate, simulate, and use probability models to represent a situation;
4.4k statistical inference and how it is used in making and evaluating predictions;	4.4s explain and use probability language to make observations and draw conclusions;
4.5k how to recognize the misuses of probability and statistics;	4.5s recognize and use appropriate graphical displays and descriptive statistics for categorical and numerical data;
4.6k how to model, construct, and solve probability and statistics problems within and outside mathematics; and	4.6s investigate and answer questions by collecting, organizing, and displaying data from real world situations;
4.7k how probability and statistics are developmental and connected across and between all grade levels.	4.7s communicate the results of a statistical investigation using appropriate language;
	4.8s investigate real-world problems by designing, administering, analyzing and interpreting surveys;
	4.9s investigate real-world problems by designing, conducting, analyzing, and interpreting statistical experiments;
	4.10s use the graph of the normal distribution as a basis for making inferences about a population;

Standard IV. **Probability and Statistics: The Master Mathematics Teacher understands probability and statistics, their applications, and the vertical alignment of probability and statistics to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).**

Application: What Master Mathematics Teachers Can Do *EC–4 Probability and Statistics* (*continued*) 4.11s   develop and justify concepts and measures of central tendency (e.g., mean, median, mode) and dispersion (e.g., range, interquartile range, variance, standard deviation) and use those measures to describe a set of data; 4.12s   calculate and interpret percentiles and quartiles; 4.13s   explore, describe, and analyze bivariate data using techniques such as scatter plots and trend lines, including the use of technology; 4.14s   make inferences about a population using the binomial distribution; 4.15s   organize, display, and interpret data in a variety of formats (e.g., tables, frequency tallies, box plots, stem-and-leaf plots, histograms) and discuss the advantages, disadvantages, and appropriateness of the representation; 4.16s   describe and apply the characteristics of a well-designed and well-conducted survey or experiment; and 4.17s   communicate the vertical alignment of probability and statistics across the grade levels.

Standard IV. **Probability and Statistics: The Master Mathematics Teacher understands probability and statistics, their applications, and the vertical alignment of probability and statistics to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).**

Application: What Master Mathematics Teachers Can Do

4–8 Probability and Statistics

The 4–8 Master Mathematics Teacher is able to:

4.18s   apply all skills specified for teachers in grades EC–4, using content and contexts appropriate for grades 4–8;

4.19s   calculate probabilities using the axioms of probability and related theorems and concepts such as the addition rule, multiplication rule, conditional probability, and independence;

4.20s   apply concepts and properties of discrete and continuous random variables to model and solve a variety of problems involving probability and probability distributions;

4.21s   explain and use probability language to make observations and draw conclusions from univariate data and to describe the level of confidence in the conclusion, including the appropriate use of technology;

4.22s   identify and understand the selection of a measurement scale (i.e., nominal, ordinal, interval, ratio) used to answer research questions and analyze data;

4.23s   describe and analyze bivariate data using various techniques (e.g., scatterplots, regression lines, outliers, residual analysis and correlation coefficients), including the appropriate use of technology; and

4.24s   analyze and interpret statistical information from the media, such as the results of polls and surveys, and recognize valid and misleading uses of statistics.

Standard IV. **Probability and Statistics: The Master Mathematics Teacher understands probability and statistics, their applications, and the vertical alignment of probability and statistics to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).**

	Application: What Master Mathematics Teachers Can Do
	8–12 Probability and Statistics
	The 8–12 Master Mathematics Teacher is able to:
	4.25s apply all skills specified for teachers in grades EC–8, using content and contexts appropriate for grades 8–12;
	4.26s summarize distributions of univariate data (e.g., median, mean, range, standard deviation, quartiles);
	4.27s interpret graphical displays of univariate and bivariate data (e.g., histograms, cumulative frequency plots, scatter plots);
	4.28s apply linear transformations (e.g., translating, stretching, shrinking) to convert data and describe the effect of linear transformations on measures of central tendency and dispersion;
	4.29s transform nonlinear data into a linear form in order to apply linear regression techniques to develop exponential, logarithmic, and power regression models, including the appropriate use of technology;
	4.30s use the law of large numbers and the central limit theorem to describe the role of probability theory in the process of statistical sampling and inference;
	4.31s use confidence interval arguments to formulate and test hypotheses;
	4.32s analyze categorical data (e.g., frequency tables, marginal and joint frequencies, association, chi square); and
	4.33s describe the importance of statistical significance and the effect of sample size when drawing conclusions.

Standard V. **Mathematical Processes: The Master Mathematics Teacher understands and uses mathematical processes to reason mathematically, to solve mathematical problems, to make mathematical connections within and outside of mathematics, and to communicate mathematically.**

Teacher Knowledge: What Master Mathematics Teachers Know	Application: What Master Mathematics Teachers Can Do
EC–12 Logical Reasoning	*EC–12 Logical Reasoning*
The EC–12 Master Mathematics Teacher knows and understands:	The EC–12 Master Mathematics Teacher is able to:
5.1k logical reasoning, justification, and proof in relation to the structure of and relationships within an axiomatic system; and	5.1s apply correct mathematical reasoning to derive valid conclusions from a set of premises;
5.2k the role of logical reasoning in mathematics, and methods and uses of informal and formal reasoning.	5.2s apply principles of inductive reasoning to make conjectures and use deductive methods to evaluate the validity of conjectures;
	5.3s use formal and informal reasoning to explore, investigate, and justify mathematical ideas;
	5.4s recognize examples of fallacious reasoning;
	5.5s evaluate mathematical arguments and proofs; and
	5.6s provide convincing arguments or proofs for mathematical theorems.

Standard V. **Mathematical Processes: The Master Mathematics Teacher understands and uses mathematical processes to reason mathematically, to solve mathematical problems, to make mathematical connections within and outside of mathematics, and to communicate mathematically.**

Teacher Knowledge: What Master Mathematics Teachers Know	Application: What Master Mathematics Teachers Can Do
EC–12 Problem Solving	*EC–12 Problem Solving*
The EC–12 Master Mathematics Teacher knows and understands:	The Master Mathematics Teacher is able to:
5.3k the process of identifying, posing, exploring, and solving mathematical problems in appropriate ways.	5.7s recognize that a mathematical problem can be solved in a variety of ways, evaluate the appropriateness of various strategies, and select an appropriate strategy for a given problem;
	5.8s evaluate the reasonableness of a solution to a given problem;
	5.9s use physical and numerical models to represent a given problem or mathematical procedure;
	5.10s recognize that assumptions are made when solving problems and identify and evaluate those assumptions;
	5.11s investigate and explore problems that have multiple solutions;
	5.12s apply content knowledge to develop a mathematical model of a real-world situation and analyze and evaluate how well the model represents the situation;
	5.13s develop and use simulations as a tool to model and solve problems; and
	5.14s develop and use iteration and recursion to model and solve problems.

Standard V. **Mathematical Processes: The Master Mathematics Teacher understands and uses mathematical processes to reason mathematically, to solve mathematical problems, to make mathematical connections within and outside of mathematics, and to communicate mathematically.**

Teacher Knowledge: What Master Mathematics Teachers Know

EC–12 Connections

The EC–12 Master Mathematics Teacher knows and understands:

5.4k   connections among mathematical concepts, procedures, and equivalent representations; and

5.5k   connections between mathematics, daily living, and other disciplines.

Application: What Master Mathematics Teachers Can Do

EC–12 Connections

The Master Mathematics Teacher is able to:

5.15s   explore problems using verbal, graphical, numerical, physical, and algebraic representations;

5.16s   recognize and use multiple representations of a mathematical concept (e.g., a point and its coordinates, the area of a circle as a quadratic function in r, probability as a ratio of two areas);

5.17s   apply mathematical methods to analyze practical situations; and

5.18s   use mathematics to model and solve problems in other disciplines, such as art, music, science, social science, and business.

Standard V. **Mathematical Processes: The Master Mathematics Teacher understands and uses mathematical processes to reason mathematically, to solve mathematical problems, to make mathematical connections within and outside of mathematics, and to communicate mathematically.**

Teacher Knowledge: What Master Mathematics Teachers Know	Application: What Master Mathematics Teachers Can Do
EC–12 Communications	*EC–12 Communication*
The EC–12 Master Mathematics Teacher knows and understands:	The Master Mathematics Teacher is able to:
5.6k how to communicate mathematical ideas and concepts in oral, written, and visual forms; and	5.19s facilitate discourse between the teacher and students and among students to explore, build, and refine mathematical ideas;
5.7k how to use mathematical manipulatives and drawings and a wide range of technological tools to develop and explore mathematical concepts and ideas.	5.20s use questioning strategies to identify, support, monitor, and challenge students' mathematical thinking;
	5.21s translate mathematical statements among developmentally appropriate language, standard English, mathematical language, and symbolic mathematics;
	5.22s provide students with opportunities to demonstrate their understanding of mathematics in a variety of ways using a variety of tools;
	5.23s use visual media such as graphs, tables, diagrams, and animations to communicate mathematical information; and
	5.24s use the language of mathematics as a precise means of expressing mathematical ideas.

Standard VI. **Instruction: The Master Mathematics Teacher applies knowledge of mathematical content, uses appropriate theories for learning mathematics, implements effective instructional approaches for teaching mathematics, including teaching students who are at-risk, and demonstrates effective classroom management techniques.**

Teacher Knowledge: What Master Mathematics Teachers Know	**Application: What Master Mathematics Teachers Can Do**
EC–12 Instruction	*EC–12 Instruction*
The EC–12 Master Mathematics Teacher knows and understands:	The EC–12 Master Mathematics Teacher is able to:
6.1k converging research evidence on how students, including students who are at risk, learn and use mathematics;	6.1s teach mathematics concepts, principles, rules, and problem solving strategies using instructional approaches supported by convergent research evidence as being effective for a range of students, including students who are at-risk (e.g., explicit, systematic instruction including maximized student participation, maximized interaction between teacher and students, corrective and positive feedback for students, and ample review);
6.2k effective instructional methodology, including instruction that is based on convergent research that supports successful use of the methodology with students who are at-risk;	
6.3k effective motivational strategies for learning mathematics;	6.2s establish deep understanding of concepts through a variety of practices that are based on convergent research that supports their effective uses with a range of students, including students who are at-risk;
6.4k connections among mathematics and other content areas;	
6.5k techniques for identifying student mathematical instructional needs and for modifying, differentiating, and integrating instruction based on those needs;	6.3s select and use technology that has demonstrated effectiveness with a range of students, including students who are at-risk;
6.6k a variety of appropriate instructional uses of technology;	6.4s determine, design, and use challenging and engaging mathematical tasks, appropriate to the instructional needs of students, that develop important mathematical content as well as the critical and analytical reasoning capacities of students;
6.7k established theories and converging research on how students, including students who are at-risk, learn and use mathematics;	
6.8k how to create a logical sequence for the introduction of concepts and skills within the priority areas of mathematics;	6.5s engage students in the learning process by using a variety of instructional formats that are based on converging research that supports their effectiveness with a range of students, including students who are at-risk;
6.9k how to sequence instruction, practice, and application so that students develop accuracy and fluency of prerequisite skills prior to their use in more complex applications;	6.6s demonstrate classroom management skills, including applying strategies that use instructional time effectively;
6.10k how to structure teacher led instruction that maximizes the amount of interaction between the teacher and the student to inform the teacher of the student's ability to correctly apply the presented material; and	6.7s use and translate among multiple representations (e.g., concrete, symbolic, verbal, tabular, graphic, pictorial) of mathematical content;

Standard VI. **Instruction: The Master Mathematics Teacher applies knowledge of mathematical content, uses appropriate theories for learning mathematics, implements effective instructional approaches for teaching mathematics, including teaching students who are at-risk, and demonstrates effective classroom management techniques.**

Teacher Knowledge: What Master Mathematics Teachers Know	Application: What Master Mathematics Teachers Can Do
EC–12 Instruction (continued)	*EC–12 Instruction (continued)*
6.11k the vertical alignment of the K–12 mathematics curriculum.	6.8s integrate mathematics into other content areas and ensure that students have developed sufficient knowledge of the underlying concepts to successfully work mathematics problems related to the integration;
	6.9s choose realistic examples and nonexamples of mathematical concepts and skills (e.g., examples that provide sufficient practice and review of mathematical concepts to maximize student learning) to model for students how to appropriately apply various concepts and skills;
	6.10s apply appropriate strategies, techniques, and procedures (e.g., pattern recognition, visual representations, formulaic approaches) for helping students understand mathematics;
	6.11s structure problem solving activities so students can recognize patterns and relationships;
	6.12s elicit prior knowledge from students and apply strategies to help students move from informal to formal knowledge;
	6.13s evaluate the appropriateness of materials, instructional strategies, terminology, and technology with respect to the instructional needs of all students, including students who are at-risk;
	6.14s plan, implement, and evaluate lessons that assist students in developing concepts and making generalizations;
	6.15s use questioning strategies that require students to provide supporting evidence, arguments, and proofs;

Standard VI. **Instruction: The Master Mathematics Teacher applies knowledge of mathematical content, uses appropriate theories for learning mathematics, implements effective instructional approaches for teaching mathematics, including teaching students who are at-risk, and demonstrates effective classroom management techniques.**

	Application: What Master Mathematics Teachers Can Do
	EC–12 Instruction (continued)
	6.16s identify all prerequisite knowledge that students need in order to work problems within a particular priority area of mathematics;
	6.17s present new content while carefully controlling the language used in the presentation (e.g., carefully defining new terms using vocabulary that the student already knows);
	6.18s analyze and use assessment results from various state mathematics diagnostic instruments to inform and adjust mathematics instruction;
	6.19s use ongoing assessment to modify instruction;
	6.20s design effective instruction that provides a variety of instructional approaches to assist individual students in closing conceptual gaps;
	6.21s provide adequate practice with corrective, positive feedback and review so that students are able to develop and maintain accuracy and needed fluency; and
	6.22s communicate effectively with stakeholders, including other teachers, about using programs and instructional techniques that are based on convergent research that supports their effectiveness with a range of students, including students who are at-risk.

Standard VII. **Creating and Promoting a Positive Learning Environment: The Master Mathematics Teacher demonstrates behavior that reflects high expectations for every student, promotes positive student attitudes towards mathematics, and provides equitable opportunities for all students to achieve at a high level.**

Teacher Knowledge: What Master Mathematics Teachers Know	Application: What Teachers Can Do
EC–12 Creating and Promoting a Positive Learning Environment	*EC–12 Creating and Promoting a Positive Learning Environment*
The EC–12 Master Mathematics Teacher knows and understands:	The EC–12 Master Mathematics Teacher is able to:
7.1k how teacher attitudes and expectations impact the learning of mathematics;	7.1s reflect upon one's own teaching behaviors and attitudes to ensure high expectations and equity in mathematics instruction for all students;
7.2k how student attitudes and expectations impact the learning of mathematics;	7.2s use a variety of instructional methods, tools, and tasks to promote students' confidence, interest, and inventiveness while learning mathematics;
7.3k how environment impacts the learning of mathematics;	7.3s listen to and respect students' ideas;
7.4k how learning mathematics enables students to compete in an increasingly complex society;	7.4s assist all students in learning mathematics through the use of manipulatives, drawings, and technological tools, based on evidence from effective practices (research); and
7.5k how students differ in their approaches to learning mathematics with respect to linguistic, cultural, socioeconomic, and developmental diversity; and	7.5s apply the results from converging research to promote and create a positive learning environment with respect to linguistic, cultural, socioeconomic, and developmental diversity.
7.6k that every child can learn mathematics at a high level consistent with the statewide mathematics curriculum (TEKS).	

Standard VIII. **Assessment: The Master Mathematics Teacher selects, constructs, and administers appropriate assessments to guide, monitor, evaluate, and report student progress to students, administrators, and parents, and develops these skills in other teachers.**

Teacher Knowledge: What Teachers Know	Application: What Master Mathematics Teachers Can Do
EC–12 Assessment	*EC–12 Assessment*
The EC–12 Master Mathematics Teacher knows and understands:	The EC–12 Master Mathematics Teacher is able to:
8.1k the purpose, characteristics, and uses of various assessment methods in mathematics, including formative and summative assessments;	8.1s design and administer a variety of appropriate assessment instruments and/or methods (e.g., formal/informal, formative/summative) to monitor student understanding of mathematics and progress over time;
8.2k the importance of carefully selecting or designing formative and summative assessments for the specific decisions they are intended to inform;	8.2s lead teacher groups in the development of assessment instruments and/or methods;
8.3k technical issues associated with testing (e.g., reliability, validity, absence of bias, clarity of language, appropriateness of level); and	8.3s develop a variety of formal and informal assessments and scoring procedures that consist of worthwhile tasks that assess mathematical understanding, common misconceptions, and error patterns;
8.4k how to communicate effectively with adults, including other teachers, about the development and use of appropriate assessment techniques.	8.4s recognize common error patterns and mathematical misconceptions, apply appropriate correction procedures, and evaluate and modify instructional approaches;
	8.5s generate assessment methods that align with state and national standards;
	8.6s interpret the results of formal and informal assessments, including the use of rubrics and holistic scoring methods;
	8.7s establish criteria consistent with current ethical and legal principles regarding the sharing of assessment results with students, parents, and appropriate school personnel;
	8.8s communicate assessment results to students' parents/guardians, mathematics teachers, and other appropriate personnel;
	8.9s develop a valid student grading system based on the results of students' assessments;

Standard VIII. **Assessment: The Master Mathematics Teacher selects, constructs, and administers appropriate assessments to guide, monitor, evaluate, and report student progress to students, administrators, and parents, and develops these skills in other teachers.**

	Application: What Master Mathematics Teachers Can Do
	EC–12 Assessment (continued)
	8.10s develop assessments that evaluate a student's knowledge and higher level thinking skills;
	8.11s evaluate an assessment for validity with respect to the objectives that the assessment measures; and
	8.12s identify and implement appropriate state mathematics diagnostic instruments.

Standard IX. **Mentoring and Leadership: The Master Mathematics Teacher facilitates appropriate standards-based mathematics instruction by communicating and collaborating with educational stake-holders; mentoring, coaching, exhibiting leadership, and consulting with colleagues; providing professional development opportunities for faculty; and making instructional decisions based on data and supported by evidence from research.**

Teacher Knowledge: What Master Mathematics Teachers Know

EC–12 Communication and Collaboration with Educational Stakeholders

The EC–12 Master Mathematics teacher knows and understands:

9.1k   the dual role of the Master Mathematics Teacher as a teacher and mentor in the school community;

9.2k   leadership, communication, and facilitation skills and strategies;

9.3k   principles, guidelines, and professional ethical standards regarding collegial and professional collaborations, including confidentiality in the mentoring relationship;

9.4k   learning processes and procedures that facilitate peer learning and self-learning;

9.5k   how to facilitate positive change in instructional practices through participation in ongoing professional development opportunities (e.g., TEXTEAMS, NCTM, CAMT, book studies, action research); and

9.6k   how local, state, and national curriculum and assessment standards are related.

Application: What Master Mathematics Teachers Can Do

EC–12 Communication and Collaboration with Educational Stakeholders

The EC–12 Master Mathematics Teacher is able to:

9.1s   assist other teachers to reflect on their own teaching behaviors and attitudes to ensure high expectations and equity in mathematics instruction for all students;

9.2s   collaborate with administrators, colleagues, families/guardians, and other members of the school community to establish and implement the roles of the Master Mathematics Teacher and ensure effective ongoing communication;

9.3s   build trust and a spirit of collaboration with other members of the school community to effect positive change in the school mathematics program and mathematics instruction;

9.4s   use leadership skills to ensure the effectiveness and ongoing improvement of the school mathematics program, encourage support for the program, and engage others in improving the program;

9.5s   collaborate with members of the school community to evaluate, negotiate, and establish priorities regarding the mathematics program, and to facilitate mentoring, professional development, and family/guardian training;

9.6s   confer with students, colleagues, administrators, families/guardians, and the community to discuss mathematics related issues; and

9.7s   apply professional principles, guidelines, and ethical standards in collegial and professional collaborations.

Standard IX. **Mentoring and Leadership: The Master Mathematics Teacher facilitates appropriate standards-based mathematics instruction by communicating and collaborating with educational stake-holders; mentoring, coaching, exhibiting leadership, and consulting with colleagues; providing professional development opportunities for faculty; and making instructional decisions based on data and supported by evidence from research.**

Teacher Knowledge: What Master Mathematics Teachers Know	Application: What Master Mathematics Teachers Can Do
EC–12 Mentoring, Coaching, and Consultation	*EC–12 Mentoring, Coaching, and Consultation*
The EC–12 Master Mathematics Teacher knows and understands:	The EC–12 Master Mathematics Teacher is able to:
9.7k skills and strategies for mentoring, coaching, and consultation in the development, implementation, and evaluation of an effective standards-based mathematics program;	9.8s apply effective mentoring, coaching, and consultation skills and strategies (e.g., observing, consensus building, providing feedback, decision making) to improve mathematics instruction for all students;
9.8k differences between consultation and supervision; and	9.9s use mentoring, coaching, and consultation to facilitate team building for identifying needs related to mathematics instruction, developing strategies for addressing those needs, and promoting mathematical development;
9.9k strategies for facilitating positive change in instructional practices through mentoring, coaching, and consultation.	9.10s use consultation to work effectively with colleagues with varying levels of skill and experience and/or different philosophical approaches to instruction to develop, implement, and monitor mathematics programs;
	9.11s select and use strategies to maximize effectiveness as a Master Mathematics Teacher, such as applying principles of time management and engaging in continuous self-assessment; and
	9.12s use consultation to improve the teacher's ability to engage all students in the learning process.

Standard IX. **Mentoring and Leadership: The Master Mathematics Teacher facilitates appropriate standards-based mathematics instruction by communicating and collaborating with educational stake-holders; mentoring, coaching, exhibiting leadership, and consulting with colleagues; providing professional development opportunities for faculty; and making instructional decisions based on data and supported by evidence from research.**

Teacher Knowledge: What Master Mathematics Teachers Know

EC–12 Professional Development for Faculty

The EC–12 Master Mathematics Teacher knows and understands:

9.10k   learning processes and procedures for facilitating adult learning;

9.11k   strategies for facilitating positive change in instructional practices through professional development; and

9.12k   models and features of effective professional development programs that promote sustained application in classroom practice (e.g., demonstration, modeling, guided practice, feedback, coaching, follow-up).

Teacher Knowledge: What Master Mathematics Teachers Can Do

EC–12 Professional Development for Faculty

The EC–12 Master Mathematics Teacher is able to:

9.13s   collaborate with teachers, administrators, and others to identify professional development needs, generate support for professional development programs, and ensure provision of effective professional development opportunities;

9.14s   design ongoing professional development opportunities that address identified student mathematics needs, are appropriate for the intended audience, and are based on data and convergent research evidence;

9.15s   use a variety of models and methods to create professional development opportunities that improve teachers' ability to implement effective mathematics instruction for all students; and

9.16s   apply principles and procedures for delivering effective professional development and follow-up to promote and sustain positive change in the mathematics program.

Standard IX. **Mentoring and Leadership: The Master Mathematics Teacher facilitates appropriate standards-based mathematics instruction by communicating and collaborating with educational stake-holders; mentoring, coaching, exhibiting leadership, and consulting with colleagues; providing professional development opportunities for faculty; and making instructional decisions based on data and supported by evidence from research.**

Teacher Knowledge: What Master Mathematics Teachers Know	**Teacher Knowledge: What Master Mathematics Teachers Know**
EC–12 Decision Making Based on Evidence from Research	*EC–12 Decision Making Based on Evidence from Research*
The EC–12 Master Mathematics Teacher knows and understands:	The EC–12 Master Mathematics Teacher is able to:
9.13k sources for locating information about converging research on mathematics learning; and	9.17s critically examine converging research on mathematics learning and analyze the usefulness of research results for addressing instructional needs; and
9.14k methods and criteria for reviewing research on mathematics learning and selecting research for educational applications.	9.18s apply appropriate procedures for translating research on mathematics learning into practice.

Standard X. **Mathematical Perspectives: The Master Mathematics Teacher understands the historical development of mathematical ideas, the interrelationship between society and mathematics, the structure of mathematics, and the evolving nature of mathematics and mathematical knowledge.**

Teacher Knowledge: What Master Mathematics Teachers Know	Application: What Master Mathematics Teachers Can Do
EC–12 Mathematical Perspectives	*EC–12 Mathematical Perspectives*
The EC–12 Master Mathematics Teacher knows and understands:	The EC–12 Master Mathematics Teacher is able to:
10.1k the history and evolution of mathematical concepts, procedures, and ideas (e.g., the development of non-Euclidean geometry);	10.1s utilize the historical development of mathematical ideas in developing mathematical concepts from concrete to abstract (e.g., Euclid's geometric representation of number operations to algebra tiles to area models);
10.2k the contributions that different cultures (e.g., Greek, Islamic) have made to the field of mathematics;	10.2s plan activities that emphasize how technology (e.g., spreadsheets, statistical software) affects the use of mathematics in various careers;
10.3k the role society plays in shaping personal views and perspectives of mathematics;	10.3s plan activities that emphasize the role of mathematics in various careers;
10.4k the impacts of technological advances on mathematical knowledge and skills (e.g., numeric versus analytic solutions) and of mathematics on technology;	10.4s use the structure of mathematical systems and their properties to make connections among mathematical concepts;
10.5k how mathematics is used in a variety of careers and professions (e.g., economics, engineering);	10.5s use current resources to plan and develop cultural, historical, and technological instruction for the classroom; and
10.6k the structural properties (e.g., mappings, inverse operations) common to the mathematics disciplines; and	10.6s use professional resources to design activities that connect society and mathematics.
10.7k the implication of current trends and research in mathematics and mathematics education.	