Exit Level

TAKS STUDY GUIDE

Revised 2007

Texas Assessment of Knowledge and Skills

Mathematics

A Student and Family Guide

Revised Based on TEKS Refinements

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Texas Assessment of Knowledge and Skills

Exit Level

Mathematics

A Student and Family Guide

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Dear Student and Parent:

The Texas Assessment of Knowledge and Skills (TAKS) is a comprehensive testing program for public school students in grades 3–11. TAKS, including TAKS (Accommodated) and Linguistically Accommodated Testing (LAT), has replaced the Texas Assessment of Academic Skills (TAAS) and is designed to measure to what extent a student has learned, understood, and is able to apply the important concepts and skills expected at each tested grade level. In addition, the test can provide valuable feedback to students, parents, and schools about student progress from grade to grade.

Students are tested in mathematics in grades 3–11; reading in grades 3–9; writing in grades 4 and 7; English language arts in grades 10 and 11; science in grades 5, 8, 10, and 11; and social studies in grades 8, 10, and 11. Every TAKS test is directly linked to the Texas Essential Knowledge and Skills (TEKS) curriculum. The TEKS is the state-mandated curriculum for Texas public school students. Essential knowledge and skills taught at each grade build upon the material learned in previous grades. By developing the academic skills specified in the TEKS, students can build a strong foundation for future success.

The Texas Education Agency has developed this study guide to help students strengthen the TEKS-based skills that are taught in class and tested on TAKS. The guide is designed for students to use on their own or for students and families to work through together. Concepts are presented in a variety of ways that will help students review the information and skills they need to be successful on TAKS. Every guide includes explanations, practice questions, detailed answer keys, and student activities. At the end of this study guide is an evaluation form for you to complete and mail back when you have finished the guide. Your comments will help us improve future versions of this guide.

There are a number of resources available for students and families who would like more information about the TAKS testing program. Information booklets are available for every TAKS subject and grade. Brochures are also available that explain the Student Success Initiative promotion requirements and the new graduation requirements for eleventh-grade students. To obtain copies of these resources or to learn more about the testing program, please contact your school or visit the Texas Education Agency website at www.tea.state.tx.us/student.assessment.

Texas is proud of the progress our students have made as they strive to reach their academic goals. We hope the study guides will help foster student learning, growth, and success in all of the TAKS subject areas.

Sincerely,

Gloria Zyskowski

Gloria Zyskowski
Deputy Associate Commissioner for Student Assessment
Texas Education Agency

Contents

Mathematics

MATHEMATICS

What Is This Book?

This is a study guide to help you strengthen the skills tested on the exit level Texas Assessment of Knowledge and Skills (TAKS). TAKS is a state-developed test administered with no time limit. It is designed to provide an accurate measure of learning in Texas schools.

By acquiring all the skills taught in eleventh grade, you will be better prepared to succeed on the exit level TAKS test.

What Are Objectives?

Objectives are goals for the knowledge and skills that you should achieve. The specific goals for instruction in Texas schools were provided by the Texas Essential Knowledge and Skills (TEKS). The objectives for TAKS were developed based on the TEKS.

How Is This Book Organized?

This study guide is divided into the ten objectives tested on TAKS. A statement at the beginning of each objective lists the mathematics skills you need to acquire. The study guide covers a large amount of material. You should not expect to complete it all at once. It may be best to work through one objective at a time.

Each objective is organized into review sections and a practice section. The review sections present examples and explanations of the mathematics skills for each objective. The practice sections feature mathematics problems that are similar to the ones used on the TAKS test.

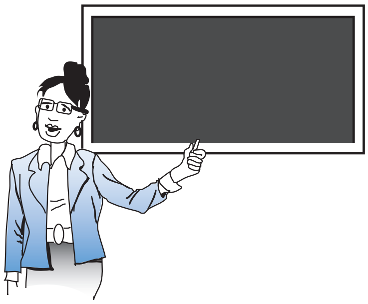
How Can I Use This Book?

First look at your Confidential Student Report. This is the report the school gave you that shows your TAKS scores. This report will tell you which TAKS subject-area test(s) you passed and which one(s) you did not pass. Use your report to determine which skills need improvement. Once you know which skills need to be improved, you can read through the instructions and examples that support those skills. You may also choose to work through all the sections. Pace yourself as you work through the study guide. Work in short sessions. If you become frustrated, stop and start again later.

What Are the Helpful Features of This Study Guide?

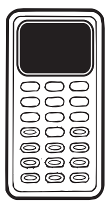
- Look for the following features in the margin:

  Ms. Mathematics provides important instructional information for a topic.

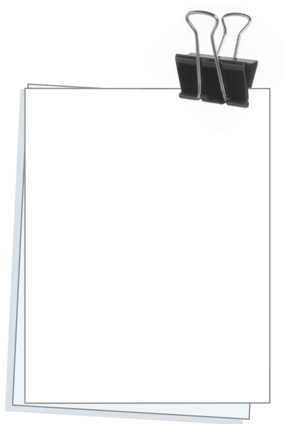


Do you see that . . . points to a significant sentence in the instruction.

Calculator suggests that using a graphing calculator might be helpful.



Memo provides page references in this study guide for additional information.



- There are several words in this study guide that are important for you to understand. These words are boldfaced in the text and are defined when they are introduced. Locate the boldfaced words and review the definitions.
- Examples are contained inside shaded boxes.
- Each objective has “Try It” problems based on the examples in the review sections.
- A Mathematics Chart for the exit level TAKS test is included on pages 8 and 9 and also as a tear-out page in the back of the book. This chart includes useful mathematics information. The tear-out Mathematics Chart in the back of the book also provides both a metric and a customary ruler to help solve problems requiring measurement of length.

How Should the “Try It” Problems Be Used?

“Try It” problems are found throughout the review sections of the mathematics study guide. These problems provide an opportunity for you to practice skills that have just been covered in the instruction. Each “Try It” problem features lines for your responses. The answers to the “Try It” problems are found immediately following each problem.

While completing a “Try It” problem, cover up the answer portion with a sheet of paper. Then check the answers.

What Kinds of Practice Questions Are in the Study Guide?

The mathematics study guide contains questions similar to those found on the exit level TAKS test. There are two types of questions in the mathematics study guide.

- Multiple-Choice Questions: Most of the practice questions are multiple choice with four answer choices. These questions present a mathematics problem using numbers, symbols, words, a table, a diagram, or a combination of these. Read each problem carefully. If there is a table or diagram, study it. You should read each answer choice carefully before choosing the best answer.
- Griddable Questions: Some practice questions use an eight-column answer grid like those used on the exit level TAKS test.

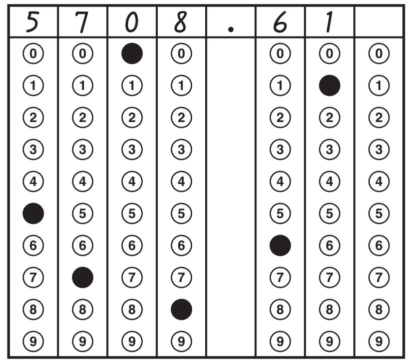
How Do You Use an Answer Grid?

The answer grid contains eight columns, which include three decimal places: tenths, hundredths, and thousandths.

Suppose 5708.61 is the answer to a problem. First write the number in the blank spaces. Be sure to use the correct place value. For example, 5 is in the thousands place, 7 is in the hundreds place, 0 is in the tens place, 8 is in the ones place, 6 is in the tenths place, and 1 is in the hundredths place.

Then fill in the correct bubble under each digit. Notice that if there is a zero in the answer, you need to fill in the bubble for the zero.

The grid shows 5708.61 correctly entered. The zero in the tens place is bubbled in because it is part of the answer. It is not necessary to bubble in the zero in the thousandths place because this zero will not affect the value of the correct answer.



Where Can Correct Answers to the Practice Questions Be Found?

The answers to the practice questions are in the answer key at the back of this book (pages 280–300). Each question includes a reference to the page number in the answer key for the answer to the problem. The answer key explains the correct answer, and it also includes some explanations for incorrect answers. After you answer the practice questions, you can check your answers.

If you still do not understand the correct answer after reading the answer explanations, ask a friend, family member, or teacher for help. Even if you have chosen the correct answer, it is a good idea to read the answer explanation because it may help you better understand why the answer is correct.

Grades 9, 10, and Exit Level Mathematics Chart

TAKS Texas Assessment of Knowledge and Skills

LENGTH

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

CAPACITY AND VOLUME

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 fluid ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 fluid ounces

MASS AND WEIGHT

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

TIME

1 year = 365 days
1 year = 12 months
1 year = 52 weeks
1 week = 7 days
1 day = 24 hours
1 hour = 60 minutes
1 minute = 60 seconds

Metric and customary rulers can be found on the tear-out Mathematics Chart in the back of this book.

Grades 9, 10, and Exit Level Mathematics Chart

Perimeter	rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	circle	$C = 2\pi r$ or $C = \pi d$
Area	rectangle	$A = lw$ or $A = bh$
	triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	regular polygon	$A = \frac{1}{2}aP$
	circle	$A = \pi r^2$
P represents the Perimeter of the Base of a three-dimensional figure.		
B represents the Area of the Base of a three-dimensional figure.		
Surface Area	cube (total)	$S = 6s^2$
	prism (lateral)	$S = Ph$
	prism (total)	$S = Ph + 2B$
	pyramid (lateral)	$S = \frac{1}{2}Pl$
	pyramid (total)	$S = \frac{1}{2}Pl + B$
	cylinder (lateral)	$S = 2\pi rh$
	cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	cone (lateral)	$S = \pi rl$
	cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	sphere	$S = 4\pi r^2$
Volume	prism or cylinder	$V = Bh$
	pyramid or cone	$V = \frac{1}{3}Bh$
	sphere	$V = \frac{4}{3}\pi r^3$
Special Right Triangles	$30^\circ, 60^\circ, 90^\circ$	$x,\ x\sqrt{3},\ 2x$
	$45^\circ, 45^\circ, 90^\circ$	$x,\ x,\ x\sqrt{2}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$

Objective 1

The student will describe functional relationships in a variety of ways.

For this objective you should be able to recognize that a function represents a dependence of one quantity on another and can be described in a number of ways.

What Is a Function?

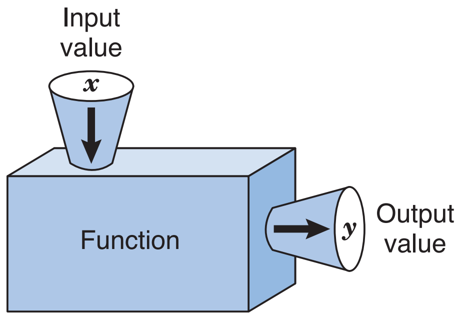
A **function** is a set of ordered pairs (x, y) in which each x-coordinate is paired with only one y-coordinate. In a list of ordered pairs belonging to a function, no x-coordinate is repeated.

The distance you can drive an automobile depends on the number of gallons of gas in the car's fuel tank. This is a good example of a function.

Fuel in Tank (gallons)	Distance (miles)
1	20.5
2	41.0
3	61.5
4	82.0
5	102.5
6	123.0

For a given number of gallons of fuel, there is exactly one distance listed.

In a functional relationship, for any given input there is a unique output.



Input an x-value and you get a y-value.

If you are given an x-value belonging to a function, you can find the corresponding y-value.

Do you see that . . .

If you input 5 gallons into the function above, the output will be 102.5 miles.

Objective 1

There are two ways to test a set of ordered pairs to see whether it is a function.

Examine the list of ordered pairs.

If a set of ordered pairs is a function, no x-coordinate in the set is repeated. No x-coordinate should be listed with two different y-coordinates.

Is this set of ordered pairs a function?

$$\{(-1, 1), (1, 5), (3, 9)\}$$

Examine the set of ordered pairs.

- The number –1 is paired with 1. The number 1 is paired with 5. The number 3 is paired with 9.
- Each x-coordinate has a unique y-coordinate. No two ordered pairs have the same x-coordinate but different y-coordinates.

This set of ordered pairs is a function.

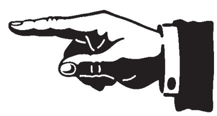
Is this set of ordered pairs a function?

$$\{(5, -2), (3, 7), (-1, -8), (8, -2)\}$$

Examine the set of ordered pairs.

- No x-coordinate is repeated.
- There are two ordered pairs that have the same y-coordinate but different x-coordinates. The number 5 is paired with –2, and the number 8 is paired with –2. This does not prevent this set of ordered pairs from being a functional relationship.

This set of ordered pairs is a function.



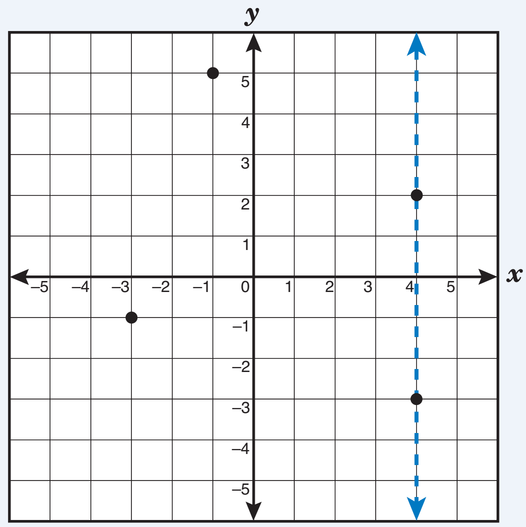
Do you see that . . .

Examine a graph of the function.

Use a vertical line to determine whether two points have the same x-coordinate. If two points in the function lie on the same vertical line, then they have the same x-coordinate, and the set of ordered pairs is not a function.

Objective 1

Do the ordered pairs graphed below represent a function?



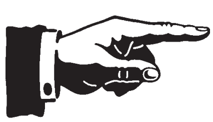
The ordered pairs $(4, -3)$ and $(4, 2)$ lie on a common vertical line.

They have the same x-coordinate, 4, but different y-coordinates, -3 and 2.

This graph of ordered pairs does not represent a function because two points lie on the same vertical line.

In a function, the y-coordinate is described in terms of the x-coordinate. The value of the y-coordinate depends on the value of the x-coordinate.

Do you see that . . .



Niyum rents a sailboat by the hour. He pays a \$27 fee and \$15 for each hour he uses the sailboat. Let c represent the total cost of renting the sailboat for h hours. Write an equation that represents the dependent variable in terms of the independent variable.

- The number of hours Niyum uses the sailboat determines how much he is charged, so the number of hours is the independent quantity.
- The total cost depends on the number of hours Niyum uses the sailboat, so the total cost of renting the sailboat is the dependent quantity.
- The total cost of renting the sailboat is \$27 plus \$15 times the number of hours.

$$c = 27 + 15h$$

The fee and the hourly rate remain the same in this situation; \$27 and \$15 are **constants** in this equation.

In this equation, the value of c depends on the value of h. Therefore, c is the **dependent** variable, and h is the **independent** variable.

When Shannon goes skydiving, she wears a device that measures her altitude in one-second intervals from the time she jumps until she opens her parachute. If she jumps from an altitude of 4000 meters, the equation $h = 4000 - 4.9t^2$ describes h, Shannon's altitude in meters, in terms of t, the number of seconds until she opens her parachute.

Identify the independent and dependent quantities and the constants in this equation.

- The number of seconds until Shannon opens her parachute is the independent quantity.
- Shannon's altitude depends on the number of seconds she has been falling; it is the dependent quantity.
- Shannon's starting altitude, 4000 meters, and 4.9 are constants in the function because they do not change.

Try It

For a science experiment Mariana measures the rate at which a liquid cools. She finds that for the first few minutes the liquid cools, its temperature can be given using the equation $t = 217 - 18m$, in which m represents the number of minutes the liquid has been cooling and t is its temperature in °C.

Which values are the dependent and independent quantities in this functional relationship? Which values are the constants in this functional relationship?

The ________________ quantity is the number of minutes the liquid has been cooling.

The ________________ quantity is the temperature of the liquid because the temperature depends on the number of minutes the liquid has been cooling.

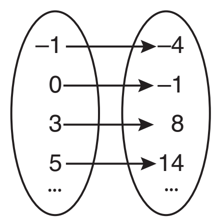
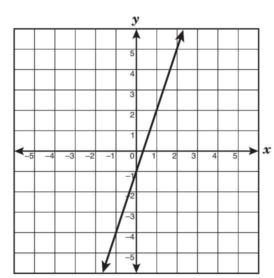
The values _______ and _______ are the constants because they do not change.

The independent quantity is the number of minutes the liquid has been cooling. The dependent quantity is the temperature of the liquid because the temperature depends on the number of minutes the liquid has been cooling. The values 217 and 18 are the constants because they do not change.

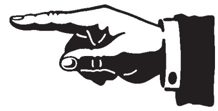
Objective 1

How Can You Represent a Function?

Functional relationships can be represented in a variety of ways.

<table>
<tr><th>Method</th><th>Description</th><th>Example</th></tr>
<tr><td>List</td><td>List the ordered pairs.</td><td>{(−1, −4), (0, −1), (3, 8), (5, 14), …}</td></tr>
<tr><td>Table</td><td>Place the ordered pairs in a table.</td><td><table><tr><th>x</th><th>y</th></tr><tr><td>−1</td><td>−4</td></tr><tr><td>0</td><td>−1</td></tr><tr><td>3</td><td>8</td></tr><tr><td>5</td><td>14</td></tr><tr><td></td><td>…</td></tr></table></td></tr>
<tr><td>Mapping</td><td>Draw a picture that shows how the ordered pairs are formed.</td><td>
</td></tr>
<tr><td>Description</td><td>Use words to describe the functional relationship.</td><td>The y-values for a set of points are 1 less than 3 times the corresponding x-values.</td></tr>
<tr><td>Equation</td><td>Write an equation that describes the y-coordinate in terms of the x-coordinate.</td><td>$y = 3x - 1$</td></tr>
<tr><td>Function notation</td><td>Write a special type of equation that uses $f(x)$ to represent y.</td><td>$f(x) = 3x - 1$</td></tr>
<tr><td>Graph</td><td>Graph the ordered pairs.</td><td>
</td></tr>
</table>

To use **function notation** to describe a function, give the function a name, typically a letter such as f, g, or h. Then use an algebraic expression to describe the y-coordinate of an ordered pair.



Do you see that . . .

Suppose $f(x) = \frac{1}{4}x + 5$.

- This function is read as "f of x equals $\frac{1}{4}$ times x plus 5."
- If you input x, the output will be $\frac{1}{4}x + 5$.
- The y-coordinate of any ordered pair for this function is $\frac{1}{4}x + 5$.

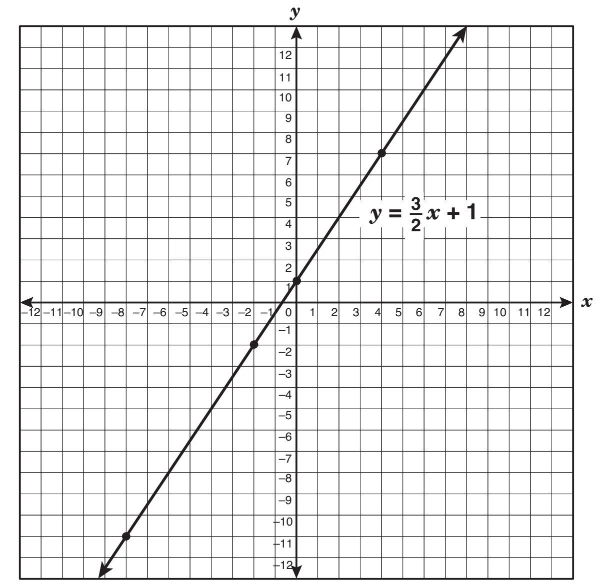
The function described by $f(x) = \frac{1}{4}x + 5$ is the same as the function described by $y = \frac{1}{4}x + 5$. Therefore, any ordered pair for this function can be represented by $(x, \frac{1}{4}x + 5)$.

Here are three methods you can use to determine whether two different representations of a function are equivalent.

Method	Action
Match a list or table of ordered pairs to a graph.	• Show that each ordered pair listed matches a point on the graph.
Match an equation to a graph.	• Determine whether they are both linear or quadratic functions. • Find points on the graph and show that their coordinates satisfy the equation. • Find points that satisfy the equation and show that they are on the graph.
Match a verbal description to a graph, an equation, or an expression written in function notation.	• Use the verbal description to find ordered pairs belonging to the function and then show that they satisfy the graph, equation, or function rule. • Find points on the graph or ordered pairs satisfying the equation or rule and show that they satisfy the verbal description.

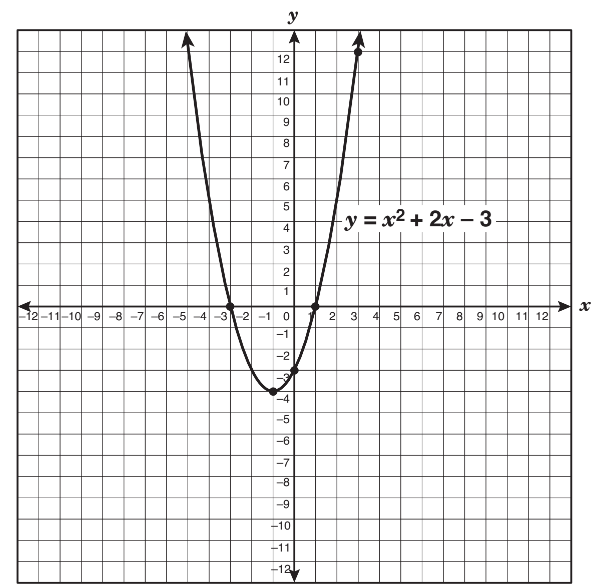
Any equation in the form $y = mx + b$ is a **linear function**. Its graph will be a line.

x	y
−8	−11
−2	−2
0	1
4	7



Any equation in the form $y = ax^2 + bx + c$, where $a \neq 0$, is a **quadratic function**. Its graph will be a parabola.

x	y
−3	0
−1	−4
0	−3
1	0
3	12



Does the ordered pair (2, 20) belong to the function $h(x) = x(x + 5) + 6$?

For the ordered pair (2, 20), $x = 2$. Determine whether $h(x) = 20$ when $x = 2$.

$$h(x) = x(x + 5) + 6$$
$$h(2) = 2(2 + 5) + 6$$
$$h(2) = 2(7) + 6$$
$$h(2) = 14 + 6$$
$$h(2) = 20$$

When x is replaced with 2, $h(x) = 20$. The ordered pair (2, 20) belongs to this function.

A variety of methods of representing a function are shown below. Which example represents a function that is different from the others?

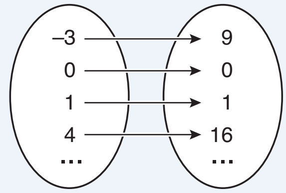
A. Verbal Description
The value of y is 1 more than the square of the value of x.

B. List of Selected Values
$\{(-2, 5), (0, 1), (1, 2), (3, 10), (5, 26), \ldots\}$

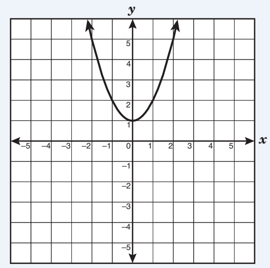
C. Table of Selected Values

x	y
2	5
0	1
−1	2
−4	17
…	…

D. Mapping of Selected Values



E. Graph



F. Equation
$y = x^2 + 1$

Look at the ordered pairs that make up each function.

- In Examples A, B, C, E, and F, each number is paired with its square increased by 1.
- For each ordered pair listed in Example D, the y-coordinate is the square of the x-coordinate, not its square increased by 1.

Only Example D represents a different function.

The total cost of attending a county fair includes the cost of admission and a charge for each ride ticket. The table below shows the number of ride tickets and the corresponding total cost of attending the fair.

Number of Ride Tickets	Total Cost
0	\$2.00
3	\$6.50
5	\$9.50
10	\$17.00

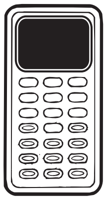
If x represents the number of ride tickets and y represents the total cost of attending the fair, does the function $y = 1.5x + 2$ describe the relationship between these quantities?

Verify that each of the data points (ordered pairs) satisfies the equation $y = 1.5x + 2$.

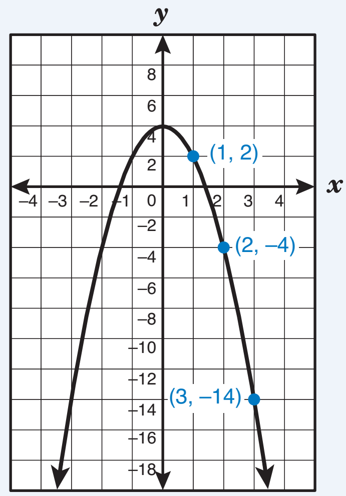
x	$y = 1.5x + 2$	y	Yes/No
0	$y = 1.5(0) + 2$ $y = 0 + 2$ $y = 2$	2.00	Yes
3	$y = 1.5(3) + 2$ $y = 4.5 + 2$ $y = 6.5$	6.50	Yes
5	$y = 1.5(5) + 2$ $y = 7.5 + 2$ $y = 9.5$	9.50	Yes
10	$y = 1.5(10) + 2$ $y = 15 + 2$ $y = 17$	17.00	Yes

The equation $y = 1.5x + 2$ describes the relationship between the number of ride tickets and the total cost.

Objective 1



Does the graph below represent the equation $y = -2x^2 + 4$?



- Verify that the equation and the graph both describe the same type of function. The equation $y = -2x^2 + 4$ is a quadratic function because there is an x^2 term in the equation. The graph is a parabola. It describes a quadratic function. Both the equation and the graph describe a quadratic function.
- Check several points on the graph to see whether their coordinates satisfy the equation $y = -2x^2 + 4$. Choose points with coordinates that are easily read from the graph. In this case, choose the points $(3, -14)$, $(1, 2)$, and $(2, -4)$.

  Substitute these values into the equation $y = -2x^2 + 4$ and determine whether the equation is true.

Point $(3, -14)$	Point $(1, 2)$	Point $(2, -4)$
$x = 3$ and $y = -14$	$x = 1$ and $y = 2$	$x = 2$ and $y = -4$
$y = -2x^2 + 4$	$y = -2x^2 + 4$	$y = -2x^2 + 4$
$-14 \stackrel{?}{=} -2(3)^2 + 4$	$2 \stackrel{?}{=} -2(1)^2 + 4$	$-4 \stackrel{?}{=} -2(2)^2 + 4$
$-14 \stackrel{?}{=} -2(9) + 4$	$2 \stackrel{?}{=} -2(1) + 4$	$-4 \stackrel{?}{=} -2(4) + 4$
$-14 \stackrel{?}{=} -18 + 4$	$2 \stackrel{?}{=} -2 + 4$	$-4 \stackrel{?}{=} -8 + 4$
$-14 = -14$	$2 = 2$	$-4 = -4$

  The coordinates of the points $(3, -14)$, $(1, 2)$, and $(2, -4)$ satisfy the equation.

- You can also find several ordered pairs of numbers that satisfy the equation and confirm that they are points on the graph. Pick values that are easy to substitute, such as $x = -1$, $x = 0$, and $x = -2$. Find the corresponding values for y.

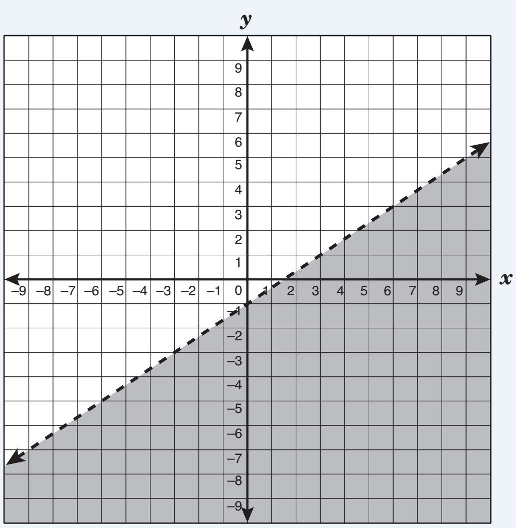
  Substitute the x-values into $y = -2x^2 + 4$ and determine the value for y.

$\underline{x = -1}$	$\underline{x = 0}$	$\underline{x = -2}$
$y = -2x^2 + 4$	$y = -2x^2 + 4$	$y = -2x^2 + 4$
$y = -2(-1)^2 + 4$	$y = -2(0)^2 + 4$	$y = -2(-2)^2 + 4$
$y = -2(1) + 4$	$y = -2(0) + 4$	$y = -2(4) + 4$
$y = -2 + 4$	$y = 0 + 4$	$y = -8 + 4$
$y = 2$	$y = 4$	$y = -4$

  The ordered pairs $(-1, 2)$, $(0, 4)$, and $(-2, -4)$ satisfy the equation. Determine whether these ordered pairs are points on the graph. Yes, all three points are on the graph.

The graph does represent the function $y = -2x^2 + 4$.

Does the graph below represent the inequality $y < \frac{2}{3}x - 1$?



- Verify that the inequality and the boundary of the graph both describe the same type of relationship. The inequality $y < \frac{2}{3}x - 1$ is in the form $y < mx + b$, which means that it is a linear inequality. The graph of the inequality is a region bounded by a line—it describes a linear inequality. Both the inequality and the graph describe a linear relationship.

Objective 1

- Check at least two points on the boundary of the graph to see whether their coordinates satisfy the equation $y = \frac{2}{3}x - 1$. Choose points with coordinates that are easily read from the graph. In this case, choose the points (3, 1) and (6, 3).

  Substitute these values into the equation $y = \frac{2}{3}x - 1$ and determine whether the equation is true.

Point (3, 1)	Point (6, 3)
$y = \frac{2}{3}x - 1$	$y = \frac{2}{3}x - 1$
$1 \stackrel{?}{=} \frac{2}{3}(3) - 1$	$3 \stackrel{?}{=} \frac{2}{3}(6) - 1$
$1 \stackrel{?}{=} 2 - 1$	$3 \stackrel{?}{=} 4 - 1$
$1 = 1$	$3 = 3$

Do you see that . . .

  The line is the correct boundary. It is dotted because the inequality $y < \frac{2}{3}x - 1$ does not include the points on the line $y = \frac{2}{3}x - 1$.

- Check at least one point within the shaded region to be certain the correct side of the line has been shaded. You can use the point (6, 0).

<u>Point (6, 0)</u>

$$y \stackrel{?}{<} \frac{2}{3}x - 1$$

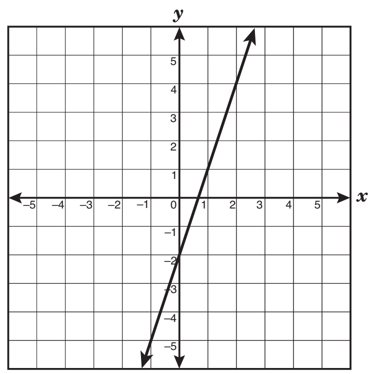
$$0 \stackrel{?}{<} \frac{2}{3}(6) - 1$$

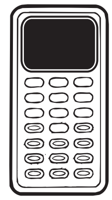
$$0 < 4 - 1$$

The shaded area is on the same side of the line as (6, 0). The graph does represent the inequality $y < \frac{2}{3}x - 1$.

Try It

Does the graph below represent the function $y = 3x - 2$?





Determine whether the equation and the graph are of the same type.

The equation is a ______________ function because it is in the form $y = mx + b$. Its graph must be a ______________.

Since the graph is a line, it is a ______________ function.

Find two points on the graph and see whether they satisfy the equation $y = 3x - 2$. Use the points $(0, -2)$ and $(2, 4)$.

If $x = 0$ and $y =$ ______, is the equation $y = 3x - 2$ true?

______ $\stackrel{?}{=}$ $3 \cdot$ ______ $-$ ______

-2 $\stackrel{?}{=}$ ______ $-$ ______

______ $=$ ______

If $x = 2$ and $y =$ ______, is the equation $y = 3x - 2$ true?

______ $\stackrel{?}{=}$ $3 \cdot$ ______ $-$ ______

4 $\stackrel{?}{=}$ ______ $-$ ______

______ $=$ ______

Both points on the graph satisfy the equation.

Objective 1

Find two points that satisfy the equation $y = 3x - 2$ and see whether they are points on the graph.

If $x = 1$, then

$y = 3 \cdot$ ______ $-$ ______

$y =$ ______ $-$ ______

$y =$ ______

The point (______, ______) satisfies the equation.

If $x = -1$, then

$y = 3 \cdot$ ______ $-$ ______

$y =$ ______ $-$ ______

$y =$ ______

The point (______, ______) satisfies the equation.

The points (______, ______) and (______, ______) are both points on the graph.

The graph represents the function $y = 3x - 2$.

The equation is a linear function because it is in the form $y = mx + b$. Its graph must be a line. Since the graph is a line, it is a linear function.

If $x = 0$ and $y = -2$, is the equation $y = 3x - 2$ true?

$-2 \stackrel{?}{=} 3 \cdot 0 - 2$

$-2 \stackrel{?}{=} 0 - 2$

$-2 = -2$

If $x = 2$ and $y = 4$, is the equation $y = 3x - 2$ true?

$4 \stackrel{?}{=} 3 \cdot 2 - 2$

$4 \stackrel{?}{=} 6 - 2$

$4 = 4$

Both points on the graph satisfy the equation. Find two points that satisfy the equation $y = 3x - 2$ and see whether they are points on the graph.

If $x = 1$, then

$y = 3 \cdot 1 - 2$

$y = 3 - 2$

$y = 1$

The point (1, 1) satisfies the equation.

If $x = -1$, then

$y = 3 \cdot -1 - 2$

$y = -3 - 2$

$y = -5$

The point $(-1, -5)$ satisfies the equation.

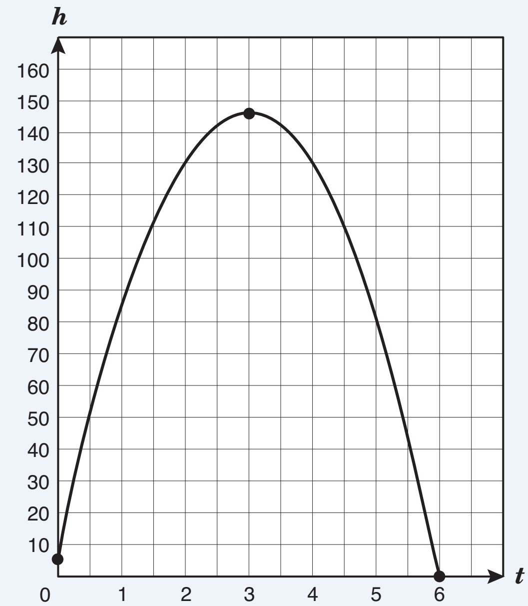
The points (1, 1) and $(-1, -5)$ are both points on the graph. The graph represents the function $y = 3x - 2$.

How Can You Draw Conclusions from a Functional Relationship?

Use these guidelines when interpreting functional relationships in a real-life problem.

- Understand the problem.
- Identify the quantities involved and any relationships between them.
- Determine what the variables in the problem represent.
- For graphs: Determine what quantity each axis on the graph represents. Look at the scale that is used on each axis.
- For tables: Determine what quantity each column in the table represents.
- Look for trends in the data. Look for maximum and minimum values in graphs.
- Look for any unusual data. For example, does a graph start at a nonzero value? Is one of the problem's variables negative at any point?
- Match the data to the equations or formulas in the problem.

A physics class is doing some outdoor experiments. The students are taking a large slingshot and tracking how long their projectiles stay in the air. The graph below shows the height, *h*, of one such projectile, in terms of *t*, the number of seconds after it was launched from the slingshot.



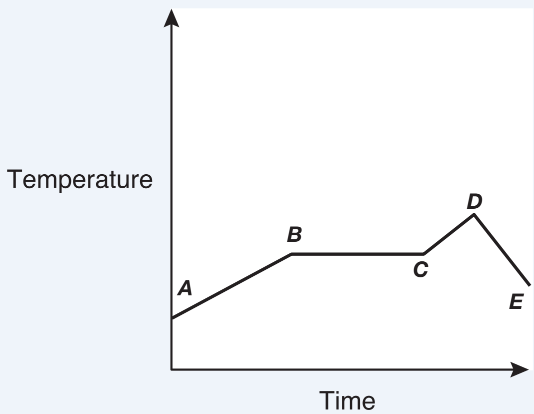
What was the approximate height of the projectile 2 seconds after it was launched? What was the approximate height of the projectile at its highest point, and how many seconds after launch did it take to get there? How many seconds after launch did the projectile return to the ground?

Examine the graph. Notice that the *x*-axis represents the time after the projectile was launched from the slingshot. The *y*-axis represents the height of the projectile, where each interval equals 10 feet.

- To find the approximate height of the projectile 2 seconds after launch, locate $t = 2$ and find the corresponding height on the *y*-axis. To do this, draw a vertical line from $t = 2$ on the *x*-axis to the graph. Draw a horizontal line from the graph to the *y*-axis. The value of *h* where the horizontal line meets the *y*-axis is just above 130 feet. The value of *h* that corresponds to $t = 2$ is approximately 131 feet. The projectile was at a height of about 131 feet at 2 seconds after launch.
- The highest point the projectile reached corresponds to the vertex (in this case, the highest point) of the graph. The graph almost reaches a height of 150 feet; we'll say approximately 147 feet. Draw a vertical line from the highest point on the graph down to the *x*-axis. The value of *t* that corresponds to $h = 147$ feet is approximately 3 seconds. The projectile reached its greatest height of about 147 feet approximately 3 seconds after launch.

- The projectile is on the ground when $h = 0$. On the graph, $h = 0$ when $t \approx 6$. Notice that the projectile is not launched from the ground; the launch point is 5 feet above the ground. The height begins to decrease approximately 3 seconds after launch. When $t \approx 6$, the projectile has hit the ground. The projectile hits the ground about 6 seconds after launch.

The graph below shows the temperature in a town over the course of one day.



During what time period did the temperature increase at the greatest rate?

The temperature in the town increased at a constant rate from point *A* to point *B*; then it remained the same from point *B* to point *C*. It increased again at a constant rate from point *C* to point *D* and then decreased at a constant rate from point *D* to point *E*.

The temperature increased during two time periods: from point *A* to point *B* and from point *C* to point *D*. The slope of the graph describes the rate of the temperature increase. The time period from point *C* to point *D* shows the greater slope.

The temperature increased at the greatest rate between point *C* and point *D*.

See Objective 3, page 69, for more information about slope.

Which of the following situations could be modeled by the equation $y = 40x + 10$?

A. The total cost in dollars, y, as a function of the number of computer games, x, where 40 represents the cost in dollars for each game and 10 represents the trade-in refund in dollars for a used game

(This cannot be modeled by $y = 40x + 10$. A refund would not be added to the total cost; it would be subtracted.)

B. The total number of hours traveled, y, as a function of the average speed in miles per hour, x, where 40 represents the number of miles traveled and 10 represents the distance in miles from the starting point

(This cannot be modeled by $y = 40x + 10$. Time varies inversely with speed, as shown in the equation $t = \frac{d}{r}$.)

C. The total cost of joining a fitness club, y, as a function of the number of months attended, x, where 40 represents the cost of each month and 10 represents the initial membership fee

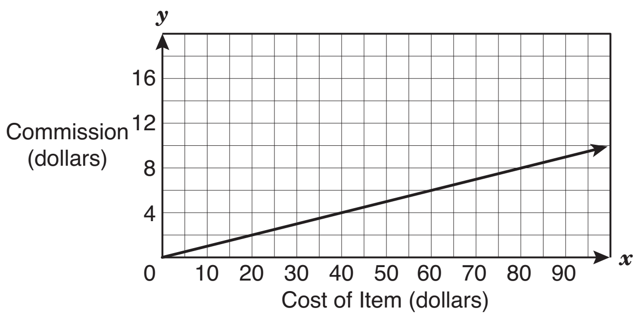
(This can be modeled by $y = 40x + 10$.)

D. The area of a trapezoid, y, as a function of one of the bases, where the other base is 40 and the height of the trapezoid is 10

(This cannot be modeled by $y = 40x + 10$. The formula for the area of a trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$.)

Try It

Sam works at an electronics store. The graph shows the commission he earns on a sale as a function of the cost of the item he sells.



What will Sam's approximate commission be if he sells an item that costs $67.50?

To determine Sam's commission, find the cost of the item on the _______-axis. Then find the corresponding commission amount on the _______-axis.

The cost of the item, $67.50, is located halfway between _______ and _______ on the *x*-axis.

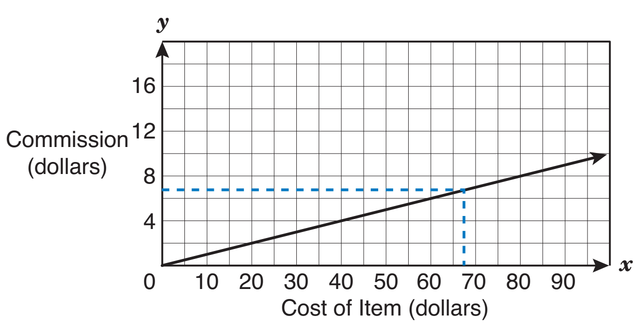
Draw a vertical line from this point on the axis to the graph.

Draw a _________________ line from this point on the graph to the _______-axis.

The line intersects the *y*-axis between _______ and _______.

Sam's commission will be about $_______.

To determine Sam's commission, find the cost of the item on the *x*-axis. Then find the corresponding commission amount on the *y*-axis. The cost of the item, $67.50, is located halfway between 65 and 70 on the *x*-axis. Draw a horizontal line from this point on the graph to the *y*-axis. The line intersects the *y*-axis between 6 and 8. Sam's commission will be about $7.00.



Now practice what you've learned.

Question 1

The basketball team is ordering T-shirts to sell for a fund-raiser. The team paid \$275 for the shirts and will sell them for \$12 each. The relationship between the number of shirts sold and the team's profit from the sale of the shirts can be represented by the function $f(n) = 12n - 275$, in which n represents the number of shirts sold. What is the dependent quantity in this functional relationship?

A The number of shirts sold

B The amount the team paid for the shirts

C The team's profit from the sale of the shirts

D The selling price of the shirts

Answer Key: page 280

Question 2

Liang has \$35 to spend on movie rentals and snacks for the weekend. He spends \$12 on popcorn and soda. If the movie store rents DVDs for \$4 each, which inequality models n, the number of DVDs Liang is able to rent?

A $4n + 12 \leq 35$

B $4n - 12 \leq 35$

C $\frac{35}{4n} \leq 12$

D $35 - 4n \leq 12$

Answer Key: page 280

Question 3

Which of the following tables represents y as a function of x?

A

x	y
2	−9
2	−6
2	−3
2	0

B

x	y
4	2
1	3
−2	4
−5	5

C

x	y
−2	0
−1	0
−2	1
−1	1

D

x	y
2	−5
3	−11
7	−8
3	−5

Answer Key: page 280

Question 4

The total cost of an item at a store is the price of the item plus 6.5% sales tax. If c, the total cost of the item, is a function of x, the price of the item, which function models this situation?

A $c = x + 6.5$

B $c = 6.5x$

C $c = 1.065x$

D $c = x + 1.065$

Answer Key: page 280

Question 5

A lawn and garden store displays the following table to show customers the amount of fertilizer needed for the corresponding lawn size.

Size of Lawn (square feet)	Fertilizer Required (pounds)
50	0.5
100	1.0
200	2.0
250	2.5
300	3.0

Which equation best represents the relationship between s, the size of the lawn, and f, the pounds of fertilizer required?

A $f = 100s$

B $f = \frac{s}{100}$

C $f = 50s$

D $f = \frac{s}{50}$

Answer Key: page 280

Question 6

Which table contains points on the graph of the function $f(x) = 4 - 2x$?

A

x	y
1	2
2	4
3	6
4	8

B

x	y
−3	−2
−1	2
0	4
2	0

C

x	y
−1	6
1	2
3	−1
5	−6

D

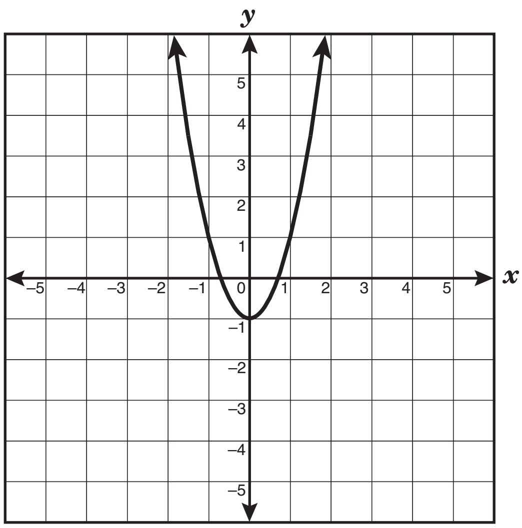
x	y
−2	8
0	4
2	0
4	−4

Answer Key: page 280

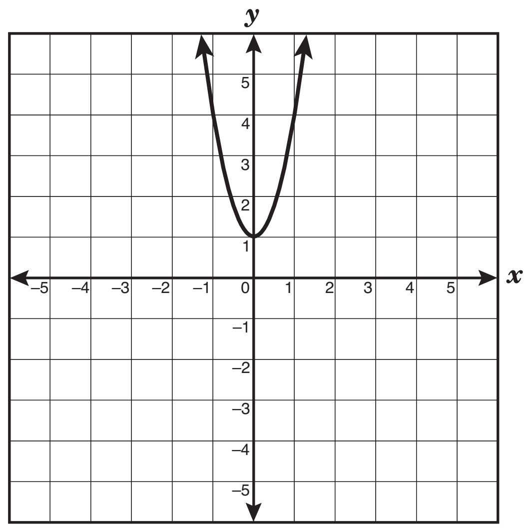
Question 7

Which graph below represents the quadratic function $y = 3x^2 + 1$?

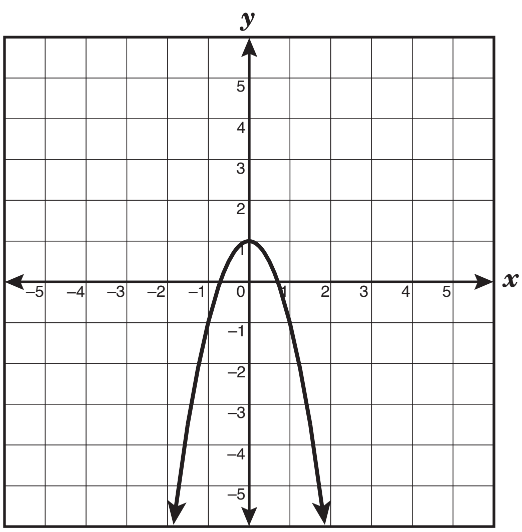
A



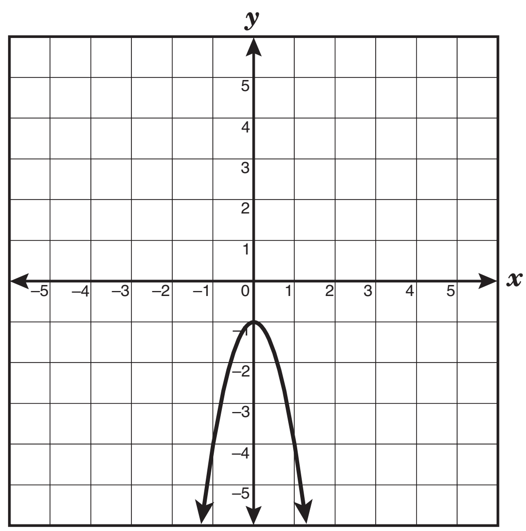
C



B



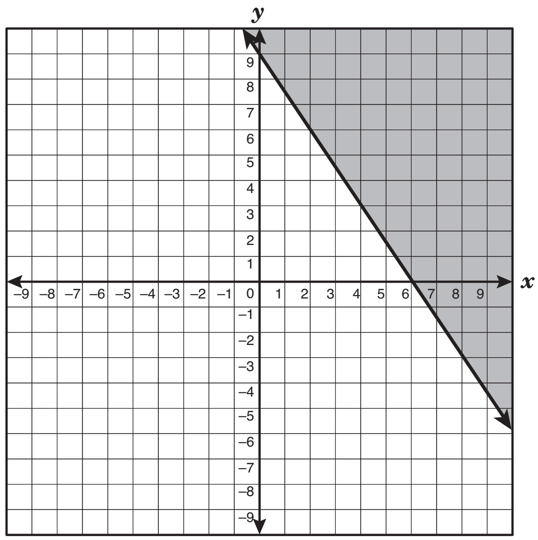
D

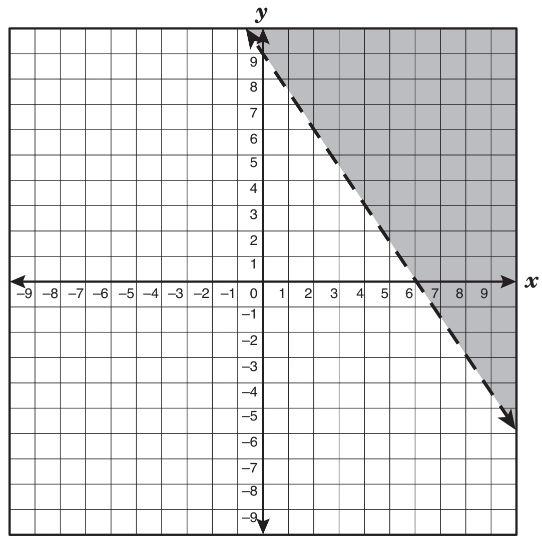


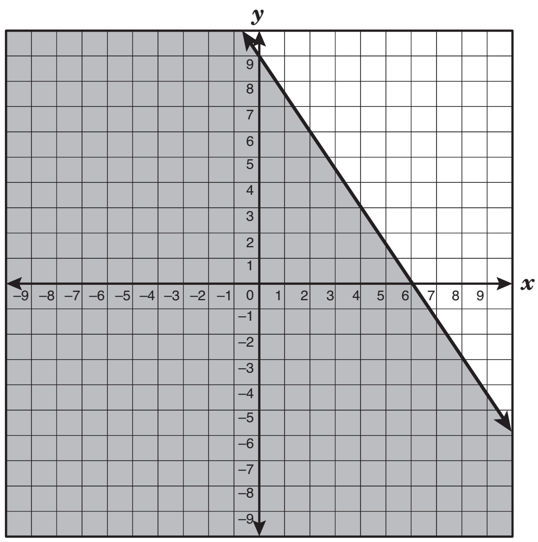
Answer Key: page 281

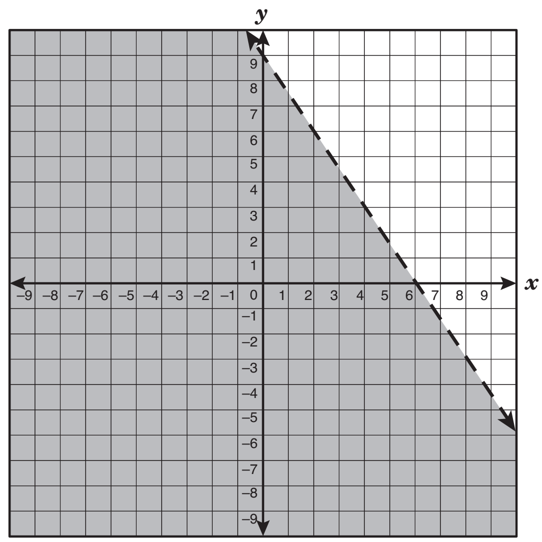
Question 8

Which of the following best represents the graph of the inequality $\frac{3}{2}x + y < 9$?

A


C


B


D


Answer Key: page 281

Question 9

The table below shows independent and dependent values in a functional relationship.

Independent	Dependent
0	5
1	6
2	9
3	14

Which function best represents this relationship?

A $f(x) = 2x^2 + 5$

B $f(x) = x^2 + 5$

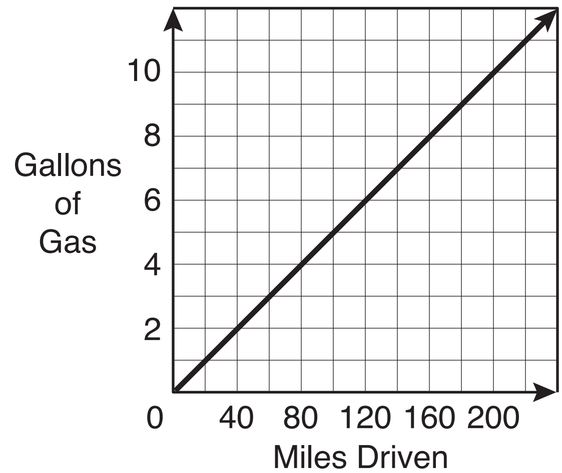
C $f(x) = 2x + 5$

D $f(x) = x + 5$

Answer Key: page 281

Question 10

The graph shows the number of gallons of gas used by Maria's car as a function of the number of miles driven.



About how many gallons of gas will Maria need for a 170-mile trip?

A 7 gal

B 8.5 gal

C 9.5 gal

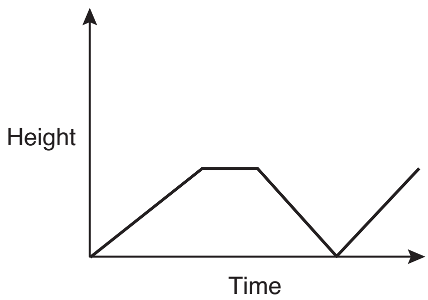
D 11 gal

Answer Key: page 282

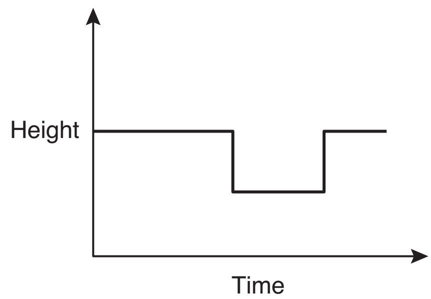
Question 11

Paul fills a bucket shaped like a cylinder with water at a constant rate. As he is carrying the bucket, he trips and spills half the water. Paul then returns to the faucet and refills the bucket at a constant rate. Which graph best represents the height of the water in the bucket as a function of time?

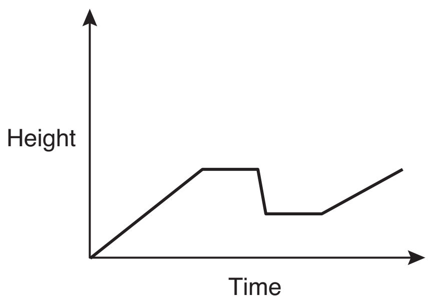
A



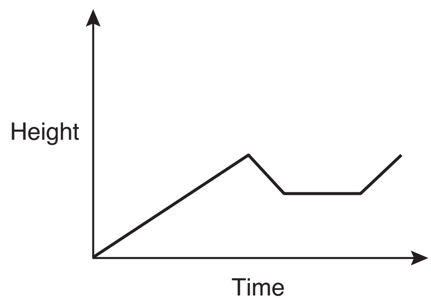
C



B



D



Answer Key: page 282

Objective 2

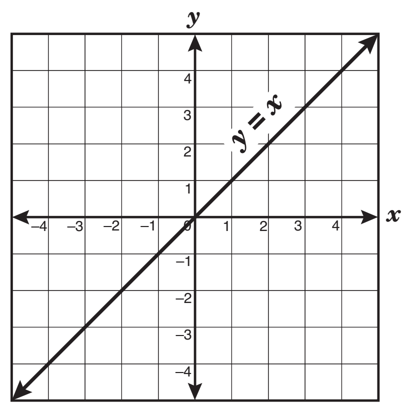
The student will demonstrate an understanding of the properties and attributes of functions.

For this objective you should be able to

- use the properties and attributes of functions;
- use algebra to express generalizations and use symbols to represent situations; and
- manipulate symbols to solve problems and use algebraic skills to simplify algebraic expressions and solve equations and inequalities in problem situations.

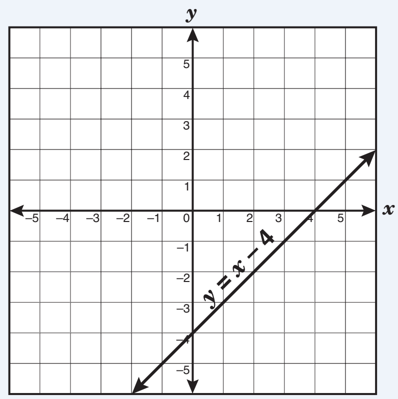
What Are Parent Functions?

The simplest linear function, $y = x$, is the **linear parent function**.

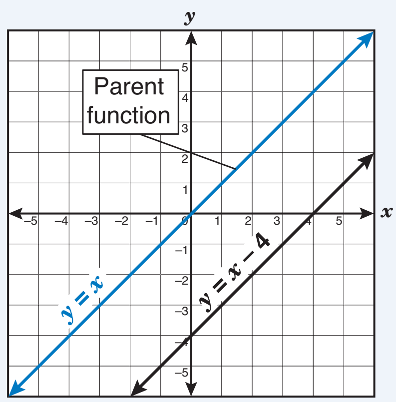


- If the graph of any function is a line, then its parent function is $y = x$.
- If a function can be written in the form $y = mx + b$, then it is linear.
- A linear function never has variables raised to a power other than 1.
- If a function is linear, then its parent function is $y = x$.
- An equation in the form $x = a$ is a linear equation, but it is not a function. Its graph is a vertical line.

What is the parent function of this graph?

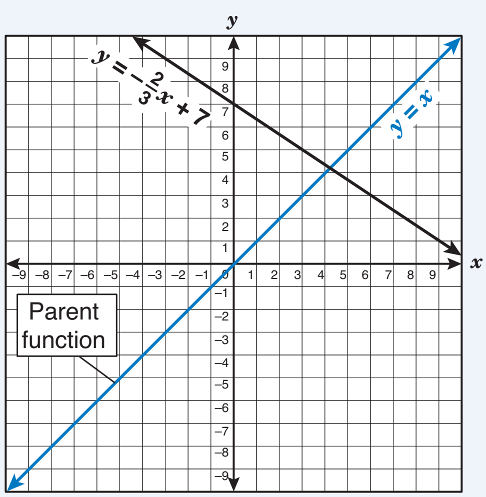


Since the graph of $y = x - 4$ is a line, its parent function is the linear parent function, $y = x$.



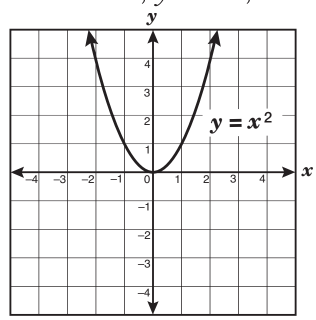
What is the parent function of the equation $y = -\frac{2}{3}x + 7$?

Since the equation $y = -\frac{2}{3}x + 7$ is a linear function, its graph is a line. Its parent function is the linear parent function, $y = x$.



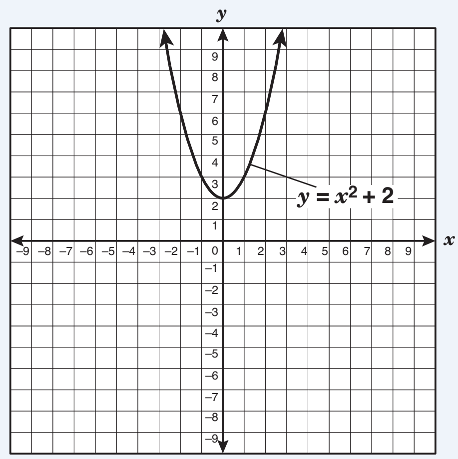
Objective 2

The simplest quadratic function, $y = x^2$, is the **quadratic parent function**.

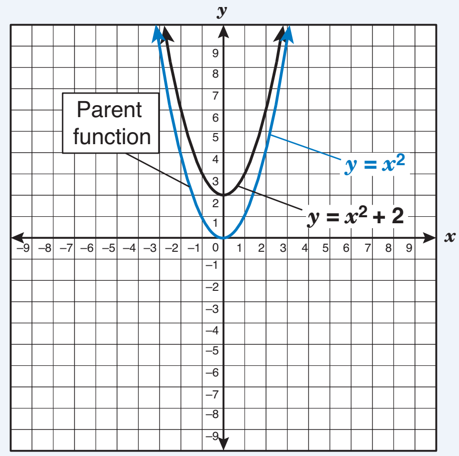


- If the graph of any function is a parabola, then its parent function is $y = x^2$.
- If an equation can be written in the form $y = ax^2 + bx + c$ where $a \neq 0$, then it is quadratic.
- If an equation can be written in this form, then its parent function is $y = x^2$.

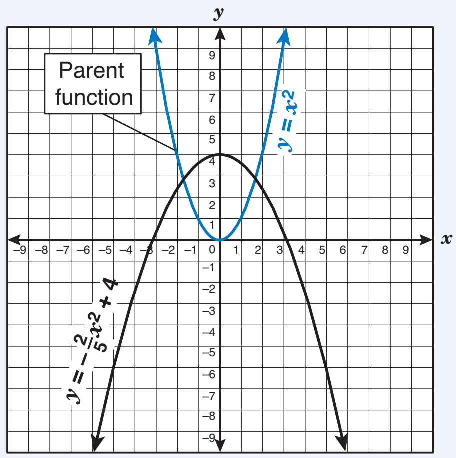
What is the parent function of this graph?



Since the graph of $y = x^2 + 2$ is a parabola, its parent function is the quadratic parent function, $y = x^2$.



What is the parent function of $y = -\frac{2}{5}x^2 + 4$?



The equation $y = -\frac{2}{5}x^2 + 4$ is a quadratic function; therefore, its parent function is the quadratic parent function, $y = x^2$.

What Are the Domain and Range of a Function?

A **function** is a set of ordered pairs of numbers (x, y) such that no x-values are repeated. The domain and range of a function are sets that describe those ordered pairs.

	Definition	Example $\{(0, 1), (2, 6), (3, 5)\}$
Domain	The set of all the x-coordinates in the function's ordered pairs	$\{0, 2, 3\}$
Range	The set of all the y-coordinates in the function's ordered pairs	$\{1, 5, 6\}$

- The **domain** is the set of all the values of the independent variable, the x-coordinate.
- The **range** is the set of all the values of the dependent variable, the y-coordinate.

See Objective 1, page 10, for more information about functions.

Identify the domain and range of the function below.

$$\{(-3, 4), (-1, 2), (2, -4), (4, -6)\}$$

The domain is the set of x-coordinates in the ordered pairs: $\{(\underline{-3}, 4), (\underline{-1}, 2), (\underline{2}, -4), (\underline{4}, -6)\}$. The domain is $\{-3, -1, 2, 4\}$.

The range is the set of y-coordinates in the ordered pairs: $\{(-3, \underline{4}), (-1, \underline{2}), (2, \underline{-4}), (4, \underline{-6})\}$. The range is $\{-6, -4, 2, 4\}$.

Objective 2

Try It

What are the domain and range of the function below?

$$\{(0, -5), (2, -1), (3, 4), (5, 20), (-3, 4)\}$$

The domain of a function is the set of all ________ -coordinates.

The domain of this function is

{________, ________, ________, ________, ________}.

The range of a function is the set of all ________-coordinates.

The range of this function is

{________, ________, ________, ________}.

The domain of a function is the set of all x-coordinates. The domain of this function is {–3, 0, 2, 3, 5}. The range of a function is the set of all y-coordinates. The range of this function is {–5, –1, 4, 20}.

The domain and range of algebraic functions are usually assumed to be the set of all real numbers. In some cases, however, the domain or range of a function may be a subset of the real numbers because certain numbers would not make sense in a real-life problem situation.

Lisa hired a moving company to help her move to a new apartment. The moving company charges \$5 for each box it moves. The relationship between c, the total the company charges in dollars, and n, the number of boxes it moves, can be expressed by the function $c = 5n$.

What is a reasonable domain and range for this function?

- The domain of this function is the set of values you may choose for n, the independent variable.

  It does not make sense for n to be negative, since Lisa cannot pack a negative number of boxes. Lisa can pack part of a box, but even though the box may be partly filled, it is still a whole box to move. There cannot be any values of n that are not whole numbers.

A reasonable domain for this function is the set $\{0, 1, 2, 3, \ldots\}$.

- The range of this function is the set of values you will obtain for the dependent variable, *c*, the total cost for moving *n* boxes.

  If Lisa packs 0 boxes, the total charge is \$0.

  If Lisa packs 1 box, the total charge is \$5.

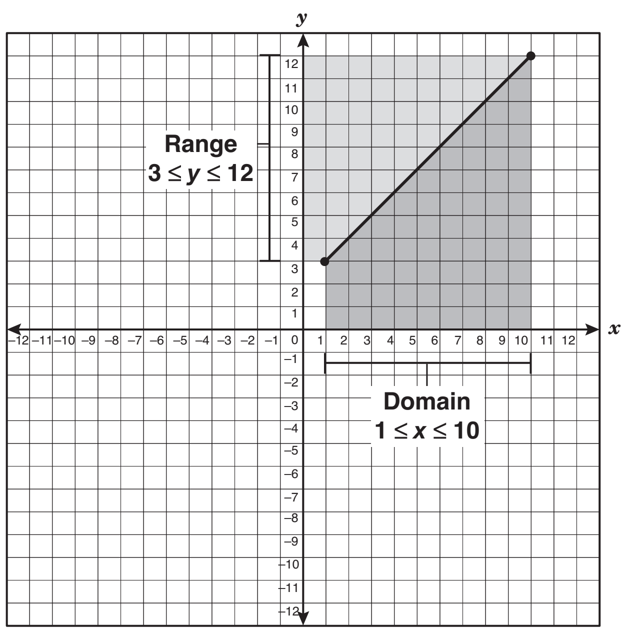
  If Lisa packs 2 boxes, the total charge is \$10.

  The range is the set of domain values multiplied by 5.

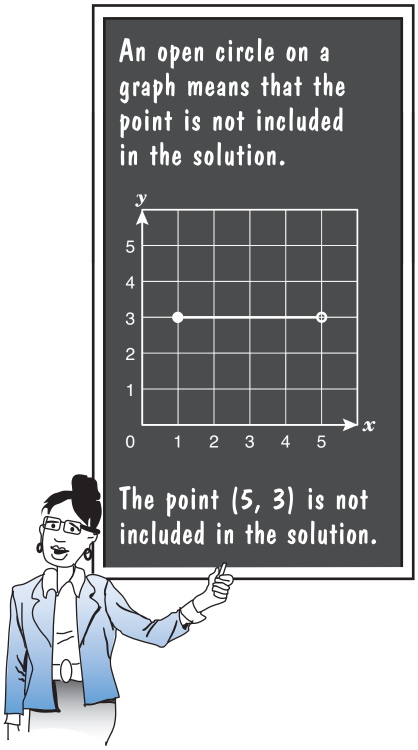
A reasonable range for this function is the set {0, 5, 10, 15, …}.

The domain and range of a function can be determined by its graph.

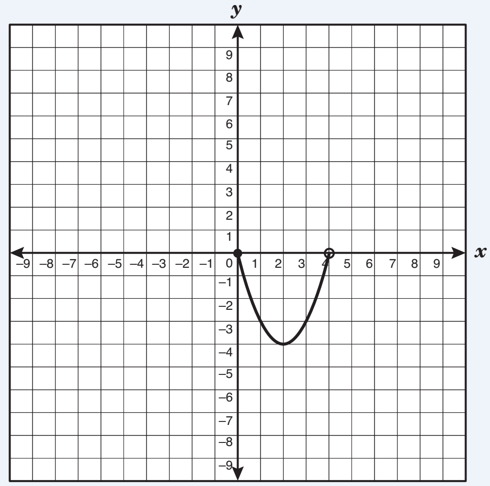
- The domain of a function is the set of all the *x*-coordinates in the function's graph.
- The range of a function is the set of all the *y*-coordinates in the function's graph.



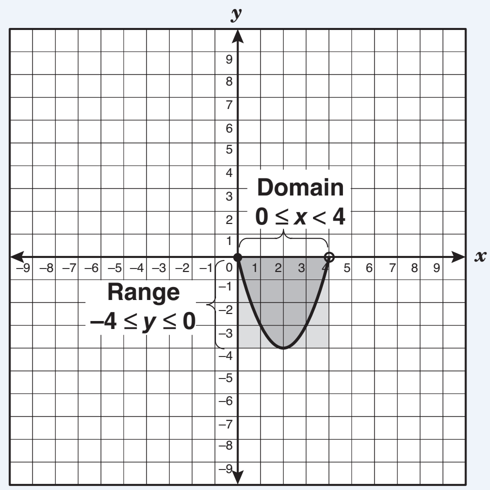
Objective 2



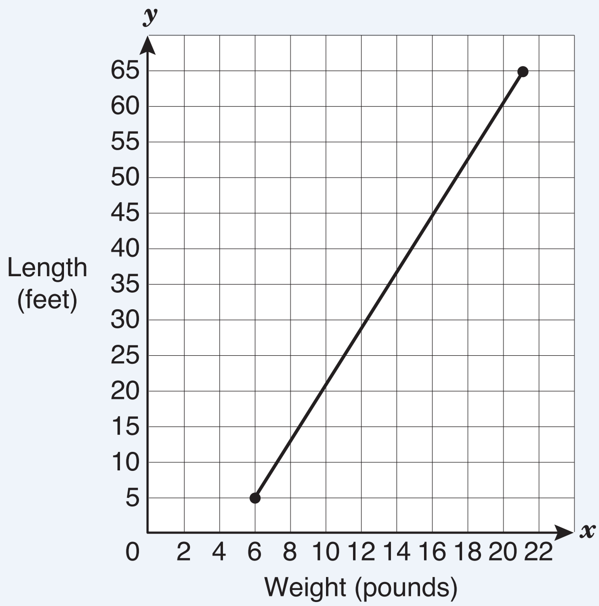
What inequalities best describe the domain and range of the function represented in this graph?



- The domain of this function is the set of *x*-values in the graph.
- The range of this function is the set of *y*-values in the graph.



A construction company uses steel ladder beams for scaffolding. The graph below represents the relationship between x, the weight of the beams in pounds, and y, their length in feet.



What is a reasonable domain and range for this function?

The domain of the function is the set of all the x-coordinates for which the function is defined. The graph includes x-values from 6 to 21 pounds. The domain of this function is $6 \le x \le 21$.

The range of the function is the set of all the y-coordinates for which the function is defined. The graph includes y-values from 5 to 65 feet. The range of this function is $5 \le y \le 65$.

Try It

A rectangular courtyard is planned as part of the design for a new building. The courtyard should be 3 feet longer than it is wide. The courtyard must be at least 12 feet wide but no more than 25 feet wide. The function $f(x) = x^2 + 3x$ describes the area of the courtyard in terms of its width in feet, x. What is a reasonable range for this function?

To determine the range for the function, determine the minimum and ____________________ values for the area of the courtyard.

The minimum area of the courtyard will occur when $x = 12$.

$$f(x) = x^2 + 3x$$
$$f(x) = (\_\_\_\_\_\_)^2 + 3(\_\_\_\_\_\_)$$
$$f(x) = \_\_\_\_\_\_ + \_\_\_\_\_\_$$
$$f(x) = \_\_\_\_\_\_$$

The maximum area of the courtyard will occur when $x =$ ________.

$$f(x) = x^2 + 3x$$
$$f(x) = (\_\_\_\_\_\_)^2 + 3(\_\_\_\_\_\_)$$
$$f(x) = \_\_\_\_\_\_ + \_\_\_\_\_\_$$
$$f(x) = \_\_\_\_\_\_$$

A reasonable range for this function is ________ $\leq f(x) \leq$ ________.

To determine the range for the function, determine the minimum and maximum values for the area of the courtyard.

The minimum area of the courtyard will occur when $x = 12$.

$$f(x) = x^2 + 3x$$
$$f(x) = (12)^2 + 3(12)$$
$$f(x) = 144 + 36$$
$$f(x) = 180$$

The maximum area of the courtyard will occur when $x = 25$.

$$f(x) = x^2 + 3x$$
$$f(x) = (25)^2 + 3(25)$$
$$f(x) = 625 + 75$$
$$f(x) = 700$$

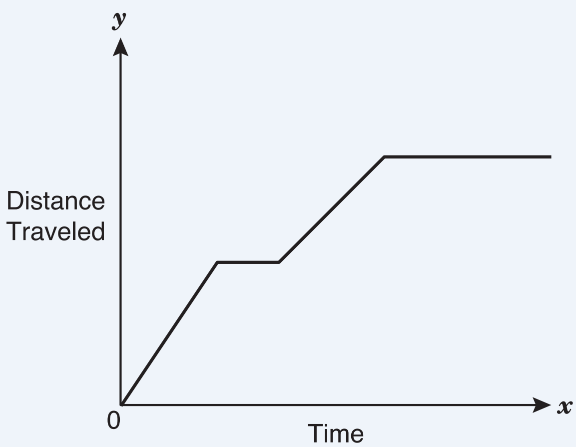
A reasonable range for this function is $180 \leq f(x) \leq 700$.

How Can You Interpret a Problem Situation from a Graph?

To interpret a problem situation described in terms of a graph, follow these guidelines.

- Identify the quantities that are being compared.
- Understand what relationship the graph is describing.
- Look at the scales used on the axes of the graph.
- Identify the domain or range of the function graphed.
- Look for patterns in the data—increases, decreases, or data that remain constant.

Patricia rides her bicycle from home to a friend's house, which is on the same street as Patricia's but on the other side of town. The graph below shows the distance she traveled in terms of the amount of time since she left home.



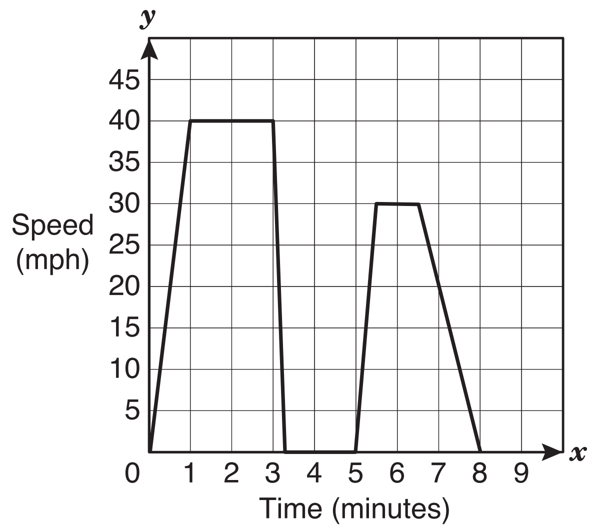
What is a reasonable interpretation of the graph?

- The graph begins at the point (0, 0). At the beginning of her ride, when time equals 0, Patricia's distance traveled is also 0.
- The first segment on the graph shows Patricia's distance traveled increasing at a constant rate; she is riding her bike at a constant speed.
- The second segment on the graph suggests that Patricia stops for a period of time; her distance traveled remains the same.
- Patricia then continues her bike ride. This is represented by the third segment of the graph. The slope of this segment is less than the slope of the first segment, which means she rides her bike at a slower constant speed.
- The last segment shows when Patricia reaches her friend's house, where she stops. Patricia's distance traveled is no longer increasing.

See Objective 3, page 69, for more information about slope.

Try It

The graph below describes the speed of an automobile over an eight-minute period as it travels from its starting point to a store.



Describe the automobile's journey in terms of its speed during the trip from its starting point to the store.

The car begins from a stop. Its speed is ________.

During the first minute, the car's speed increases to ________ mph.

The car's speed then remains the same for the next ________ minutes.

The car then brakes and comes quickly to a stop, remaining stopped for nearly ________ minutes.

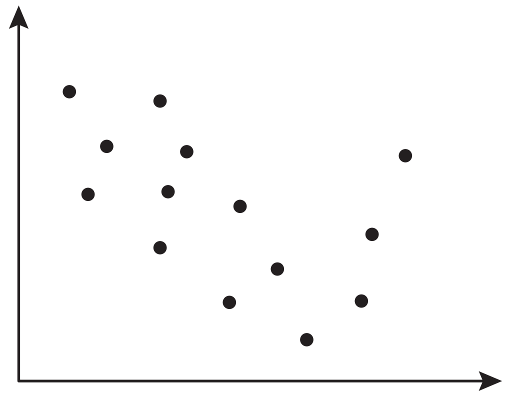
The car then accelerates to ________ mph and travels at that speed for ________ minute.

The car then slows down steadily and finally stops at the end of ________ minutes.

The car begins from a stop. Its speed is 0. During the first minute, the car's speed increases to 40 mph. The car's speed then remains the same for the next 2 minutes. The car then brakes and comes quickly to a stop, remaining stopped for nearly 2 minutes. The car then accelerates to 30 mph and travels at that speed for 1 minute. The car then slows down steadily and finally stops at the end of 8 minutes.

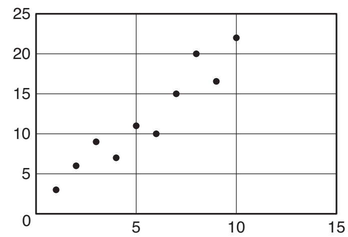
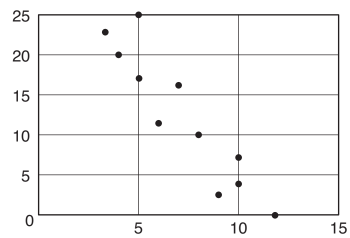
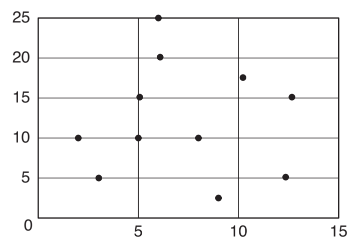
What Is a Correlation in a Scatterplot?

One way to represent a set of related data is to graph the data using a scatterplot. In a **scatterplot** each pair of corresponding values in the data set is represented by a point on a graph.

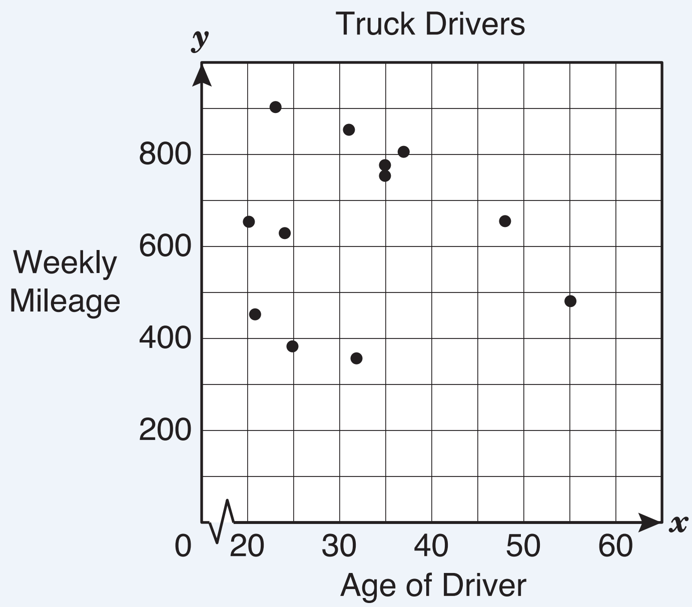


To make predictions using a scatterplot, look for a **correlation**, or pattern, in the data. The pattern may not be true for every point, but look for the overall pattern the data seem to best fit.

Objective 2

<table>
<tr><th>As you move from left to right on the graph, if the data points …</th><th>as shown in this scatterplot …</th><th>they show this type of correlation:</th></tr>
<tr><td>move up</td><td>
</td><td>positive correlation.</td></tr>
<tr><td>move down</td><td>
</td><td>negative correlation.</td></tr>
<tr><td>show no pattern</td><td>
</td><td>no correlation.</td></tr>
</table>

A trucking company keeps track of the number of miles each of its drivers logs each week. The scatterplot below shows the relationship between a driver's age and the number of miles the driver drove last week.

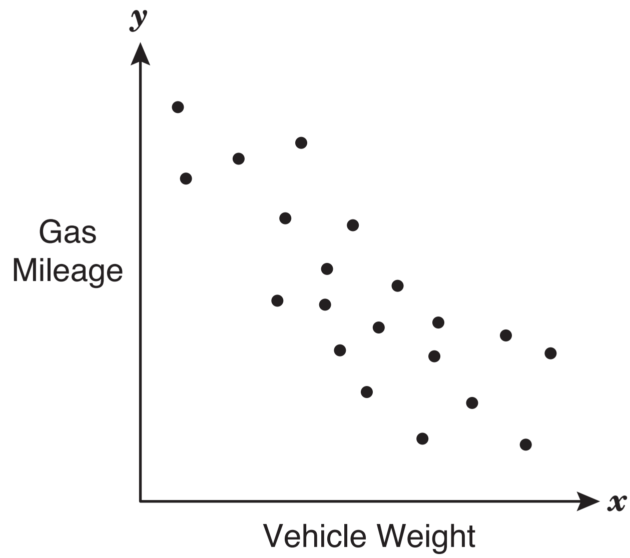


Describe the correlation between the number of miles a driver logs and the driver's age.

Look for a pattern in the graph. There is no pattern to the graph. The graph shows no correlation between the number of hours a driver logs and the driver's age.

Try It

The scatterplot shows the weight of a vehicle compared with the average gas mileage the vehicle gets.



What conclusion can you draw about the relationship between vehicle weight and gas mileage?

As the vehicle weight increases, the gas mileage ______________.

The gas mileage for a vehicle has a ______________ correlation

with ____.

As the vehicle weight increases, the gas mileage decreases. The gas mileage for a vehicle has a negative correlation with the weight of the vehicle.

How Do You Use Symbols to Represent Unknown Quantities?

Represent unknown quantities with **variables**, or letters such as x or y. Use variables in expressions, equations, or inequalities.

Jessica has been saving quarters and dimes in a jar. Write an expression that represents the amount of money in the jar in dollars and cents.

- Represent the quantities.

  Let q = the number of quarters in the jar.

  Let d = the number of dimes in the jar.

  Let $0.25q$ = the value of the quarters in the jar.

  Let $0.10d$ = the value of the dimes in the jar.

- Represent the total amount of money in the jar.

$$\text{value of quarters} + \text{value of dimes}$$

$$0.25q + 0.10d$$

The expression $0.25q + 0.10d$ represents the amount of money in the jar.

The length of Karen's rectangular swimming pool is 15 feet greater than its width. Her neighbor's pool is twice as long and 3 feet wider than Karen's pool. Write an expression that could be used to represent the perimeter of the neighbor's pool.

- Represent the quantities.

  Let w = the width of Karen's pool.

  Let $(w + 15)$ = the length of Karen's pool.

  Let $(w + 3)$ = the width of the neighbor's pool.

  Let $2(w + 15)$ = the length of the neighbor's pool.

- Represent the perimeter of the neighbor's pool.

$$2(\text{length}) + 2(\text{width})$$

$$2[2(w + 15)] + 2(w + 3)$$

$$= 2(2w + 30) + 2(w + 3)$$

$$= 4w + 60 + 2w + 6$$

$$= 6w + 66$$

The perimeter of the neighbor's pool can be represented by the expression $6w + 66$.

Objective 2

How Do You Represent Patterns in Data Algebraically?

To represent patterns in data algebraically, follow these guidelines.

- Identify what quantities the data represent.
- Identify the relationships between those quantities.
- Look for patterns in the data.
- Use symbols to translate the patterns into an algebraic expression or equation.

See Objective 3, page 69, for more information about slope.

The table shows the functional relationship between x and y.

x	y
1	0
2	3
3	8
4	15

Use function notation to write a rule that represents this relationship.

Look for a pattern between the ordered pairs of the function.

- Look at the ordered pairs (1, 0) and (2, 3).
  The change in y is $3 - 0 = 3$.
  The change in x is $2 - 1 = 1$.
  The slope is 3.
- Look at the ordered pairs (2, 3) and (3, 8).
  The change in y is $8 - 3 = 5$.
  The change in x is $3 - 2 = 1$.
  The slope is 5.

The slopes between these ordered pairs are not equal. This means that the function is not linear.

The relationship between x and y may be quadratic. Look for a pattern between the values of x^2 and y.

x	x^2	y
1	1	0
2	4	3
3	9	8
4	16	15

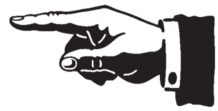
The y-value appears to be 1 less than x^2. Does the function $y = x^2 - 1$ describe the relationship between these quantities?

Verify that each of the data points (ordered pairs) satisfies the equation $y = x^2 - 1$.

Replace y with $f(x)$. The rule expressed in function notation is $f(x) = x^2 - 1$.

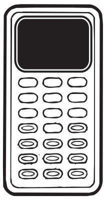
x	$f(x) = x^2 - 1$	y	Yes/No
1	$f(1) = (1)^2 - 1$ $f(1) = 1 - 1$ $f(1) = 0$	0	Yes
2	$f(2) = (2)^2 - 1$ $f(2) = 4 - 1$ $f(2) = 3$	3	Yes
3	$f(3) = (3)^2 - 1$ $f(3) = 9 - 1$ $f(3) = 8$	8	Yes
4	$f(4) = (4)^2 - 1$ $f(4) = 16 - 1$ $f(4) = 15$	15	Yes

The rule $f(x) = x^2 - 1$ represents this relationship.



Do you see that . . .

Objective 2



Look at the pattern of numbers below.

3, 7, 13, 21, …

Does the expression $n^2 + n + 1$ represent the nth number in this pattern?

To test whether an expression represents a numerical pattern, it may be helpful to organize the information in a table.

Test the proposed rule for each term. For the first term, $n = 1$; for the second term, $n = 2$; and so on.

Position	$n^2 + n + 1$	Value	Yes/No
1	$1^2 + 1 + 1 = 1 + 1 + 1 = 3$	3	Yes
2	$2^2 + 2 + 1 = 4 + 2 + 1 = 7$	7	Yes
3	$3^2 + 3 + 1 = 9 + 3 + 1 = 13$	13	Yes
4	$4^2 + 4 + 1 = 16 + 4 + 1 = 21$	21	Yes

Yes, the nth number in this pattern is represented by the rule $n^2 + n + 1$.

Students in a math class used a pattern to construct a series of increasingly larger quadrilaterals, and then they measured the perimeters of the quadrilaterals. The perimeters are given in the table below.

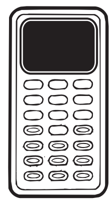
Quadrilateral

Position	1	2	3	4
Perimeter (cm)	5	8	13	20

If the pattern continues, what expression can be used to represent the perimeter of the nth quadrilateral in this series?

The perimeters do not increase by a constant amount, so this is not a linear relationship.

Look for a quadratic pattern.



Position (n)	Square of position (n^2)	Perimeter (cm)
1	1	5
2	4	8
3	9	13
4	16	20

The perimeter of each quadrilateral is 4 more than the square of its position number in the series. The perimeter of the nth quadrilateral should be $n^2 + 4$.

See whether this expression works for the known values.

Position	$n^2 + 4$	Perimeter (cm)	Yes/No
1	$1^2 + 4 = 1 + 4 = 5$	5	Yes
2	$2^2 + 4 = 4 + 4 = 8$	8	Yes
3	$3^2 + 4 = 9 + 4 = 13$	13	Yes
4	$4^2 + 4 = 16 + 4 = 20$	20	Yes

If the pattern continues, the perimeter of the nth quadrilateral could be represented by the expression $n^2 + 4$.

Objective 2

How Do You Simplify Algebraic Expressions?

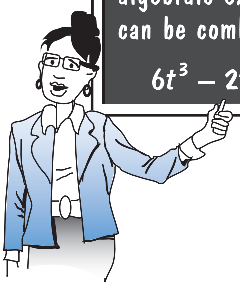
You can use the commutative, associative, and distributive properties to simplify algebraic expressions. Use these properties to remove parentheses and combine like terms.

Like terms are terms in an algebraic expression that use the same variable raised to the same power.

For example, in the expression $6t^3 - 2t^3$, $6t^3$ and $2t^3$ are like terms because they are both expressed in terms of the same variable, t, raised to the same power, 3.

Like terms in an algebraic expression can be combined.

$6t^3 - 2t^3 = 4t^3$



Name	Property (If a, b, and c are any three real numbers, then…)	Examples
Commutative Property	$a + b = b + a$ or $ab = ba$	$12 + 5y = 5y + 12$ or $b^2a = ab^2$
Associative Property	$a + (b + c) = (a + b) + c$ or $a(bc) = (ab)c$	$(5 + 2m) + 5m = 5 + (2m + 5m)$ $= 5 + 7m$ or $(6v)v = 6(v \cdot v)$ $= 6v^2$
Distributive Property	$a(b + c) = ab + ac$	$y(2y - 3) = y(2y) + y(-3)$ $= 2y^2 - 3y$

- Use the commutative property to change the order of the terms in addition or multiplication.
- Use the associative property to change the groupings in addition or multiplication.
- Use the distributive property to remove the parentheses by multiplying the term outside the parentheses by each term inside the parentheses.

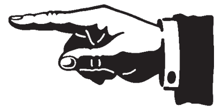
Objective 2

Simplify the expression $12x(x - 1) - (x^2 - 5)$.

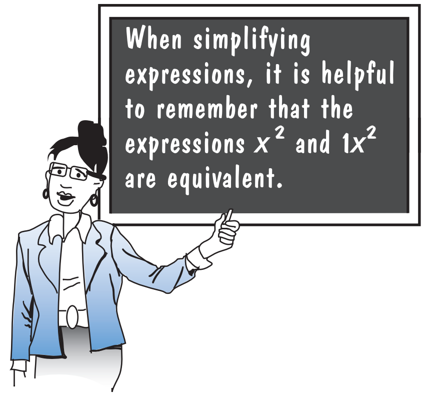
When parentheses are preceded by a negative sign, it means the quantity in parentheses is multiplied by -1.

$$\begin{aligned} 12x(x - 1) - (x^2 - 5) &= 12x(x - 1) + -1(x^2 - 5) \\ &= 12x(x) + 12x(-1) - x^2 + 5 \\ &= 12x^2 - 12x - x^2 + 5 \\ &= 12x^2 - x^2 - 12x + 5 \\ &= 11x^2 - 12x + 5 \end{aligned}$$

When simplified, the expression $12x(x - 1) - (x^2 - 5)$ equals $11x^2 - 12x + 5$.



Do you see that . . .



The length of the base of a triangle is 6 inches less than its height. Write an expression for the area of the triangle in terms of its height, h.

- The height of the triangle is represented by h.

  The length of the base is 6 inches less than the height, or $h - 6$.
- The area is one-half times the base times the height.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(h - 6)h$$

- Simplify the expression.

$$A = \frac{1}{2}h(h - 6)$$

$$A = \frac{1}{2}h(h) - \frac{1}{2}h(6)$$

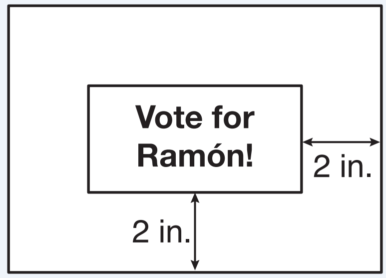
$$A = \frac{1}{2}h^2 - \frac{1}{2}h(6)$$

$$A = \frac{1}{2}h^2 - 3h$$

The expression $\frac{1}{2}h^2 - 3h$ represents the area of the triangle in terms of h.

Objective 2

Ramón is designing his campaign posters for a school election. He plans to put a 2-inch border completely around each rectangular poster. Ramón wants the part of the poster used for his campaign message, the smaller rectangle, to be 5 inches longer than it is wide. Write an expression that represents the area of the poster including the border.

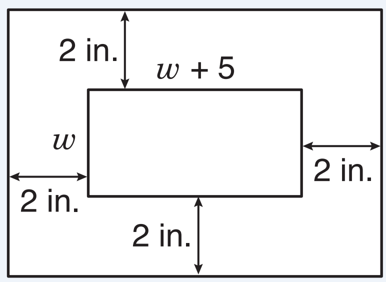


- Write two expressions that represent the length and width of the message section of the poster.

$$\text{width} = w$$

$$\text{length} = w + 5$$

- Write two expressions that represent the length and width of the poster, including the border.



$$\text{width} = 2 + w + 2 = w + 4$$

$$\text{length} = 2 + (w + 5) + 2 = w + 9$$

- Find the area of the poster board by multiplying the length of the board by the width.

$$A = (w + 9)(w + 4)$$

$$A = (w)(w) + 4(w) + 9(w) + (9)(4)$$

$$A = w^2 + 4w + 9w + 36$$

$$A = w^2 + 13w + 36$$

The expression $w^2 + 13w + 36$ represents the area of the poster including the border.

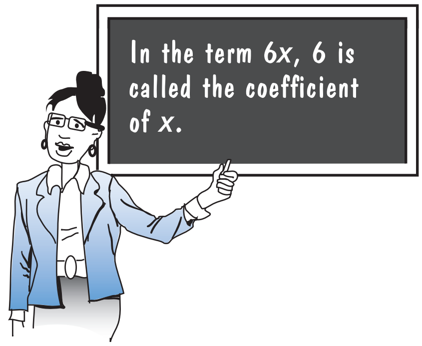
See Objective 5, page 129, for more information about the FOIL method.

Objective 2

How Do You Solve Algebraic Equations?

To solve algebraic equations, follow these guidelines.

- Simplify any expressions in the equation.
- Add or subtract on both sides of the equation to get variable terms on one side and constant terms on the other.
- Simplify again if necessary.
- Multiply or divide to obtain an equation that has the variable isolated with a coefficient of 1.



Solve the equation $x + 4 + 2(x - 1) = 23$.

$$\begin{aligned}
x + 4 + 2(x - 1) &= 23 \\
x + 4 + 2x - 2 &= 23 \\
3x + 2 &= 23 \\
-2 &= -2 \\
\hline
3x \qquad &= 21 \\
\frac{3x}{3} &= \frac{21}{3} \\
x &= 7
\end{aligned}$$

The solution to the equation is $x = 7$.

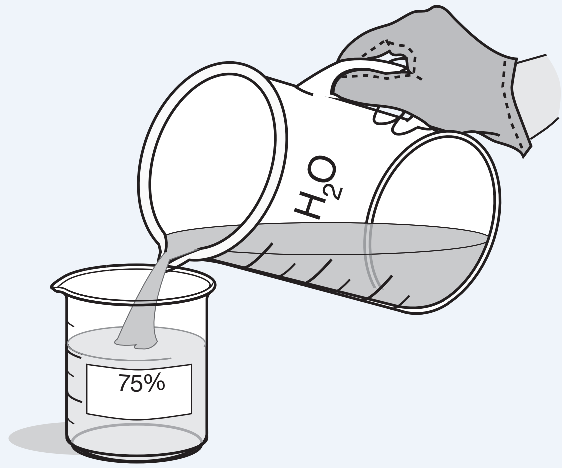
Solve the equation $2(m - 1) = 5(m + 2) - 6$.

$$\begin{aligned}
2(m - 1) &= 5(m + 2) - 6 \\
2m - 2 &= 5m + 10 - 6 \\
2m - 2 &= 5m + 4 \\
-2m \qquad &= -2m \\
\hline
-2 &= 3m + 4 \\
-4 &= \qquad -4 \\
\hline
-6 &= 3m \\
\frac{-6}{3} &= \frac{3m}{3} \\
-2 &= m
\end{aligned}$$

The solution to the equation is $m = -2$.

Objective 2

How many milliliters of water must be added to a beaker containing 200 milliliters of a solution that is 75% ethanol to produce a solution that is 50% ethanol?



- There are 200 milliliters of solution in the beaker to start.
- Let n represent the number of milliliters of water to be added.

  Then $n + 200$ represents the total number of milliliters in the combined solution.
- Represent the amount of ethanol in each solution.

  The ethanol in the original beaker:
  75% of 200 milliliters $= 0.75(200) = 150$

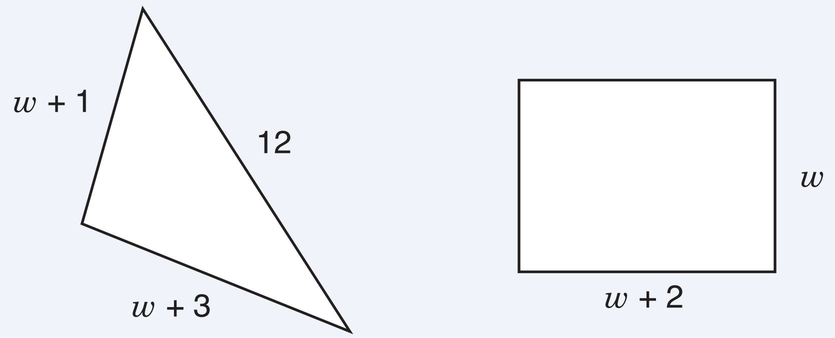
  The ethanol in the combined solution:
  50% of $(n + 200) = 0.50(n + 200)$
- The water added contains no ethanol, so the ethanol in the combined solution equals the ethanol in the beaker originally. Write an equation showing this relationship.

$$\begin{aligned} 0.50(n + 200) &= 150 \\ 0.50n + 100 &= 150 \\ -100 &= -100 \\ \hline 0.5n &= 50 \\ \frac{0.5n}{0.5} &= \frac{50}{0.5} \\ n &= 100 \end{aligned}$$

So 100 milliliters of water must be added to produce a solution that is 50% ethanol.

The length of a rectangle is 2 inches greater than its width. A triangle is drawn such that one of its sides is 1 inch greater than the width of the rectangle, another is 3 inches greater than the width of the rectangle, and the third side is 12 inches long. Find the dimensions of the rectangle if the triangle and rectangle have the same perimeter.

Draw a picture and label the dimensions of the two figures.



Represent the perimeters of the two figures with expressions and simplify the expressions.

Perimeter of triangle:
$$\begin{aligned}(w + 1) + (w + 3) + 12 &=\\ w + w + 1 + 3 + 12 &=\\ 2w + 16\end{aligned}$$

Perimeter of rectangle:
$$\begin{aligned}2w + 2(w + 2) &=\\ 2w + 2w + 4 &=\\ 4w + 4\end{aligned}$$

Write an equation that shows that the two perimeters are equal.

$$\begin{aligned}2w + 16 &= 4w + 4\\ -2w \qquad &= -2w\\ \hline 16 &= 2w + 4\\ -4 &= \qquad -4\\ \hline 12 &= 2w\\ \frac{12}{2} &= \frac{2w}{2}\\ 6 &= w\end{aligned}$$

The width of the rectangle is 6 inches.

The length of the rectangle is $w + 2 = 6 + 2 = 8$ inches.

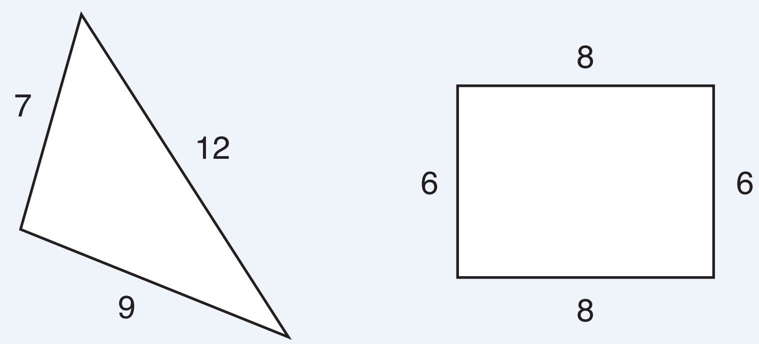
The dimensions of the rectangle are 6 inches by 8 inches.

One side of the triangle is $w + 1 = 6 + 1 = 7$ inches.

Another side is $w + 3 = 6 + 3 = 9$ inches.

The remaining side is 12 inches.

Check to see whether your answer is correct.



Perimeter of triangle: $7 + 9 + 12 = 28$ inches

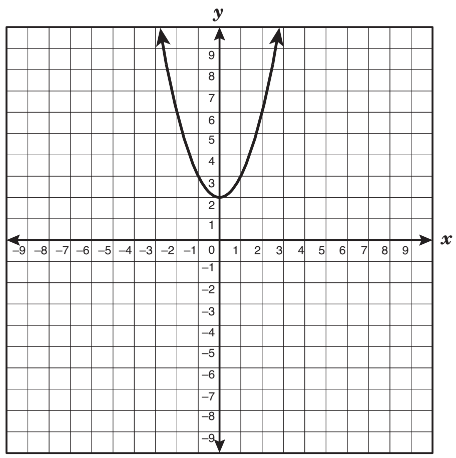
Perimeter of rectangle: $2 \cdot 6 + 2 \cdot 8 = 12 + 16 = 28$ inches

The two figures have equal perimeters.

Now practice what you've learned.

Question 12

Which function is the parent function of the graph below?



A $y = x$

B $y = 2$

C $y = x^2$

D $y = x + 2$

Answer Key: page 282

Question 13

Sam is in line to buy concert tickets and will buy at least 1 ticket. Tickets cost \$20 each, with a limit of 6 tickets per person. If x represents the number of tickets Sam purchases and y represents the amount in dollars he pays for the tickets, this function can be represented by the equation $y = 20x$. What is a reasonable domain for this function?

A $\{x: x \leq 6\}$

B $\{y: 0 \leq y \leq 120\}$

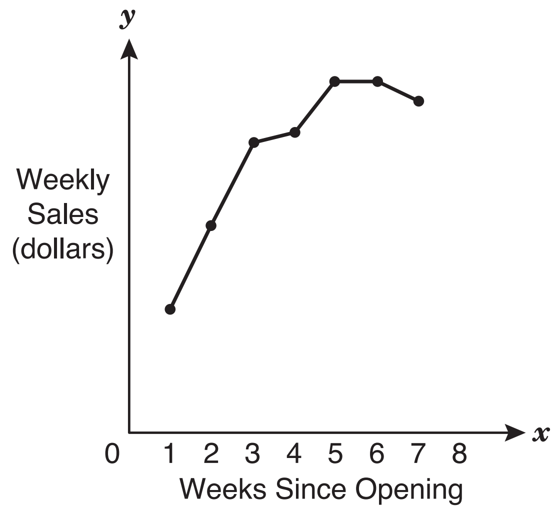
C $\{1, 2, 3, 4, 5, 6\}$

D $\{20, 40, 60, 80, 100, 120\}$

Answer Key: page 282

Question 14

Jim recently opened a store. The graph below shows the store's weekly sales since it opened.



What can be inferred from this graph?

A The store's sales have been increasing since it opened.

B The store's sales increased for the first 2 weeks after it opened, but then they began to decrease.

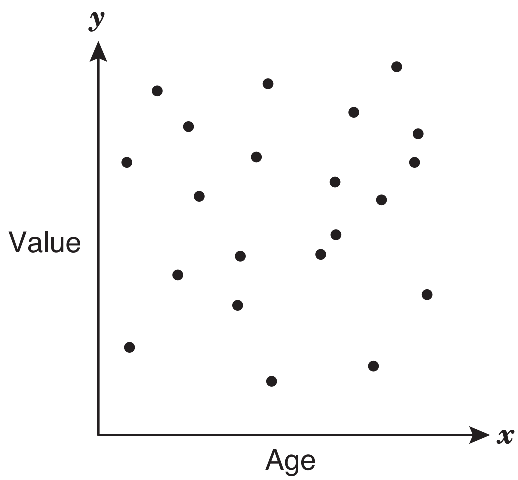
C The store's sales increased for the first 5 weeks after it opened, but then they decreased the next 2 weeks.

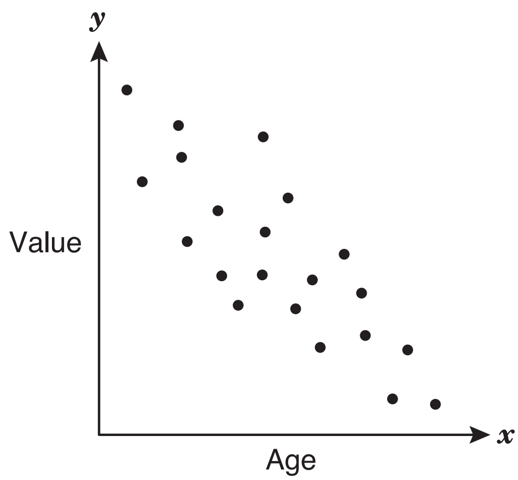
D The store's sales increased for the first 5 weeks after it opened, remained constant the next week, and then decreased the last week shown.

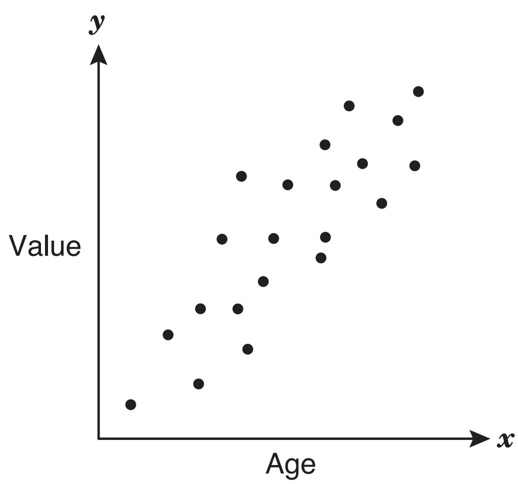
Answer Key: page 282

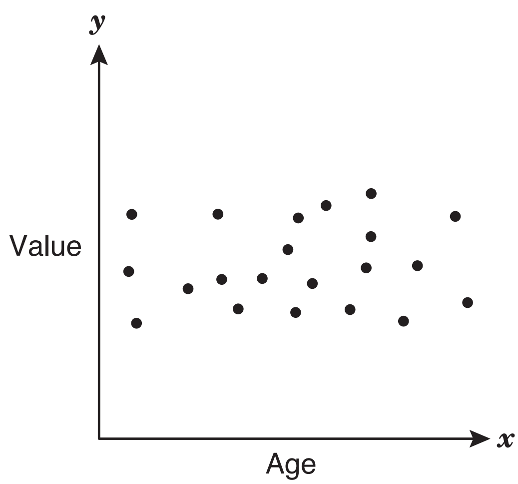
Question 15

As the age of a car increases, its value decreases. Which scatterplot best represents this relationship?

A


C


B


D


Answer Key: page 282

Question 16

The table shows the average tuition per semester at a community college over a 25-year period.

Year	Average Tuition
1980	\$385
1985	\$510
1990	\$605
1995	\$730
2000	\$825
2005	\$950

If the trend continues, what is the best estimate of the tuition at this community college in 2010?

A \$1045

B \$1110

C \$1075

D \$1055

Answer Key: page 282

Question 17

Nancy purchased 5 more pounds of pecans than she did of almonds. Pecans cost \$4.25 per pound, and almonds cost \$5.50 per pound. If x represents the number of pounds of almonds Nancy bought, which expression represents the total cost of the nuts she purchased?

A $19.50x + 21.25$

B $19.50x + 48.75$

C $9.75x + 48.75$

D $9.75x + 21.25$

Answer Key: page 282

Question 18

A personal trainer at a local gym charges the following fees for a one-hour training session for 1 to 4 people.

Number of People	1	2	3	4
Cost (\$)	20	24	28	32

Which equation best represents the relationship between n, the number of people, and c, the cost?

A $n = c + 19$

B $c = 20n$

C $c = 4n + 16$

D $c = 5n + 14$

Answer Key: page 283

Question 19

Which algebraic expression best represents the relationship between the terms in the following sequence and their position, n, in the sequence?

$$0, 6, 16, 30, 48, \ldots$$

A $n - 2$

B $2n^2 - 2$

C $2n - 2$

D $n^2 - 2$

Answer Key: page 283

Question 20

Barbara is 3 times as old as Mark. If the sum of their ages is 52, which equation can be used to find their ages?

A $3x^2 = 52$

B $2x + 3 = 52$

C $4x = 52$

D $3x + 4 = 52$

Answer Key: page 283

Question 21

Which of the following expressions is equivalent to $4(n - 3)(n + 2)$?

A $4n^2 - 4n - 24$

B $4n^2 + 4n + 24$

C $4n^2 - 24$

D $4n^2 + 4n - 24$

Answer Key: page 283

Objective 3

The student will demonstrate an understanding of linear functions.

For this objective you should be able to

- represent linear functions in different ways and translate among their various representations; and
- interpret the meaning of slope and intercepts of a linear function and describe the effects of changes in slope and *y*-intercept in real-world and mathematical situations.

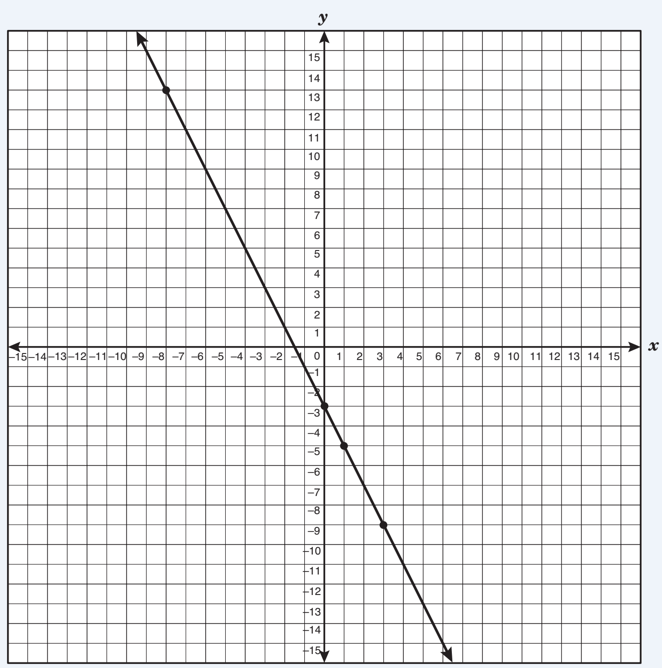
What Is a Linear Function?

A **linear function** is any function whose graph is a nonvertical line.

Is the function represented by the following table of values a linear function?

x	y
−8	13
0	−3
1	−5
3	−9

Graph the ordered pairs in the table on a coordinate grid.



The points lie on a line. Therefore, the function they represent is a linear function.

See Objective 1, page 10, for more information about functions.

Try It

The table below describes a linear relationship.

x	y
−2	−3
4	0
8	2

Does the equation $x - 2y = 4$ represent the same linear function?

To confirm that the equation $x - 2y = 4$ represents the same linear function as the table, substitute the ordered pairs from the table into the equation.

x	y	Does $x - 2y = 4$?	Yes/No
−2	−3	____ $- 2($____$) \stackrel{?}{=} 4$ ____ $+$ ____ $\stackrel{?}{=} 4$ ____ $= 4$	
4	0	____ $- 2($____$) \stackrel{?}{=} 4$ ____ $-$ ____ $\stackrel{?}{=} 4$ ____ $= 4$	
8	2	____ $- 2($____$) \stackrel{?}{=} 4$ ____ $-$ ____ $\stackrel{?}{=} 4$ ____ $= 4$	

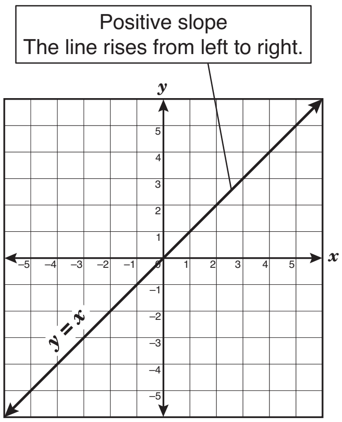
The table and the equation describe the same function.

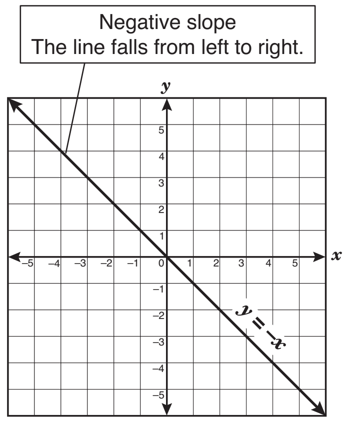
---

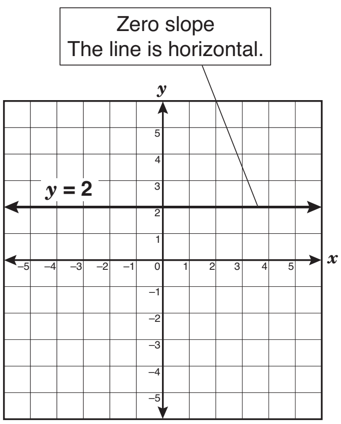
x	y	Does $x - 2y = 4$?	Yes/No
−2	−3	$-2 - 2(-3) \stackrel{?}{=} 4$ $-2 + 6 \stackrel{?}{=} 4$ $4 = 4$	Yes
4	0	$4 - 2(0) \stackrel{?}{=} 4$ $4 - 0 \stackrel{?}{=} 4$ $4 = 4$	Yes
8	2	$8 - 2(2) \stackrel{?}{=} 4$ $8 - 4 \stackrel{?}{=} 4$ $4 = 4$	Yes

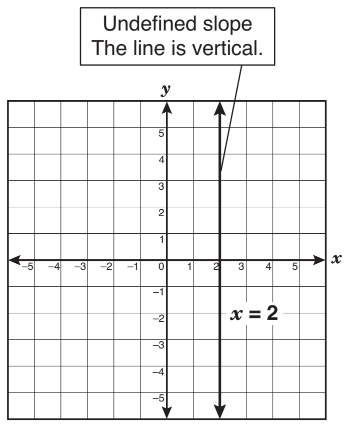
What Is Slope?

The **slope** of a linear graph is its rate of change. The slope shows how fast the graph increases or decreases. The slope of a line can also be described as its steepness, how fast the line rises or falls.





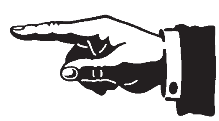




The rate of change of a line is the ratio that compares the change in y-values to the corresponding change in x-values for any two points on the graph.

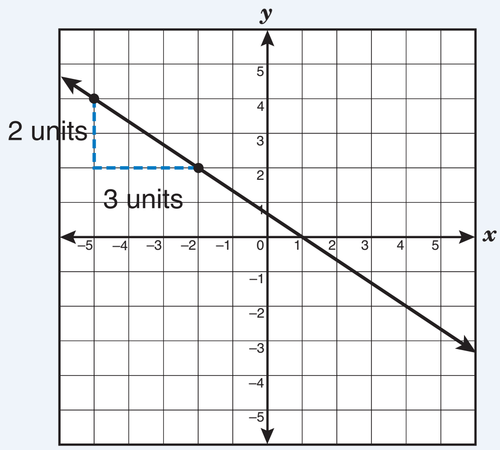
A graph's slope is often described as its **rise** (change in y) over **run** (change in x). To find the slope of a line from its graph:

- Pick any two points on the graph.
- Find the change in y-values, $y_2 - y_1$, or the rise.
  Count the number of units up or down between the two points.
- Find the change in x-values, $x_2 - x_1$, or the run.
  Count the number of units left to right between the two points.
- Determine whether the slope is positive or negative.
  As you go from left to right:
  if the line points up, the slope is positive.
  if the line points down, the slope is negative.
- Write the slope as a ratio: $\frac{\text{rise}}{\text{run}}$ or $\frac{\text{change in } y}{\text{change in } x}$.



Do you see that . . .

In the graph below you can find the slope by counting the change in y-values and the change in x-values between any two points.



The y-value changes 2 units for every 3 units the x-value changes. As you go from left to right, the line points down, so the slope is negative. The rate of change is given by the ratio below.

$$\frac{\text{rise}}{\text{run}} = -\frac{2}{3}$$

The slope of the graph is $-\frac{2}{3}$.

Another way to find the slope is to use the slope formula.

Slope Formula

For any two points (x_1, y_1) and (x_2, y_2) on a graph, the slope, m, of the line that passes through them is:

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

← change in y-values
← change in x-values

What is the slope of the line passing through the points (8, 3) and (6, −2)?

Let (x_1, y_1) be (8, 3) and (x_2, y_2) be (6, −2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 3}{6 - 8} = \frac{-5}{-2} = \frac{5}{2}$$

The slope of the line is $\frac{5}{2}$.

The set of ordered pairs below represents a linear function. What is the rate of change of the function?

$$\{(0, 5), (1, 3), (2, 1), (3, -1), (4, -3)\}$$

Since this is a linear function, you can use any two ordered pairs belonging to the function to find its rate of change.

- You could use the ordered pairs (0, 5) and (2, 1).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 5}{2 - 0} = \frac{-4}{2} = -2$$

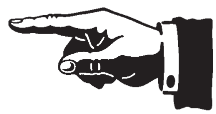
The rate of change of the function is -2.

- Or you could use the ordered pairs (1, 3) and (4, -3).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 3}{4 - 1} = \frac{-6}{3} = -2$$

The rate of change of the function is still -2.

The rate of change, or slope, of a linear function does not depend on which two points you pick to calculate the slope.



Do you see that . . .

Try It

What is the slope of the line passing through the points (5, 2) and (2, 8)?

Let (x_1, y_1) be (5, 2) and (x_2, y_2) be (2, 8).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - \square}{2 - \square} = \frac{\square}{\square} = \underline{\qquad\qquad}$$

If the slope is -2, then the graph's y-values decrease ________ units for every 1 unit the x-values increase.

Let (x_1, y_1) be (5, 2) and (x_2, y_2) be (2, 8).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 2}{2 - 5} = \frac{6}{-3} = -2$$

If the slope is -2, then the graph's y-values decrease 2 units for every 1 unit the x-values increase.

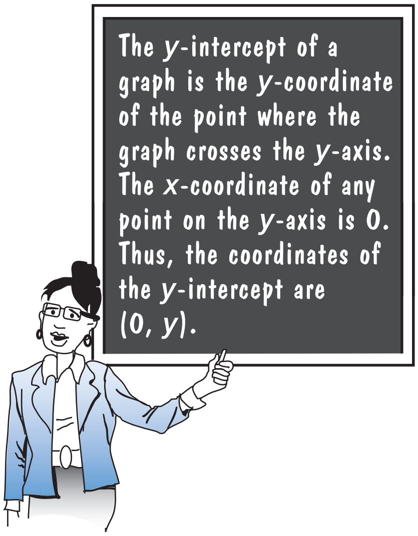
Objective 3

What Is the Slope-Intercept Form of a Linear Function?

One form of the equation of a linear function is $y = mx + b$. This form is called the **slope-intercept form** of a linear function.

When the equation of a linear function is written in the form $y = mx + b$:

- $\boldsymbol{m}$ is the **slope** of the graph of the function; m is the value by which the x-term is being multiplied.
- $\boldsymbol{b}$ is the y-coordinate of the ***y*-intercept** of the graph of the function; b is the value that is being added to the x-term.



What is the slope of the graph of $3y = 2x - 6$? What is the y-intercept of the graph?

First write the equation in slope-intercept form, $y = mx + b$.

$$3y = 2x - 6$$

$$\frac{3y}{3} = \frac{2}{3}x - \frac{6}{3}$$

$$y = \frac{2}{3}x - 2$$

Read the values of m and b from the equation.

For the function $y = \frac{2}{3}x - 2$,

- the value by which the x-term is being multiplied is $\frac{2}{3}$, so $m = \frac{2}{3}$.
- the value that is being added to the x-term is -2, so $b = -2$.

The slope, m, of the graph of the function is $\frac{2}{3}$.

The y-coordinate of the y-intercept, b, of the function is -2. Therefore, the coordinates of the y-intercept are $(0, -2)$.

What are the slope and y-intercept of the graph of $5x - 2y = 30$?

- To determine the slope and y-intercept, transform the equation $5x - 2y = 30$ into slope-intercept form, $y = mx + b$.

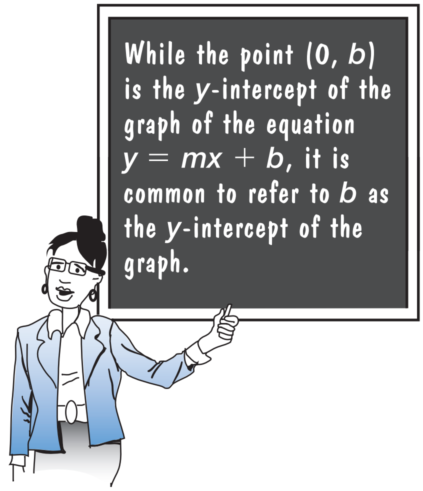
$$\begin{aligned} 5x - 2y &= 30 \\ -5x \qquad &= -5x \\ \hline -2y &= -5x + 30 \\ \frac{-2y}{-2} &= \frac{-5}{-2}x + \frac{30}{-2} \\ y &= \frac{5}{2}x - 15 \end{aligned}$$

The equation is now in the form $y = mx + b$.

- Read the values of m and b from the revised equation. For this function, $m = \frac{5}{2}$ and $b = -15$.

The slope, m, of the graph of the function is $\frac{5}{2}$.

The y-intercept, b, of the graph of the function is -15.



Objective 3

Try It

Find the slope and y-intercept of the graph of the function $6x + 2y = 12$.

To determine the slope and y-intercept, transform the equation $6x + 2y = 12$ into slope-intercept form, $y = mx + b$.

$$\begin{array}{rcl} 6x \quad + 2y & = & 12 \\ -\_\_\_\_\_\_ \qquad & = & -\_\_\_\_\_\_ \\ \hline 2y & = & \_\_\_\_\_\_ + 12 \end{array}$$

$$\frac{2y}{\square} = \frac{-6x}{\square} + \frac{12}{\square}$$

$$y = \_\_\_\_\_\_ x + \_\_\_\_\_\_$$

Read the values of ________ and ________ from the revised equation.

For this function, m = ________ and b = ________.

The slope of the graph of the function is ________.

The y-coordinate of the y-intercept of the graph of the function is ________.

$$\begin{array}{rcl} 6x + 2y & = & 12 \\ -6x \qquad & = & -6x \\ \hline 2y & = & -6x + 12 \end{array}$$

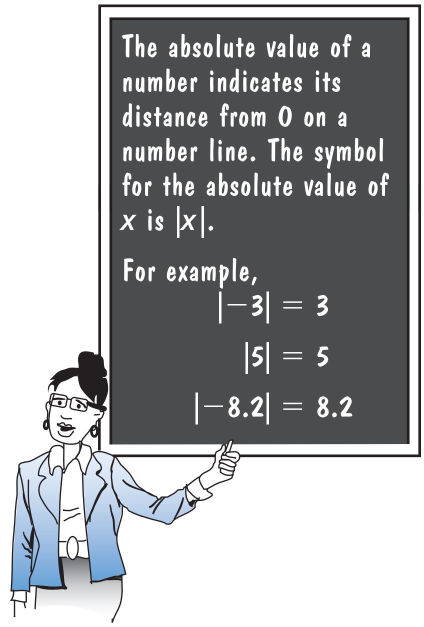
$$\frac{2y}{2} = \frac{-6x}{2} + \frac{12}{2}$$

$$y = -3x + 6$$

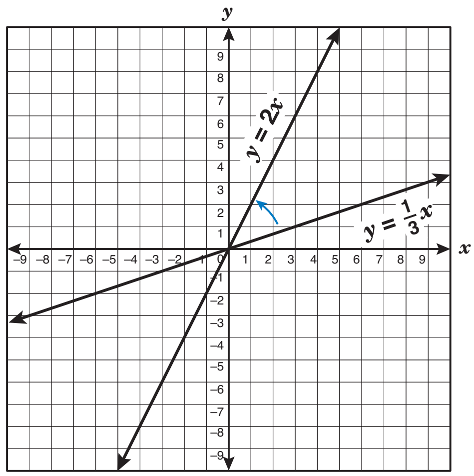
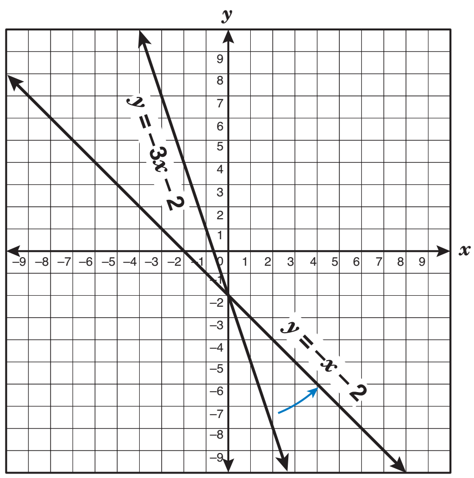
Read the values of m and b from the revised equation. For this function, $m = -3$ and $b = 6$. The slope of the graph of the function is -3. The y-coordinate of the y-intercept of the graph of the function is 6.

What Are the Effects on the Graph of a Linear Function If the Values of *m* and *b* Are Changed in the Equation $y = mx + b$?

In the equation $y = mx + b$, m represents the slope of the graph, and b represents the y-intercept of the graph. Changing either of these two constants, m or b, will produce a new graph. The new graph is related to the original graph in predictable ways.



Change in Slope, *m*

Function 1	Function 2	Effect
$y = \frac{1}{3}x$	$y = 2x$	 Since $\lvert 2 \rvert > \lvert \frac{1}{3} \rvert$, the graph of function 2 is steeper than the graph of function 1.
$y = -3x - 2$	$y = -x - 2$	 Since $\lvert -1 \rvert < \lvert -3 \rvert$, the graph of function 2 is less steep than the graph of function 1.

Do you see that . . .

If the function $y = x + 5$ is changed to $y = 3x + 5$, how would the graph of the new function compare with the original graph?

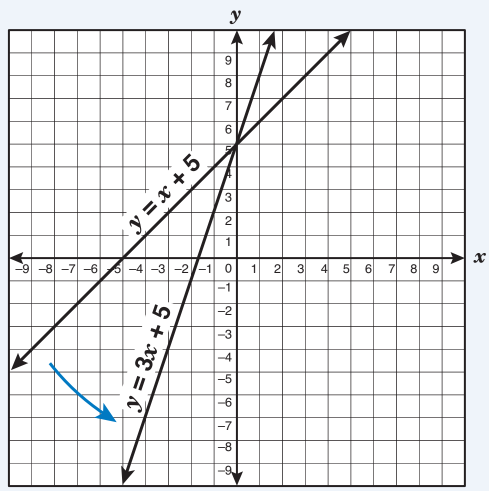
The equations are both in slope-intercept form, $y = mx + b$. In this form, m represents the graph's slope.

- Since x is the same as $1x$, the coefficient $m = 1$.

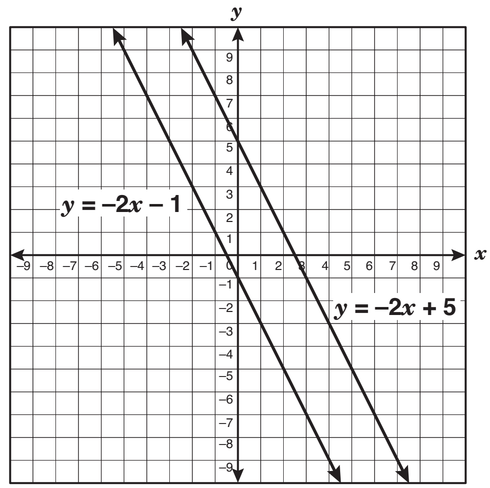
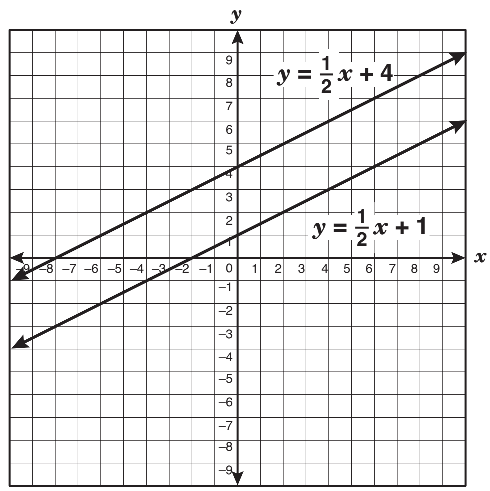
  The slope of the original function is 1.

- If the equation is changed to $y = 3x + 5$, the new value of m will be 3.

The graph of the new function will be steeper because $|3| > |1|$.



Change in *y*-Intercept, *b*

Function 1	Function 2	Effect
$y = -2x - 1$	$y = -2x + 5$	 Function 2 crosses the *y*-axis at a point 6 units higher than function 1 because $5 - (-1) = 6$.
$y = \frac{1}{2}x + 4$	$y = \frac{1}{2}x + 1$	 Function 2 crosses the *y*-axis at a point 3 units lower than function 1 because $1 - 4 = -3$.

Objective 3

If the y-intercept of the function $y = \frac{3}{2}x + 4$ were decreased by 3, what would be the equation of the new function?

The equation is in slope-intercept form, $y = mx + b$. In this form, b represents the graph's y-intercept.

- The y-intercept of the original function is 4.
- If the y-intercept of the function were decreased by 3, it would be $4 - 3 = 1$. The new value for b would be 1.

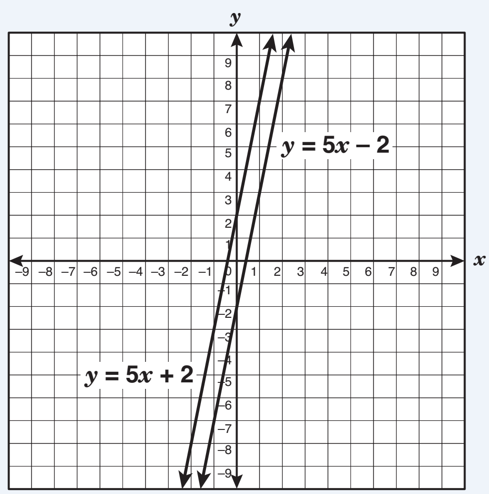
The equation of the new function would be $y = \frac{3}{2}x + 1$.

The function $y = 5x + 2$ was changed to $y = 5x - 2$. How would the graph of the new function compare with the graph of the original function?

When the equation of a linear function is written in the form $y = mx + b$, b represents the y-intercept of the graph.

- The value for b in the first function is 2. Its graph crosses the y-axis at $(0, 2)$.
- The value for b in the second function is -2. Its graph crosses the y-axis at $(0, -2)$.

Since $-2 - 2 = -4$, the graph of $y = 5x - 2$ is 4 units below the graph of $y = 5x + 2$.



Try It

If the slope of the function $y = 0.5x + 4$ is increased by 3, how will the graph of the new function compare with the graph of the original function?

The equation of the given function is in slope-intercept form.

The slope of the original line is ________, and the y-coordinate of the y-intercept is ________.

If the slope is increased by 3, the new slope becomes

________ + ________ = ________.

The new line will have the ________________ y-intercept as the original line.

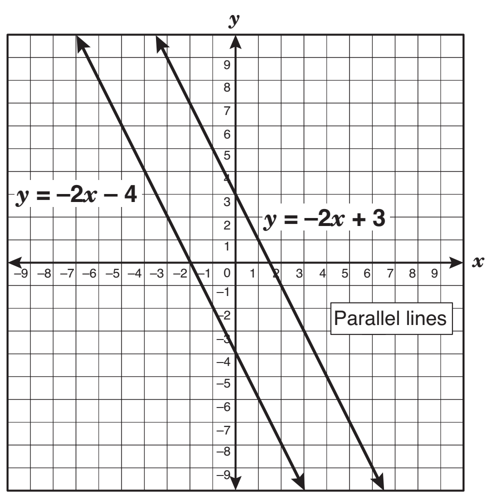
Since $|3.5| > |0.5|$, the graph of the new line will be ________________ than the original line.

The slope of the original line is 0.5, and the y-coordinate of the y-intercept is 4. If the slope is increased by 3, the new slope becomes $0.5 + 3 = 3.5$. The new line will have the same y-intercept as the original line.
Since $|3.5| > |0.5|$, the graph of the new line will be steeper than the original line.

Slopes of lines can tell you whether the lines are parallel or perpendicular.

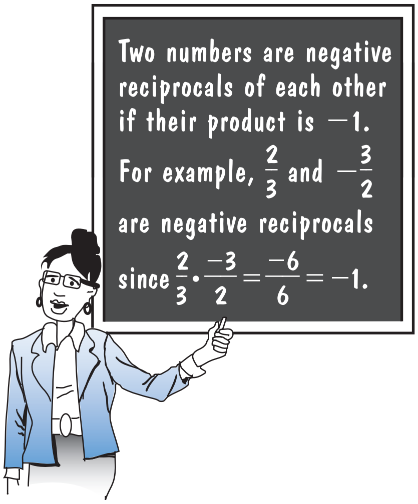
- If two lines are parallel, then they have the same slope, and their equations have the same value for m.

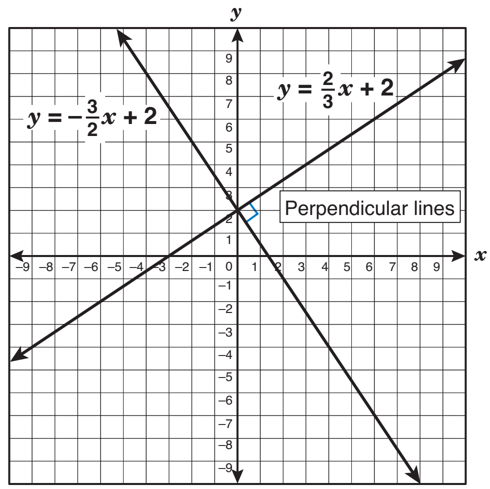
Look at the graphs of these two equations: $y = -2x - 4$ and $y = -2x + 3$.



- If two lines are perpendicular, then they have negative reciprocal slopes, and their equations have negative reciprocal values for m.

  Look at the graphs of these two equations: $y = \frac{2}{3}x + 2$ and $y = -\frac{3}{2}x + 2$.





How Do You Write Linear Equations?

You can write linear equations in slope-intercept form, $y = mx + b$, or in **standard form**, $Ax + By = C$. In standard form, A, B, and C are integers, and A is usually greater than zero.

You can find the equation of a line given any of the following information:

- the slope and the y-intercept of the graph
- the slope and a point on the graph
- two points on the graph

Given the slope and the y-intercept

Identify the values for both m, the slope, and b, the y-intercept. Write the equation in slope-intercept form, $y = mx + b$, using these values.

What is the equation of the line with a slope of 4 and a y-intercept of -2?

Find the values you should substitute into the equation $y = mx + b$.

- If the slope is 4, then $m = 4$.
- If the y-intercept is -2, then $b = -2$.

The equation of the line is:

$$y = mx + b$$
$$y = 4x - 2$$

Try It

Write the equation of the line with a slope of $-\frac{1}{5}$ and a y-coordinate of the y-intercept of 8.

If the slope is $-\frac{1}{5}$, then $m =$ ________.

If the y-coordinate of the y-intercept is 8, then $b =$ ________.
The equation of the function is:

$$y = mx + b$$

$$y = \_\_\_\_\_\_ x + \_\_\_\_\_\_$$

If the slope is $-\frac{1}{5}$, then $m = -\frac{1}{5}$. If the y-coordinate of the y-intercept is 8, then $b = 8$.

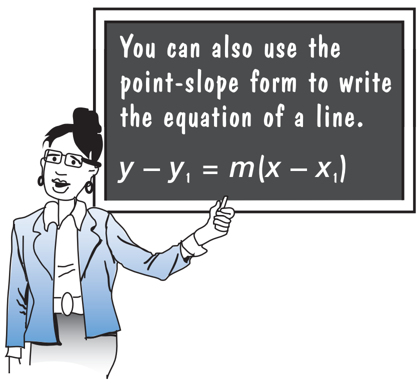
The equation of the function is:

$$y = mx + b$$

$$y = -\frac{1}{5}x + 8$$

Given the slope and a point on the graph

- Substitute the given values (the x- and y-coordinates of the given point and m, the slope) into the slope-intercept form of the equation, $y = mx + b$.
- Solve the equation for b.
- Substitute the values for m and b into the slope-intercept form, $y = mx + b$.



What is the equation of the line with a slope of $\frac{1}{3}$ that passes through the point $(-3, -6)$?

- Substitute $x = -3$, $y = -6$, and $m = \frac{1}{3}$ into the equation $y = mx + b$.

$$y = mx + b$$

$$-6 = \frac{1}{3}(-3) + b$$

- Solve for b.

$$-6 = \frac{1}{3}(-3) + b$$

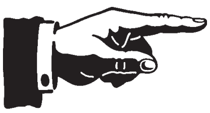
$$-6 = -1 + b$$

$$\underline{+1 = +1}$$

$$-5 = b$$

Objective 3

Do you see that . . .



- Substitute the given value for m, $\frac{1}{3}$, and the value you have just found for b, -5, into the slope-intercept form of the equation.

$$y = mx + b$$

$$y = \frac{1}{3}x - 5$$

The equation of the line is $y = \frac{1}{3}x - 5$.

This equation could also be written in standard form, $Ax + By = C$, in which A is usually positive.

$$\begin{aligned} y &= \tfrac{1}{3}x - 5 \\ (3)y &= 3(\tfrac{1}{3}x) - 3(5) \\ 3y &= x - 15 \\ -x \qquad &= -x \\ \hline -x + 3y &= -15 \end{aligned}$$

To change -1 to a positive value, multiply both sides of the equation by -1.

$$x - 3y = 15$$

In standard form, the equation of this line is $x - 3y = 15$.

Try It

Write the equation of the linear function whose graph has a slope of 5 and contains the point $(-1, 12)$.

Begin by substituting the given values into the slope-intercept form.

The slope of the line is ________; therefore, $m =$ ________.

The line passes through the point (________, ________); therefore,

$x =$ ________ and $y =$ ________.

$$y = mx + b$$

$$\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_ \cdot \_\_\_\_\_\_\_\_ + b$$

$$\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_ + b$$

$$\underline{+5 = \quad +5 \qquad}$$

$$\_\_\_\_\_\_\_\_ = \qquad\qquad b$$

Finally, substitute the given value for m, ________, and the value you have just found for b, ________, into the slope-intercept form of the equation.

$$y = mx + b$$

$$y = \_\_\_\_\_\_\_\_x + \_\_\_\_\_\_\_\_$$

The slope of the line is 5; therefore, $m = 5$. The line passes through the point $(-1, 12)$; therefore, $x = -1$ and $y = 12$.

$$y = mx + b$$

$$12 = 5 \cdot -1 + b$$

$$12 = -5 + b$$

$$\underline{+5 = +5 \qquad}$$

$$17 = \qquad b$$

Finally, substitute the given value for m, 5, and the value you have just found for b, 17, into the slope-intercept form of the equation.

$$y = mx + b$$

$$y = 5x + 17$$

Objective 3

Given two points on the graph

- Use the x-coordinates and y-coordinates of the two given points to find the slope, m, of the line.
- Substitute the coordinates of one of the known points and the slope you just found into the slope-intercept form of the equation, $y = mx + b$.
- Solve the equation for b.
- Substitute m and b into the slope-intercept form, $y = mx + b$.

Find the equation of the line passing through the points $(-2, -6)$ and $(2, 14)$.

- Begin by finding the slope of the graph using the slope formula. The line passes through the points $(-2, -6)$ and $(2, 14)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - (-6)}{2 - (-2)} = \frac{20}{4} = 5$$

If $m = 5$, the slope of the line is 5.

- Next, substitute the coordinates of either of the given points and the value you just found for m, 5, into the slope-intercept form of the equation. Find the value of b. Use $(-2, -6)$ or $(2, 14)$.

$x = -2, y = -6$	$x = 2, y = 14$
$y = mx + b$	$y = mx + b$
$-6 = 5(-2) + b$	$14 = 5(2) + b$
$-6 = -10 + b$	$14 = 10 + b$
$+10 = +10$	$-10 = -10$
$4 = b$	$4 = b$

- You now know the values of m and b; substitute them into the slope-intercept form of the equation.

$$y = mx + b$$
$$y = 5x + 4$$

The equation of the line passing through the points $(-2, -6)$ and $(2, 14)$ is $y = 5x + 4$.

Try It

Write the equation of the linear function that contains the points (2, 5) and (−3, 15).

Begin by finding the ______________ of the graph.

The line passes through the points (________, ________) and (________, ________).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\boxed{} - \boxed{}}{\boxed{} - \boxed{}} = \frac{\boxed{}}{\boxed{}} = \underline{\qquad\qquad}$$

The slope of the line is ________.

Next, substitute the coordinates of one of the given points, (2, ________), and the slope you just found, ________, into the slope-intercept form of the equation of a line to find the value of b.

$$\begin{aligned} y &= mx + b \\ \underline{\qquad\quad} &= \underline{\qquad\quad} \cdot \underline{\qquad\quad} + b \\ 5 &= \underline{\qquad\quad} + b \\ +4 &= +4 \\ \hline \underline{\qquad\quad} &= \qquad b \end{aligned}$$

Finally, substitute the values of m and b into the slope-intercept form.

$$y = \underline{\qquad\quad}x + \underline{\qquad\quad}$$

Begin by finding the slope of the graph. The line passes through the points (2, 5) and (−3, 15).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15 - 5}{-3 - 2} = \frac{10}{-5} = -2$$

The slope of the line is −2.

Next, substitute the coordinates of one of the given points, (2, 5), and the slope you just found, −2, into the slope-intercept form of the equation of a line to find the value of b.

$$\begin{aligned} y &= mx + b \\ 5 &= -2 \cdot 2 + b \\ 5 &= -4 + b \\ +4 &= +4 \\ \hline 9 &= \qquad b \end{aligned}$$

Finally, substitute the values of m and b into the slope-intercept form.

$$y = -2x + 9$$

Objective 3

How Do You Find the x-Intercept and y-Intercept for the Graph of an Equation?

Finding the x-intercept

The x-intercept is the point where the graph of a line intersects the x-axis. The x-intercept has the coordinates $(x, 0)$.

To find the x-coordinate of the x-intercept, substitute $y = 0$ into the equation and solve for x. The value of x is the x-coordinate of the x-intercept. The graph of the line intersects the x-axis at the point $(x, 0)$.

What is the x-intercept of the graph of $2x - 5y = 20$?

Substitute $y = 0$ into the equation and solve for x.

$$\begin{aligned} 2x - 5y &= 20 \\ 2x - 5(0) &= 20 \\ 2x - 0 &= 20 \\ 2x &= 20 \\ x &= 10 \end{aligned}$$

The x-coordinate of the x-intercept of the graph is 10. The graph of the line intersects the x-axis at $(10, 0)$.

Finding the y-intercept

The y-intercept is the point where the graph of a line intersects the y-axis. The y-intercept has the coordinates $(0, y)$.

One way to find the y-intercept is to write the equation in slope-intercept form, $y = mx + b$.

The value of b is the y-coordinate of the y-intercept. The graph of the line intersects the y-axis at the point $(0, b)$.

Another way to find the y-intercept is to substitute $x = 0$ into the equation and solve for y.

Here are two ways to find the y-intercept of the graph of $7y + 14 = 2x$.

Write the equation in the form $y = mx + b$.

$$\begin{array}{rcr} 7y + 14 & = & 2x \\ -14 & = & -14 \\ \hline 7y & = & 2x - 14 \end{array}$$

$$\frac{7y}{7} = \frac{2}{7}x - \frac{14}{7}$$

$$y = \frac{2}{7}x - 2$$

Therefore, $b = -2$

Substitute $x = 0$ and solve for y.

$$\begin{array}{rcr} 7y + 14 & = & 2x \\ 7y + 14 & = & 2(0) \\ 7y + 14 & = & 0 \\ -14 & = & -14 \\ \hline 7y & = & -14 \end{array}$$

$$\frac{7y}{7} = -\frac{14}{7}$$

$$y = -2$$

The y-intercept of the graph is -2. The graph intersects the y-axis at $(0, -2)$.

Find the x- and y-intercepts of $3x + 6y = 12$.

To find the x-intercept, substitute $y = 0$ into the equation and solve for x.

$$\begin{aligned} 3x + 6y &= 12 \\ 3x + 6(0) &= 12 \\ 3x + 0 &= 12 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

The x-intercept is 4. The graph intersects the x-axis at $(4, 0)$.

Find the y-intercept by writing the equation in slope-intercept form.

$$\begin{array}{rcr} 3x + 6y & = & 12 \\ -3x & = & -3x \\ \hline 6y & = & -3x + 12 \end{array}$$

$$\frac{6y}{6} = \frac{-3}{6}x + \frac{12}{6}$$

$$y = -\frac{1}{2}x + 2$$

The y-intercept is 2. The graph intersects the y-axis at $(0, 2)$.

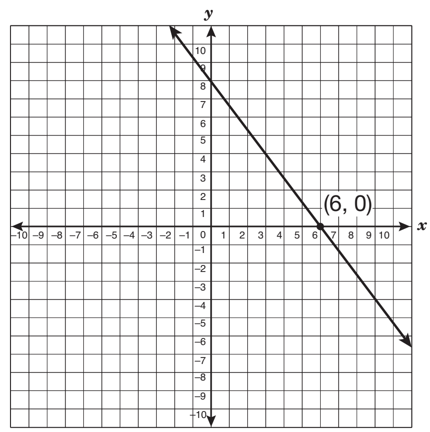
Or, find the y-intercept by substituting $x = 0$ into the equation.

$$\begin{aligned} 3x + 6y &= 12 \\ 3(0) + 6y &= 12 \\ 0 + 6y &= 12 \\ 6y &= 12 \\ y &= 2 \end{aligned}$$

The y-intercept is 2. The graph intersects the y-axis at $(0, 2)$.

Objective 3

For any equation, such as $y = -\frac{4}{3}x + 8$, a **root** is a value of x that makes $y = 0$. For any function, a value of x that makes $f(x)$ equal to zero is called a **zero of a function**. Both the root of an equation and the zero of a function can be found by locating the x-coordinate of the x-intercept of the graph of a function, the point at which the graph intercepts the x-axis.

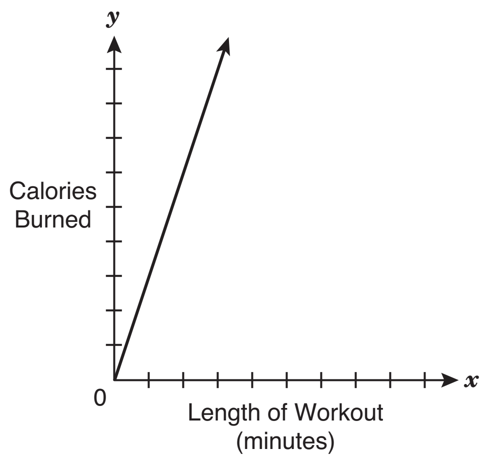
Roots of Equations	Zeros of Functions	x-Intercepts of Graphs
$y = -\frac{4}{3}x + 8$	$f(x) = -\frac{4}{3}x + 8$	$y = -\frac{4}{3}x + 8$
$0 = -\frac{4}{3}x + 8$ $\underline{-8 = \quad\quad -8}$ $-8 = -\frac{4}{3}x$ $(-\frac{3}{4}) \cdot (-8) = (-\frac{3}{4}) \cdot (-\frac{4}{3})x$ $6 = x$ $x = 6$	$0 = -\frac{4}{3}x + 8$ $\underline{-8 = \quad\quad -8}$ $-8 = -\frac{4}{3}x$ $(-\frac{3}{4}) \cdot (-8) = (-\frac{3}{4}) \cdot (-\frac{4}{3})x$ $6 = x$ $x = 6$	
The root of $y = -\frac{4}{3}x + 8$ is 6.	The zero of $f(x) = -\frac{4}{3}x + 8$ is 6.	The coordinates of the x-intercept of $y = -\frac{4}{3}x + 8$ are (6, 0).

How Do You Interpret the Meaning of Slopes and Intercepts?

To interpret the meaning of the slope or the x- or y-intercept of a function in a real-life problem, follow these guidelines.

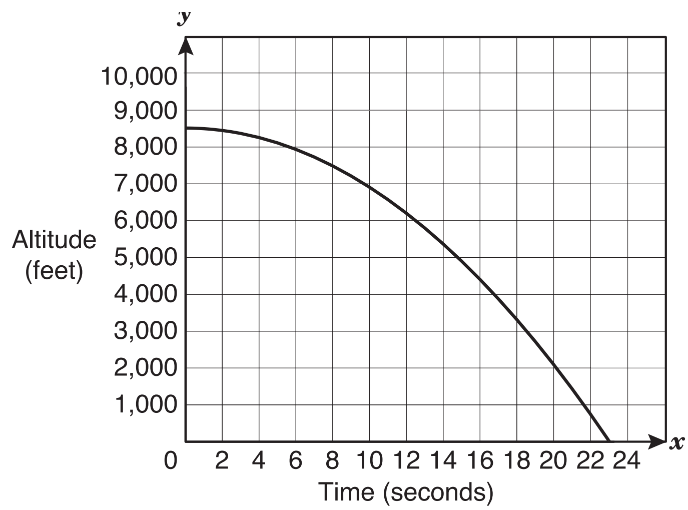
- The slope of the function is the function's rate of change. A graph's slope tells you how fast the function's dependent variable is changing for every unit change in the independent variable.

  For example, if a graph compares calories you burn to the number of minutes you work out, the slope tells you the rate at which you are burning calories, or how many calories you burn per minute.



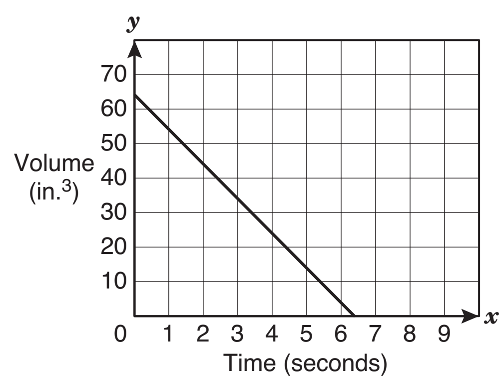
- The y-intercept is the point where the graph of a function intersects the y-axis. The y-intercept has the coordinates $(0, y)$. It is the point in the function where the independent quantity, x, has a value of 0. The y-intercept is often the starting point in a problem situation.

  For example, if a graph describes the height of a free-falling object in terms of the number of seconds the object falls, then the y-intercept tells you the object's altitude when it began to fall, when $t = 0$.

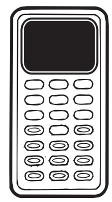


- The x-intercept is the point where the graph of a function intersects the x-axis. The x-intercept has the coordinates $(x, 0)$. It is the point in the problem where the dependent quantity, y, has a value of 0.

  For example, if a graph describes the volume of a balloon losing air at a constant rate in terms of the number of seconds the air escapes, then the x-intercept (when the volume of the balloon is 0) tells you the number of seconds the balloon takes to deflate completely.



The domestic production of crude oil in the United States from 1985 to 2002 can be approximated by the following graph.



What do the slope and y-intercept of this graph tell you about the average U.S. production of crude oil during this time period?

- The y-intercept of the line is at 10.6. This represents 10.6 million barrels of oil being produced per day. Since the x-axis represents "Years Since 1985," the y-intercept tells you that the number of barrels of oil being produced per day in 1985 was 10.6 million barrels.

- The slope of the graph is negative because the line falls to the right. This means that the United States has been producing less crude oil each year, and the decline in oil production has been at a fairly steady rate.

  By estimating the coordinates of any two points on the graph, you can approximate this rate of decline in oil production.

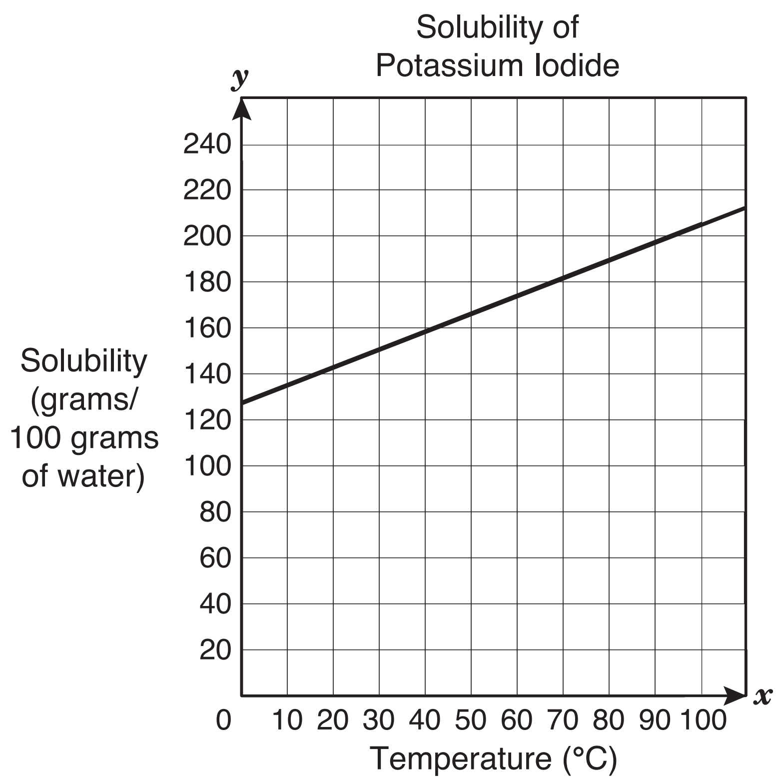
  The following two points appear to be on the graph: (0, 10.6) and (17, 8.2).

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8.2 - 10.6}{17 - 0} = \frac{-2.4}{17} \approx -0.14$$

The slope of -0.14 represents a decline in average oil production of -0.14 million barrels per day per year (or approximately 140,000 barrels per day fewer each year).

Try It

The graph below describes the number of grams of potassium iodide that will dissolve in 100 grams of water in terms of the temperature of the water in degrees Celsius.



What does the slope of this graph tell you about the solubility of potassium iodide?

The slope of the graph is ______________. It rises to the right.

This means that as the temperature of the water increases, the number of grams of potassium iodide that will dissolve in 100 grams of water

________________.

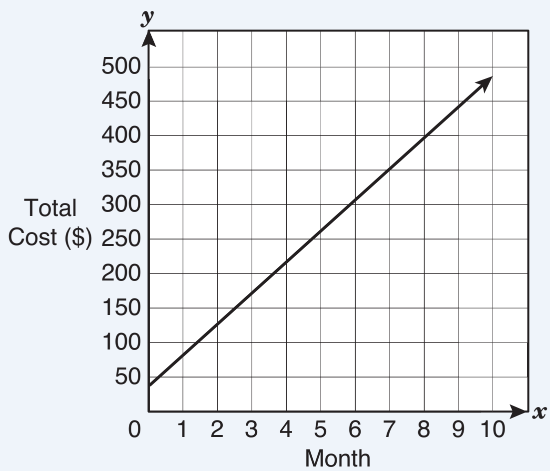
The slope of the graph is positive. It rises to the right. This means that as the temperature of the water increases, the number of grams of potassium iodide that will dissolve in 100 grams of water increases.

How Do You Predict the Effects of Changing Slopes and *y*-Intercepts in Applied Situations?

Many real-life problems can be modeled with linear functions. To analyze such problems, it is often helpful to identify the slope and the y-intercept of the linear function. Interpreting the meaning of these values will help you predict the effect changing them will have on the quantities in the problem.

- If the slope is changed, the rate of change in the problem will increase or decrease.
- If the y-intercept is changed, an initial condition will change.

A cable-television company charges a $35 initial installation fee and then $45 per month for cable service. The graph below describes the total cost of this cable service in terms of the number of months it is used.



If the company discounted the charge to $39 per month, how would this change affect the slope and y-intercept of the graph?

The price per month is the rate of change of the function. This is the slope of the graph. Decreasing the monthly rate from $45 to $39 will decrease the slope of the graph. This change would make the graph less steep.

The discount does not affect the installation fee. The y-intercept of the graph would remain at $35.

Try It

Bruce purchased a used truck for $8200. Trucks of this model are expected to decrease in value by $960 each year. How would a graph showing the value of the truck over time change if Bruce had purchased the same truck for $9200 instead of $8200?

The ______________ of the graph represents the rate of change of the function. The rate of change of the original function is −$_______ per year.

The rate of change of the new function will be −$_______ per year.

The ______________ of the graph does not change.

The ______________ of the graph represents the purchase price of the truck.

The y-intercept of the original graph is (_______, _______).

Since the initial purchase price changes, the y-intercept of the new graph will be (_______, _______).

This would cause the graph to be shifted _______ by $_______.

The slope of the graph represents the rate of change of the function. The rate of change of the original function is −$960 per year. The rate of change of the new function will be −$960 per year. The slope of the graph does not change.

The y-intercept of the graph represents the purchase price of the truck. The y-intercept of the original graph is (0, 8200). Since the initial purchase price changes, the y-intercept of the new graph will be (0, 9200). This would cause the graph to be shifted up by $1000.

How Do You Solve Problems Involving Direct Variation or Proportional Change?

If a quantity y varies directly with a quantity x, then the linear equation representing the relationship between the two quantities is $y = kx$. In this equation, k is called the **proportionality constant**.

To say "y varies directly with x" is to say "y is directly proportional to x."

If the equation $y = kx$ were graphed, k would be the slope of the graph.

In a science lab Kim found that the distance a spring stretched was directly proportional to the mass she hung from it.

If the spring Kim used stretched 1.5 centimeters when 25 grams were hung from it, how many centimeters will it stretch if 125 grams are hung from it?

- Write an equation that compares the number of centimeters the spring stretches to the number of grams hung from it.

  Let d = the number of centimeters the spring stretches.

  Let g = the number of grams hung from it.

  The direct variation equation is $d = kg$.

- Substitute the known values for d and g to find the proportionality constant, k.

  1.5 centimeters ($d = 1.5$) for 25 grams ($g = 25$)

  $$d = kg$$
  $$1.5 = k(25)$$
  $$k = \frac{1.5}{25} = 0.06$$

  If $k = 0.06$, then the proportionality constant is 0.06.
  If $k = 0.06$, the slope of the graph is 0.06.

- Find the number of centimeters the spring will stretch when 125 grams are hung from it. Substitute $g = 125$ into the equation $d = 0.06g$.

  $$d = 0.06g$$
  $$d = 0.06(125)$$
  $$d = 7.5$$

The spring will stretch 7.5 centimeters if a mass of 125 grams is hung from it.

Now practice what you've learned.

Question 22

Which of the following cannot be described by a linear function?

A The amount spent on n shirts that cost \$20 each

B The number of miles driven for h hours at a constant speed of 60 miles per hour

C The total amount saved after making an initial deposit of \$100 and depositing \$30 a month thereafter for n months

D The area of a rectangular garden that is x feet wide and has a length equal to twice its width

Answer Key: page 283

Question 23

Which equation describes the same linear function as the one shown in the table below?

x	-5	3	0	1	4
y	4	0	$\frac{3}{2}$	1	$-\frac{1}{2}$

A $y = \frac{3}{2}x + \frac{3}{2}$

B $y = \frac{3}{2}x - 3$

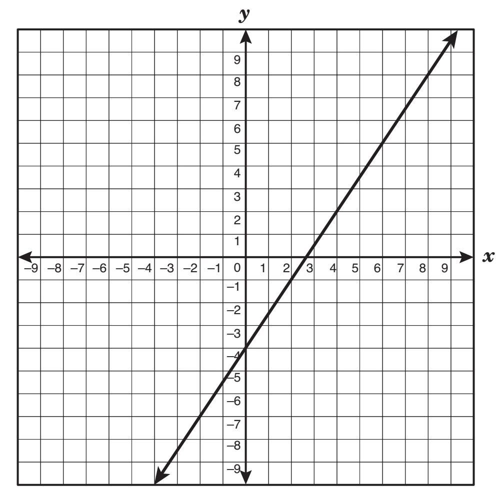
C $y = -\frac{1}{2}x - 3$

D $y = -\frac{1}{2}x + \frac{3}{2}$

Answer Key: page 283

Question 24

What is the rate of change for the function graphed below?



A $-\frac{3}{2}$

B $\frac{3}{2}$

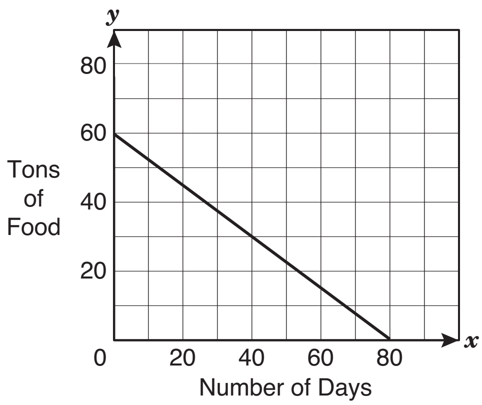
C $\frac{2}{3}$

D $-\frac{2}{3}$

Answer Key: page 284

Question 25

The graph below shows the relationship between the number of tons of elephant food remaining in the zoo's food storage bin and the number of days since the food was last purchased.



How many tons of elephant food were in the bin immediately after the last purchase of food was placed in the bin?

A 90 tons

B 40 tons

C 60 tons

D 50 tons

Answer Key: page 284

Question 26

The junior class will spend \$500 on decorations for a school party. The juniors will purchase x cases of balloons and y cases of streamers. Balloons cost \$25 for each case, and streamers cost \$125 for each case. If this information is graphed as a linear equation, what is the x-intercept of the line?

A The x-intercept is (4, 0).

B The x-intercept is (20, 0).

C The x-intercept is (25, 0).

D The x-intercept is (125, 0).

Answer Key: page 284

Question 27

Which of the following pairs of equations describes a pair of parallel lines?

A $y = -x + 1$ and $y = x + 1$

B $y = -x - 1$ and $y = -2x - 1$

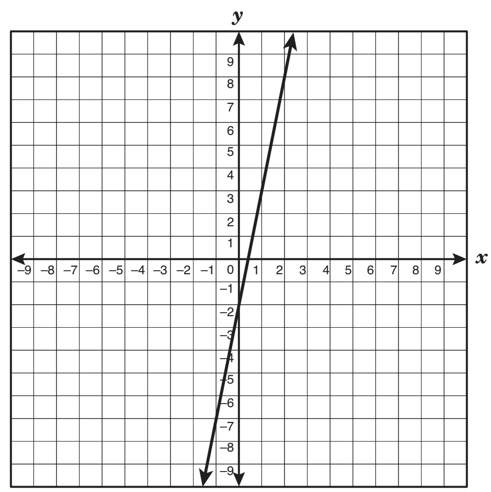
C $y = x + 1$ and $y = x - 25$

D $y = 2x + 1$ and $y = -\frac{1}{2}x + 1$

Answer Key: page 284

Question 28

The equation $y = 5x - 2$ is graphed below.



If the value of m in the equation is changed to $-\frac{1}{5}$, which of the following best describes the effect on the graph?

A The new line would be translated down.

B The new line would be perpendicular to the original line.

C The new line would be parallel to the original line.

D The new line would intersect the original line on the x-axis.

Answer Key: page 284

Question 29

Which is the equation of the line that contains the points $(-1, -5)$ and $(2, 1)$?

A $y = 2x - 3$

B $y = \frac{1}{2}x + \frac{9}{2}$

C $y = \frac{1}{4}x + \frac{1}{2}$

D $y = -4x + 9$

Answer Key: page 284

Question 30

Josh fills his swimming pool at the rate of 2000 gallons per hour. There were 3000 gallons of water in the pool when he started filling it. The total number of gallons of water in the pool after Josh fills it for x hours can be represented by the equation $y = 2000x + 3000$. If Josh adds a second hose, he can fill the pool twice as fast. How will this affect the graph of the equation?

A The line will be translated up.

B The line will be translated down.

C The line will be steeper.

D The line will be less steep.

Answer Key: page 284

Question 31

At a candy store selling gourmet jelly beans, the amount you pay varies directly with the weight of the beans. The cost of 5 pounds of jelly beans is \$12. During the past month \$7200 worth of jelly beans have been purchased. How many pounds of jelly beans were purchased during the past month?

A 120

B 600

C 1200

D 3000

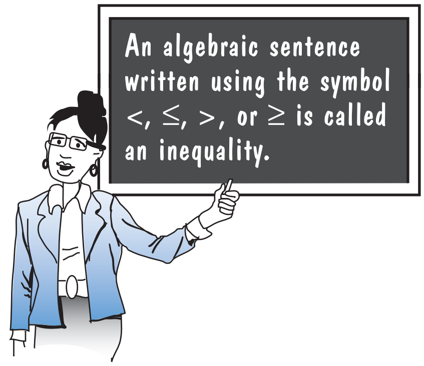
Answer Key: page 284

Objective 4

The student will formulate and use linear equations and inequalities.

For this objective you should be able to

- formulate linear equations and inequalities from problem situations, use a variety of methods to solve them, and analyze the solutions; and
- formulate systems of linear equations from problem situations, use a variety of methods to solve them, and analyze the solutions.



How Do You Solve Problems Using Linear Equations or Inequalities?

Many real-life problems can be solved using either a linear equation or inequality. To solve the equation or inequality, follow these steps:

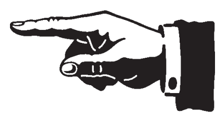
- Simplify the expressions in the equation or inequality by removing parentheses and combining like terms.
- Isolate the variable as a single term on one side of the equation by adding or subtracting expressions on both sides of the equation or inequality.
- Use multiplication or division to produce a coefficient of 1 for the variable term.
- When solving an inequality, you must reverse the inequality symbol if you multiply or divide both sides by a negative number.

$$-4x < 12$$

$$\frac{-4x}{-4} > \frac{12}{-4} \qquad \text{Divide both sides by a negative number.}$$

$$x > -3$$

  The inequality symbol reversed; it went from $<$ to $>$.
- Use the solution of the equation or inequality to find the answer to the question asked.
- See whether your answer is reasonable.



Do you see that . . .

Objective 4

Jennifer works at a warehouse. She is packing a box of cookbooks and Spanish books to ship to a bookstore. The boxes Jennifer uses can hold no more than 35 pounds of books each. The cookbooks weigh 2 pounds each, and the Spanish books weigh 1.5 pounds each. Write an inequality that represents the number of cookbooks, x, and Spanish books, y, that Jennifer can pack in one box.

- Represent the quantities involved with variables or expressions. The number of cookbooks is represented by x. The cookbooks weigh 2 pounds each. The expression $2x$ represents the total weight of the cookbooks.

  The number of Spanish books is represented by y. The Spanish books weigh 1.5 pounds each. The expression $1.5y$ represents the total weight of the Spanish books.

- Write an inequality showing that the total weight cannot be more than 35 pounds.

$$2x + 1.5y \leq 35$$

The inequality $2x + 1.5y \leq 35$ can be used to find the number of each type of book Jennifer can pack in one box.

A company that manufactures games spends \$1200 per month to maintain its offices and production facilities. The games cost \$4 each to produce. The games are sold for \$8 each. How many games must the company make and sell each month to make a profit of at least \$1000 per month?

- Represent the unknown quantities with variables and expressions. Let x represent the number of games made and sold in a month. The expression $4x$ represents the total cost of producing x games.

  The expression $8x$ represents the amount earned from selling x games.

- The company's monthly profit is equal to the amount of money the company earns in a month (its revenue) minus both the overhead cost and the cost of producing the games.

$$\begin{aligned} \text{Profit} &= \text{revenue} - \text{overhead} - \text{production costs} \\ \text{Profit} &= 8x - 1200 - 4x \end{aligned}$$

- The profit must be at least \$1000. This means the profit must be greater than or equal to \$1000. Use an inequality to represent this situation.

$$8x - 1200 - 4x \geq 1000$$

- Solve the inequality.

$$\begin{aligned} 8x - 1200 - 4x &\geq 1000 \\ 8x - 4x - 1200 &\geq 1000 \\ 4x - 1200 &\geq 1000 \\ +1200 &= +1200 \\ \hline 4x &\geq 2200 \\ \frac{4x}{4} &\geq \frac{2200}{4} \\ x &\geq 550 \end{aligned}$$

The company must sell at least 550 games each month to make a profit of $1000 each month.

Jeremy likes to scuba dive. He knows that the deeper a scuba diver goes, the fewer minutes the diver can remain at that depth.

The inequality $m \leq 18 - \frac{3}{50}d$ represents the number of minutes, m, that Jeremy can remain at a depth of d feet. What is the maximum amount of time that Jeremy can spend at a depth of 200 feet?

In the above inequality, d represents his depth in feet, and m represents the number of minutes he can remain at that depth.

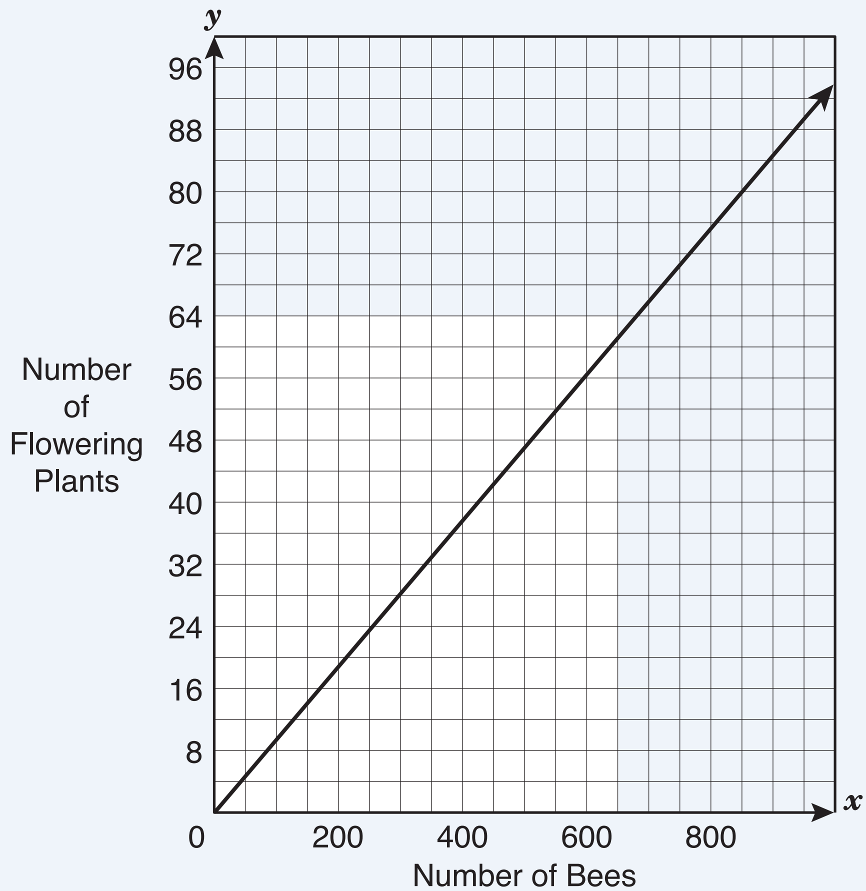
To find the maximum number of minutes Jeremy can remain at a depth of 200 feet, substitute $d = 200$ into the inequality and solve for m.

$$\begin{aligned} m &\leq 18 - \frac{3}{50}d \\ m &\leq 18 - \frac{3}{50}(200) \\ m &\leq 18 - \frac{600}{50} \\ m &\leq 18 - 12 \\ m &\leq 6 \end{aligned}$$

Jeremy can spend a maximum of 6 minutes at a depth of 200 feet.

Emmett's new hobby is beekeeping. He knows that in order for the bees to make honey, they must have available to them a sufficient number of flowering plants. Emmett is expecting a shipment of 800 bees and needs to know how many flowering plants they will require.

The bee company included the graph below to help customers determine the number of flowering plants needed. Use the graph to determine the number of plants Emmett should purchase so that he will have enough plants for the 800 bees he is expecting.



The x-axis represents the number of bees, and the y-axis represents the number of plants needed. Find the point on the x-axis that represents 800 bees. Go up to the graph of the line. Read the corresponding y-value of this point. If you can't find an exact value on the graph for y, round up so that Emmett is sure to have enough plants. The y-coordinate is approximately 76. Emmett should purchase at least 76 plants.

Try It

Six friends decide to combine their money so they can all go to the movies together and buy refreshments. Their total amount of money is $57. The movie tickets cost $6.50 each. Each friend agrees to spend the same amount of money. What is the maximum number of dollars each friend can spend on refreshments?

Represent the number of dollars each friend can spend on refreshments with x.

Write and solve an equation that can be used to find x.

$$\begin{aligned}
\text{Total \$} &= \underline{\qquad\qquad} \cdot (\text{\$ tickets} + \text{\$ refreshments}) \\
\underline{\qquad\qquad} &= \underline{\qquad\qquad}(6.50 + x) \\
\underline{\qquad\qquad} &= \underline{\qquad\qquad} + \underline{\qquad\qquad}\,x \\
-39 &= \quad -39 \\
\underline{\qquad\qquad} &= 6x \\
\frac{\square}{\square} &= \frac{6x}{\square} \\
\underline{\qquad\qquad} &= x
\end{aligned}$$

Each friend can spend at most $________ on refreshments.

$$\begin{aligned}
\text{Total \$} &= 6 \cdot (\text{\$ tickets} + \text{\$ refreshments}) \\
57 &= 6(6.50 + x) \\
57 &= 39 + 6x \\
-39 &= -39 \\
18 &= 6x \\
\frac{18}{6} &= \frac{6x}{6} \\
3 &= x
\end{aligned}$$

Each friend can spend at most $3 on refreshments.

Objective 4

A system of linear equations is two or more linear equations that use two or more variables.

$$3x - y = 12$$

$$x + y = 36$$

The equations above are an example of a system of two linear equations in two unknowns.

How Do You Represent Problems Using a System of Linear Equations?

Many real-life problems can be solved using a system of two or more linear equations. To represent a problem using a system of linear equations, follow these guidelines.

- Identify the quantities involved and the relationships between them.
- Represent the quantities involved with two different variables or with expressions involving two variables.
- Write two independent equations that can be used to solve the problem.

Zola the zookeeper is in charge of feeding the apes. She feeds them apple pieces that are 20 calories each and banana pieces that are 30 calories each. Zola feeds Arlo the ape 24 pieces of fruit for a total of 520 calories. Write a system of linear equations that could be used to find the number of pieces of each type of fruit, apples and bananas, that Arlo eats.

Represent the number of apple pieces Arlo eats with a and the number of banana pieces with b.

You know two different things about the quantities.

- The total number of pieces of fruit is 24.

$$a + b = 24$$

- Each apple piece has 20 calories. The expression $20a$ represents the number of calories from a pieces of apple.
  Each banana piece has 30 calories. The expression $30b$ represents the number of calories from b pieces of banana.
  The total number of calories from all the fruit pieces is 520.

$$20a + 30b = 520$$

The following system of linear equations could be used to find the number of pieces of apple and the number of pieces of banana Arlo eats:

$$a + b = 24$$

$$20a + 30b = 520$$

Try It

A rectangle has a perimeter of 50 inches. The length of the rectangle is six inches less than three times the width. Write a system of equations that could be used to find the length and the width of the rectangle.

The problem involves two quantities, the __________________ and the __________________ of the rectangle.

Represent the length of the rectangle with l and the width with w.

Write an equation showing that the perimeter of the rectangle is _________ inches.

$$\_\_\_\_\_\_ \cdot \_\_\_\_\_\_ + \_\_\_\_\_\_ \cdot \_\_\_\_\_\_ = \_\_\_\_\_\_$$

Write an equation showing that the length of the rectangle is six inches less than three times its width.

$$\text{length} = (3 \cdot \text{width}) - 6$$

$$l = \_\_\_\_\_\_ \cdot \_\_\_\_\_\_ - \_\_\_\_\_\_$$

The system of equations

____

____

could be used to find the length and the width of the rectangle.

The problem involves two quantities, the length and the width of the rectangle.

Write an equation showing that the perimeter of the rectangle is 50 inches.

$$2 \cdot l + 2 \cdot w = 50$$

Write an equation showing that the length of the rectangle is six inches less than three times its width.

$$l = 3 \cdot w - 6$$

The system of equations

$$2l + 2w = 50$$
$$l = 3w - 6$$

could be used to find the length and the width of the rectangle.

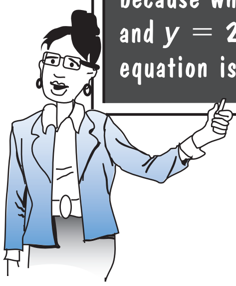
Objective 4

The solution to a system of linear equations is a pair of numbers that makes both equations true.

For example, the ordered pair (12, 24) is a solution to the following system of equations:

$$3x - y = 12$$
$$x + y = 36$$

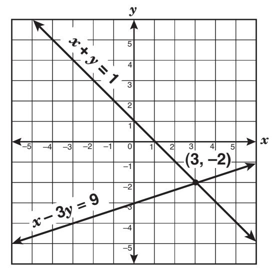
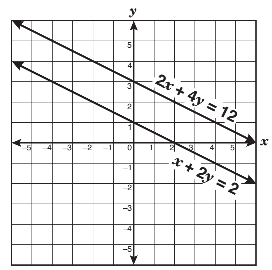
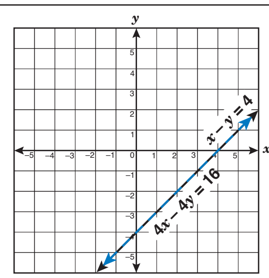
because when $x = 12$ and $y = 24$, each equation is true.



How Do You Solve a System of Linear Equations?

You can solve a system of linear equations algebraically and graphically. Two algebraic methods are substitution and elimination.

A graph can show you how many solutions a system of equations has.

If the two lines …	… as shown by this graph …	… then the system of equations …
intersect at a single point (intersecting lines)		has one solution: $x = 3$ and $y = -2$ or $(3, -2)$.
do not intersect (parallel lines)		has no solution.
intersect at every point (coincident lines)		has infinitely many solutions.

You can also determine the number of solutions a system of equations has by writing the equations in slope-intercept form, $y = mx + b$.

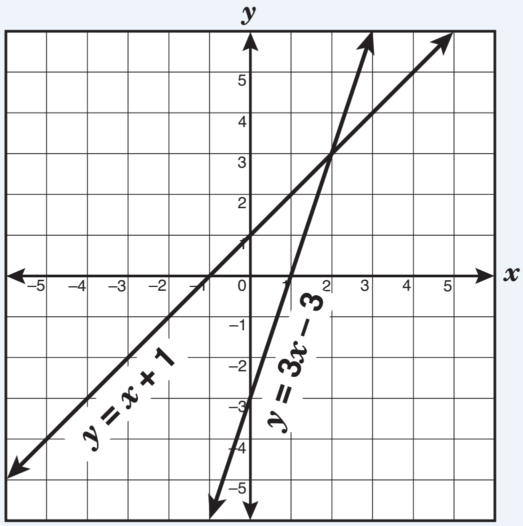
If the two equations …	… as shown by this system …	… then the system of equations …
have different values for m	$x - 3y = 9 \longrightarrow y = \frac{1}{3}x - 3$ $x + y = 1 \longrightarrow y = -x + 1$	has one solution: $x = 3$ and $y = -2$.
have different values for b but the same value for m	$x + 2y = 2 \longrightarrow y = -\frac{1}{2}x + 1$ $(b = 1)$ $2x + 4y = 12 \longrightarrow y = -\frac{1}{2}x + 3$ $(b = 3)$	has no solution.
have the same values for m and b	$x - y = 4 \longrightarrow y = x - 4$ $4x - 4y = 16 \longrightarrow y = x - 4$	has infinitely many solutions.

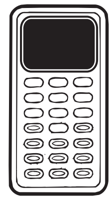
Solve this system of equations using the graphical method.

$$-x + y = 1$$

$$3x - y = 3$$

The two equations are graphed below.





The point where the two lines intersect, (2, 3), represents the solution of the system of equations. Its coordinates satisfy both equations.

$x = 2$ and $y = 3$	$-x + y = 1$ $-(2) + 3 = 1$ $1 = 1$	$3x - y = 3$ $3(2) - 3 = 3$ $6 - 3 = 3$ $3 = 3$

The solution to the system of equations is (2, 3).

Objective 4

When you use the substitution method to solve a system of equations, solve one equation for one of the two variables. Then substitute into the second equation.

Solve the following system of equations using the substitution method.

$$m + n = 35$$
$$4m - 2n = 20$$

This system of equations lends itself to being solved using substitution because the first equation can be solved easily for m in terms of n.

To solve the first equation for m, subtract n from both sides.

$$\begin{array}{rcl} m + n &=& 35 \\ -\,n &=& \quad -\,n \\ \hline m \quad &=& 35 - n \end{array}$$

Now substitute $(35 - n)$ for m in the second equation.
The result will be a linear equation with just one variable.

Solve that equation.

$$\begin{array}{rcl} 4m - 2n &=& 20 \\ 4(35 - n) - 2n &=& 20 \\ 140 - 4n - 2n &=& 20 \\ 140 - 6n &=& 20 \\ -140 \qquad &=& -140 \\ \hline -6n &=& -120 \\ n &=& 20 \end{array}$$

Now that you know the value of n, you can use this value to find the value of m by substituting the value of n into either of the two original equations.

In this system, the first equation is the easier equation to use.

$$m + n = 35$$
$$m + 20 = 35$$
$$m = 15$$

The solution to the system of equations is $m = 15$ and $n = 20$, or $(15, 20)$.

Objective 4

The elimination method is also called the addition method because you eliminate one of the variables by adding. Before you add, you may need to multiply one or both equations so that one of the variables has opposite coefficients.

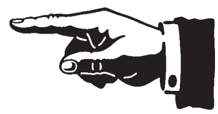
Solve the same system of equations using the elimination method.

$$\begin{aligned} m + n &= 35 \\ 4m - 2n &= 20 \end{aligned}$$

In the given system of equations, if you multiply both sides of the first equation by 2, you get the following equation:

$$2m + 2n = 70$$

The variable n has a coefficient of 2 in the new first equation and a coefficient of -2 in the original second equation. When you add those two equations, the variable n will disappear because it will have a 0 coefficient.



Do you see that . . .

$$\begin{array}{r} 2m + 2n = 70 \\ \underline{+\ 4m - 2n = 20} \\ 6m \qquad\ = 90 \\ m = 15 \end{array}$$

To find n, substitute 15 for m into the first equation.

$$\begin{aligned} m + n &= 35 \\ 15 + n &= 35 \\ n &= 20 \end{aligned}$$

The solution to the system of equations is $m = 15$ and $n = 20$, or (15, 20). This is the same solution obtained by using the substitution method.

Objective 4

Not all systems of equations have a unique solution. Their graphs may be parallel lines, or they may be the same line.

Solve the following system of equations.

$$\begin{aligned} 2x + y &= 6 \\ 4(x - 1) - 1 &= 2(2 - y) + 3 \end{aligned}$$

First, simplify the second equation:

$$\begin{aligned} 4(x - 1) - 1 &= 2(2 - y) + 3 \\ 4x - 4 - 1 &= 4 - 2y + 3 \\ 4x - 5 &= -2y + 7 \\ 4x + 2y &= 12 \end{aligned}$$

Now the system of equations you must solve looks like this:

$$\begin{aligned} 2x + y &= 6 \\ 4x + 2y &= 12 \end{aligned}$$

Notice that the variable y has a coefficient of 2 in the new second equation and 1 in the first equation. If you multiply the first equation by -2, the variable y will have a coefficient of -2 in one equation and 2 in the other.

$$\begin{aligned} -2(2x + y) &= -2(6) \\ -4x - 2y &= -12 \end{aligned}$$

Now, when you add the two equations the y terms should disappear.

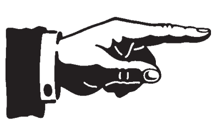
$$\begin{array}{r} -4x - 2y = -12 \\ \underline{+\ 4x + 2y = 12} \\ 0x + 0y = 0 \\ 0 = 0 \end{array}$$

But everything disappeared! This system of equations does not have a unique solution.

To see whether two equations have no solution or an infinite number of solutions, write them in slope-intercept form:

$$\begin{aligned} 2x + y &= 6 \;\longrightarrow\; y = -2x + 6 \;\longrightarrow\; y = -2x + 6 \\ 4x + 2y &= 12 \;\longrightarrow\; 2y = -4x + 12 \;\longrightarrow\; y = -2x + 6 \end{aligned}$$

Do you see that . . .



The two equations are in fact the same equation—their graphs are the same line. These are called coincident lines. That is why you can't find a unique solution. This system of equations has an infinite number of solutions.

Try It

Describe the solution set for this system of linear equations. How many solutions does the system of equations have?

$$3x + y = 1$$
$$-6x - 2y = 4$$

You can determine the number of solutions a system of equations has by writing the equations in ________________, $y = mx + b$.

Solve each equation for y.

$3x + y = 1$	$-6x - 2y = 4$
$y =$ ____________	$-2y =$ ____________
	$y =$ ____________

The slope of the first equation is ________, and its y-intercept is ________.

The slope of the second equation is ________, and its y-intercept is ________.

Since both equations have the same __________ but different __________, their graphs will be __________ lines. The graphs will not __________.

The system of linear equations has ________ solution(s).

You can determine the number of solutions a system of equations has by writing the equations in slope-intercept form, $y = mx + b$.

$3x + y = 1$	$-6x - 2y = 4$
$y = -3x + 1$	$-2y = 6x + 4$
	$y = -3x - 2$

The slope of the first equation is -3, and its y-intercept is 1. The slope of the second equation is -3, and its y-intercept is -2. Since both equations have the same slope but different y-intercepts, their graphs will be parallel lines. The graphs will not intersect. The system of linear equations has no solution(s).

Now practice what you've learned.

Question 32

Fido's vet has placed him on a diet; he is allowed no more than 1000 calories a day. Fido's dry dog food has 200 calories per cup, and his biscuits have 400 calories each. Which inequality could be used to find d, the number of cups of dry food, and b, the number of biscuits, that Fido is allowed to eat each day?

A $(200 + 400)(d + b) > 1000$

B $200d + 400b \leq 1000$

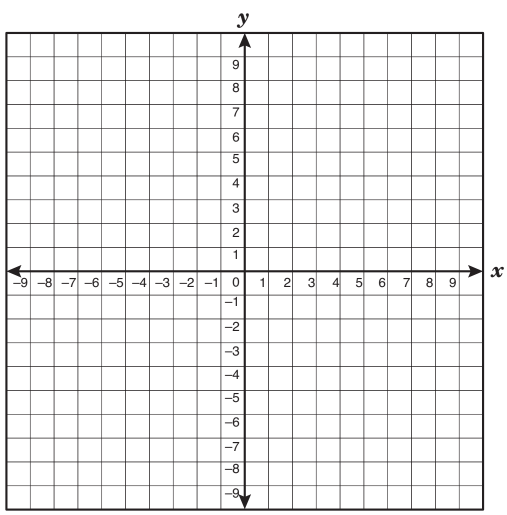
C $(d + 400)(b + 200) \leq 1000$

D $400d + 200b > 1000$

Answer Key: page 285

Question 33

Which of the following ordered pairs is in the solution set of $-8y < 3x - 40$?



A $(-4, 6\frac{3}{4})$

B $(5, 3)$

C $(-2, 5\frac{1}{2})$

D $(0, 5)$

Answer Key: page 285

Question 34

The Seashell Hotel charges a different rate for its large rooms than for its small rooms. When 20 large rooms are occupied, the total amount of money collected from the large rooms is $1800. When 10 large rooms and 21 small rooms are occupied, the hotel collects $2370. How much does the hotel charge for each small room?

A $90

B $60

C $80

D $70

Answer Key: page 285

Question 35

Jamal often orders books from an on-line bookstore. Paperback books cost $6 each, and hardback books cost $12 each. Jamal wants to order at least one hardback book. If he plans to spend no more than $72, which inequality best represents the number of paperback books, p, Jamal can order?

A $0 \leq p \leq 10$

B $0 \leq p \leq 11$

C $1 \leq p \leq 9$

D $1 \leq p \leq 12$

Answer Key: page 285

Question 36

The science club members must raise at least $60 for a field trip. To do so, they are selling chocolates for $3 a box and mints for $2 a box.

If the inequality $3x + 2y \geq 60$ represents the possible solutions to this problem, which of the following points represents a reasonable number of boxes of chocolates and boxes of mints that the club members could sell to raise enough money for their field trip?

A (11, 12)

B (12, 14)

C (14, 8)

D (10, 14)

Answer Key: page 286

Question 37

Andrea is making cookies that contain both peanut butter chips and chocolate chips. She used a total of 16 ounces of chips. There are 8 chocolate chips in each ounce of chocolate chips, and 12 peanut butter chips in each ounce of peanut butter chips. If Andrea used a total of 300 chips in her cookies, which system of linear equations could be used to find the number of ounces of chocolate chips, c, and the number of ounces of peanut butter chips, p, Andrea used?

A $c + p = 16$
$8c + 12p = 300$

B $c + p = 300$
$8c + 12p = 16$

C $16c + 8p = 300$
$c + p = 16$

D $16c - 8p = 12$
$c + p = 300$

Answer Key: page 286

Question 38

Julie is planning to put a fence around a rectangular garden. The length of the garden is 3 feet more than 1.5 times its width. If Julie uses a total of 36 feet of fencing around the edge of the garden, what is the length of the garden?

A 6 ft

B 13.2 ft

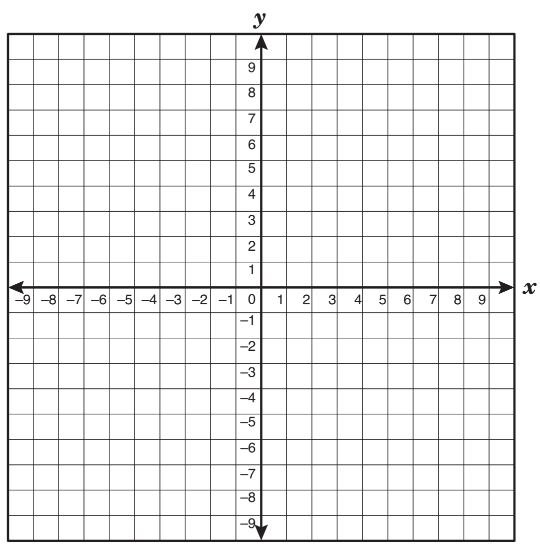
C 12 ft

D 10.5 ft

Answer Key: page 286

Question 39

The graph of line r passes through the points (3, 0) and (0, 3). The graph of line s has a slope of $-\frac{2}{3}$ and intercepts the origin.



Which of the following is the solution to the system of linear equations represented by the information above?

A (2, 1)

B (10, −7)

C (6, −4)

D (9, −6)

Answer Key: page 287

Question 40

Which is the correct solution of the system of linear equations?

$$2x - 4y = 6$$
$$6x = 12y + 18$$

A $(0, -\frac{3}{2})$

B $(-1, 5)$

C There are an infinite number of solutions.

D There is no solution.

Answer Key: page 287

Question 41

The system of linear equations below has how many solutions?

$$-3x + y = 2$$
$$6x - 2y = 2$$

A One solution

B Two solutions

C Infinitely many solutions

D No solution

Answer Key: page 287

Objective 5

The student will demonstrate an understanding of quadratic and other nonlinear functions.

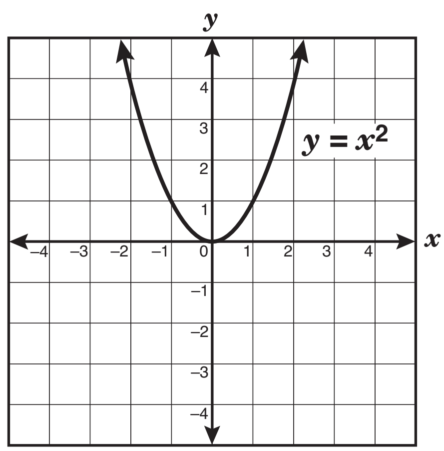
For this objective you should be able to

- interpret and describe the effects of changes in the parameters of quadratic functions;
- solve quadratic equations using appropriate methods; and
- apply the laws of exponents in problem-solving situations.

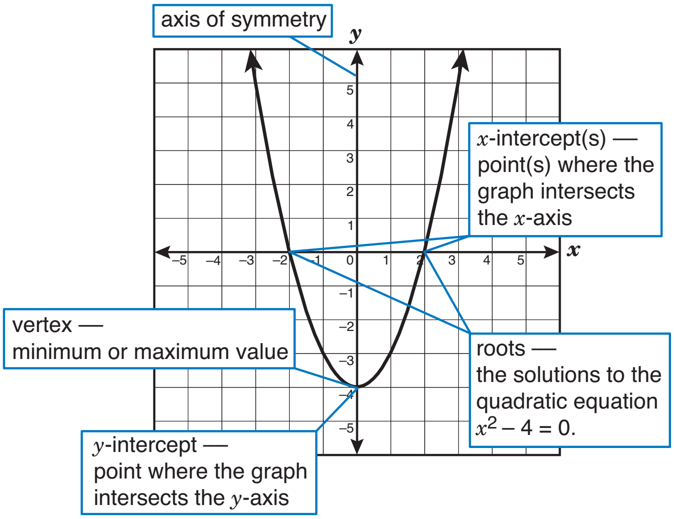
What Is a Quadratic Function?

- A **quadratic function** is any function that can be written in the form $y = ax^2 + bx + c$ where $a \neq 0$. Its graph is a parabola.
- A **quadratic equation** is any equation that can be written in the form $0 = ax^2 + bx + c$. The constants a, b, and c are called the **parameters** of the equation. When you know their values, you can describe the shape and location of the parabola.

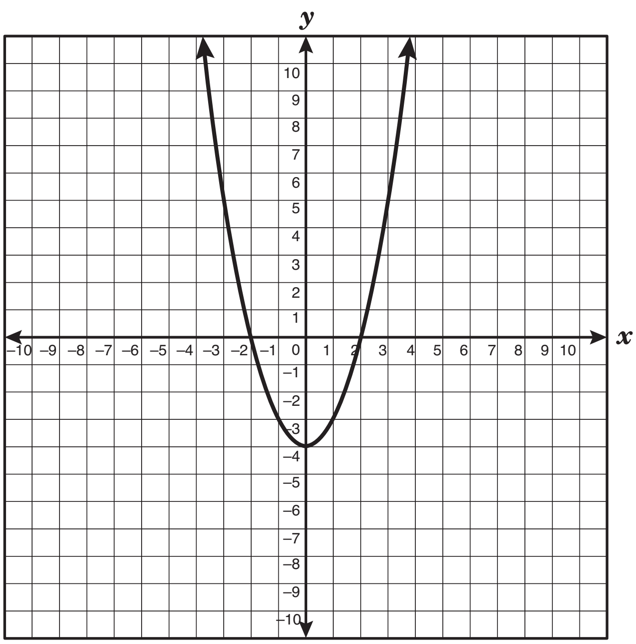
The simplest quadratic function is $y = x^2$. It is the quadratic parent function.



The graph of the quadratic function $y = x^2 - 4$ is shown below.



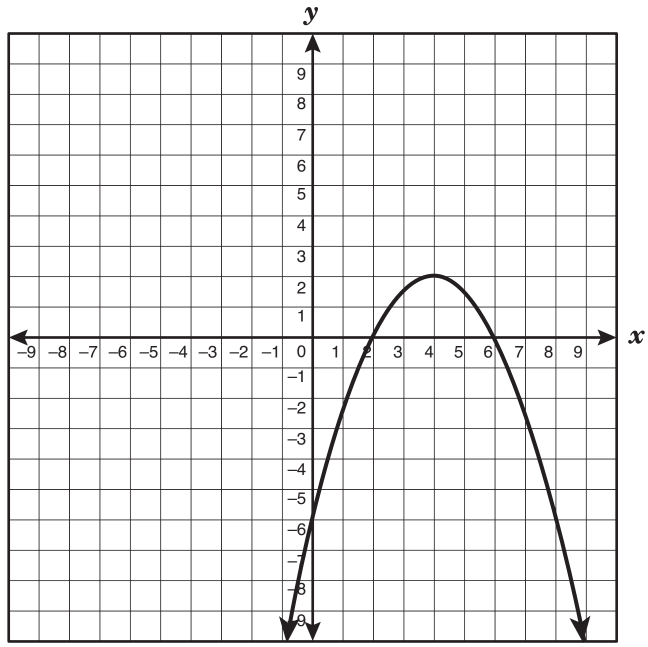
The characteristics of the graph of $y = x^2 - 4$ are shown below.



Vertex	(0, –4)
Roots	$x = -2$ and $x = 2$
Zeros	$x = -2$ and $x = 2$
x-intercepts	(–2, 0) and (2, 0)
y-intercept	(0, –4)
Axis of Symmetry	$x = 0$

Try It

The graph of $f(x) = -\frac{1}{2}x^2 + 4x - 6$ is shown below.



What are the characteristics of this graph?

The zeros of the function are __________ and __________.

The vertex is located at __________.

The y-intercept of the graph is at __________.

The equation of the axis of symmetry is __________.

The zeros of the function are at 2 and 6. The vertex is located at (4, 2). The y-intercept of the graph is at (0, –6). The equation of the axis of symmetry is $x = 4$.

What Happens to the Graph of $y = ax^2$ When a Is Changed?

If two quadratic functions of the form $y = ax^2$ differ only in the sign of the coefficient of x^2, then one graph will be a reflection of the other graph across the x-axis.

- If $a > 0$, then the parabola opens upward.
- If $a < 0$, then the parabola opens downward.

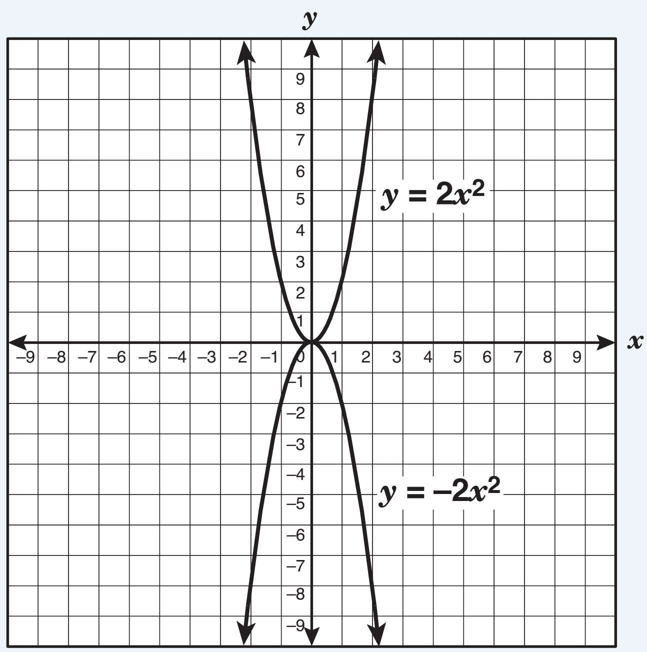
How do the graphs of $y = 2x^2$ and $y = -2x^2$ compare?

In one function, $a = 2$. In the other function, $a = -2$.

Since the coefficients of x^2 differ only in sign, one positive and the other negative, one graph is a reflection of the other across the x-axis.

The graph of $y = 2x^2$ opens upward because $2 > 0$.

The graph of $y = -2x^2$ opens downward because $-2 < 0$.



Objective 5

See Objective 3, page 75, for more information about absolute value.

If two quadratic functions of the form $y = ax^2$ have different coefficients of x^2, then one graph will be wider than the other. The smaller the absolute value of a, the coefficient of x^2, the wider the graph.

Which of these three functions produces the widest graph?

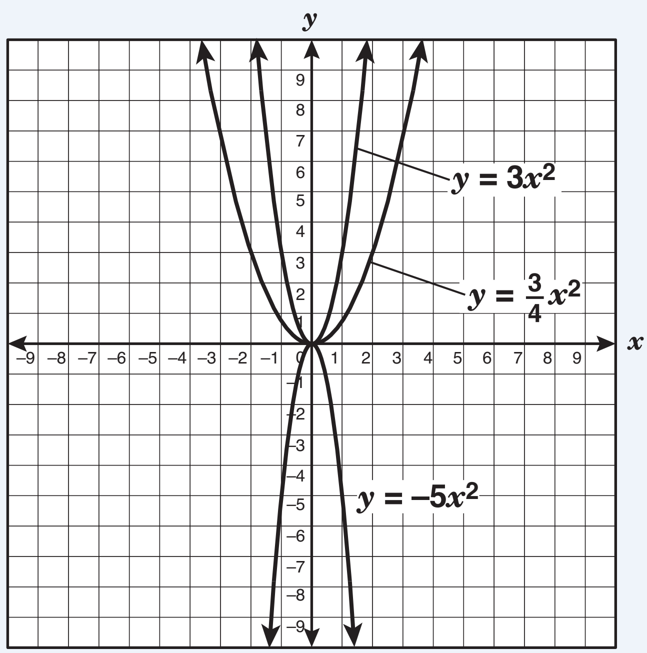
$$y = 3x^2 \qquad y = -5x^2 \qquad y = \frac{3}{4}x^2$$

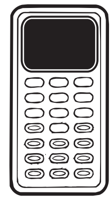
Compare the absolute values of the three coefficients of x^2.

- The three coefficients of x^2 are 3, -5, and $\frac{3}{4}$.
- The absolute values of the coefficients are 3, 5, and $\frac{3}{4}$.
- The coefficient -5 has the greatest absolute value.

  The function $y = -5x^2$ has the narrowest graph.
- The coefficient $\frac{3}{4}$ has the least absolute value.

  The function $y = \frac{3}{4}x^2$ has the widest graph.

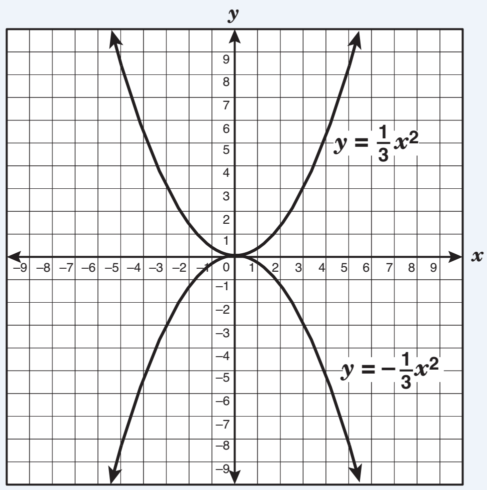




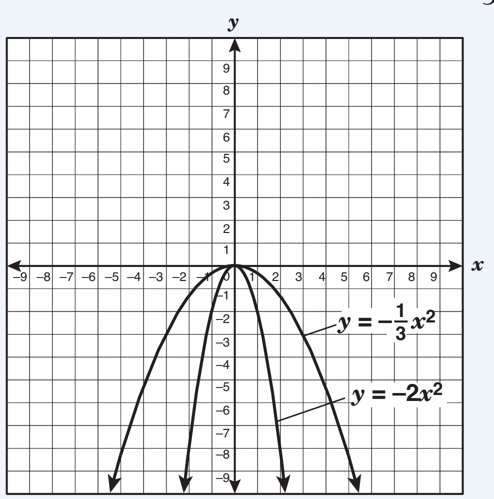
If the coefficient of x^2 in the function $y = \frac{1}{3}x^2$ is changed to -2, how does the new graph compare with the original graph?

First find the effect of changing the sign of a. Then find the effect of changing the value of a.

- If the coefficient of x^2 changes from positive $\frac{1}{3}$ to negative $\frac{1}{3}$, the new graph will be a reflection of the original graph.
  The graph of $y = -\frac{1}{3}x^2$ is a reflection of the graph of $y = \frac{1}{3}x^2$ across the x-axis.



- If the coefficient of x^2 next changes from $-\frac{1}{3}$ to -2, it increases in size because $|-2| > |-\frac{1}{3}|$. If the absolute value of the coefficient increases in size, the graph gets narrower. The graph of $y = -2x^2$ is narrower than the graph of $y = -\frac{1}{3}x^2$.



The graph of $y = -2x^2$ is narrower than the graph of $y = \frac{1}{3}x^2$, and it opens downward, not upward.

Objective 5

Try It

If the graphs of $y = -x^2$, $y = \frac{1}{4}x^2$, and $y = -\frac{1}{4}x^2$ are graphed on the same coordinate grid, which two graphs will be congruent? Which graph will be the narrowest?

Because their coefficients of x^2 differ only in sign, the graphs of $y =$ ______x^2 and $y =$ ______x^2 are reflections of each other. Their graphs will be ____________________________ .

The coefficients of x^2 in the three equations are ______ , ______ , and ______ .

The absolute values of the coefficients of x^2 are ______ , ______ , and ______ .

The coefficient ______ has the greatest absolute value.

Therefore, the graph of $y =$ ______ will be the narrowest.

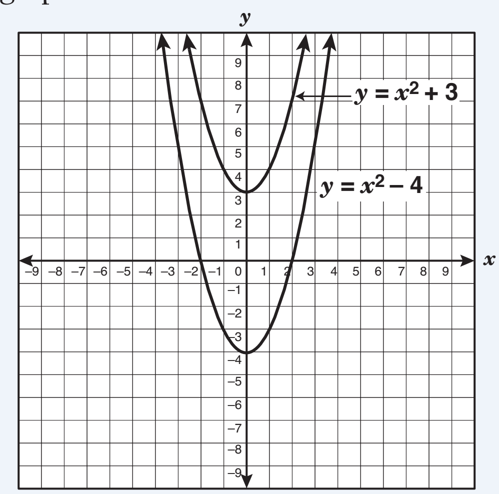
Because their coefficients of x^2 differ only in sign, the graphs of $y = \frac{1}{4}x^2$ and $y = -\frac{1}{4}x^2$ are reflections of each other. Their graphs will be congruent. The coefficients of x^2 in the three equations are -1, $\frac{1}{4}$, and $-\frac{1}{4}$. The absolute values of the coefficients of x^2 are 1, $\frac{1}{4}$, and $\frac{1}{4}$. The coefficient -1 has the greatest absolute value. Therefore, the graph of $y = -x^2$ will be the narrowest.

What Happens to the Graph of $y = x^2 + c$ When c Is Changed?

If two quadratic functions of the form $y = x^2 + c$ have different constants, c, then one graph will be a translation up or down of the other graph.

How does the graph of $y = x^2 + 3$ compare with the graph of $y = x^2 - 4$?

- In the function $y = x^2 + 3$, the constant 3 has been added to the parent function $y = x^2$. Adding the constant $c = 3$ means the graph of $y = x^2$ is translated up 3 units.
- In the function $y = x^2 - 4$, the constant -4 has been added to the parent function $y = x^2$. Adding the constant $c = -4$ means the graph of $y = x^2$ is translated down 4 units.
- Look at the graphs.



The vertex of the graph of $y = x^2 + 3$ is 7 units above the vertex of the graph of $y = x^2 - 4$.

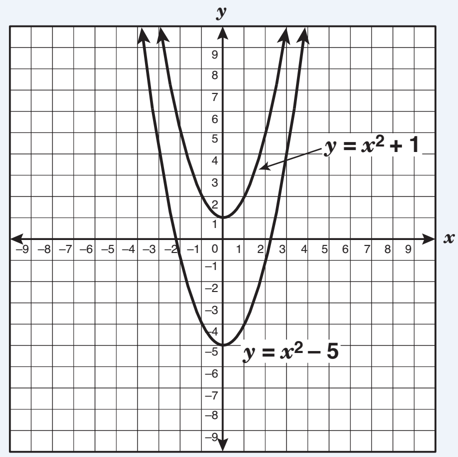
If the value of c in the function $y = x^2 + 1$ is changed to -5, how does the graph of the new function compare to the graph of the original function?

In the function $y = x^2 + 1$, the constant $c = 1$.

If the value of c is changed to -5, the function becomes $y = x^2 - 5$.

Since $-5 - 1 = -6$, the graph of $y = x^2 - 5$ will be 6 units below the graph of $y = x^2 + 1$.

Since $a = 1$ in both functions, the two graphs are congruent.



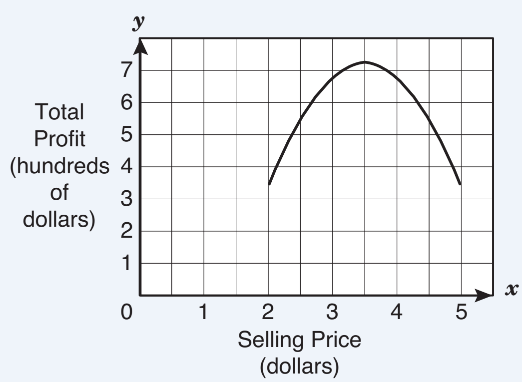
How Do You Draw Conclusions from the Graphs of Quadratic Functions?

To analyze graphs of quadratic functions and draw conclusions from them, consider the following.

- Understand the problem. Identify the quantities involved and the relationship between them.
- Identify the quantities represented in the graph by using the horizontal and vertical axes and looking at the scales used.
- Find the *x*-intercepts and *y*-intercept of the graph and determine what these values represent in the problem.
- Decide whether the graph has a minimum or maximum point and determine what this value represents in the problem.

The community service club at a school sells hamburgers at football games to raise money. The club has tried different selling prices for burgers to see how the different prices affect its profit.

The graph below shows the total profit the club expects to earn depending on the price it charges for burgers.



For what amount should the club sell the hamburgers in order to maximize its profit?

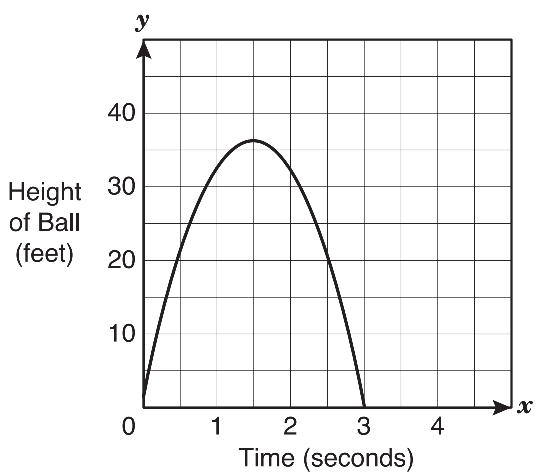
The horizontal axis represents the selling price in dollars. The vertical axis represents the profit in hundreds of dollars.

- As the selling price increases, the profit increases and then decreases.
- The greatest value of the function is at the vertex, (3.5, 7.25). This means that the maximum profit the club can earn is \$725, when the selling price is \$3.50 per hamburger.
- Notice that a lower or higher selling price will result in a smaller profit.

The club should sell the hamburgers for \$3.50 in order to maximize its profit.

Try It

Jason is practicing punting a football. He drops the football and then kicks the ball at a height of about one foot. The graph below shows the height of the football over time.



What conclusions about the flight of the ball can you draw from the graph?

The x-axis represents the number of ________________ that have passed since Jason kicked the ball.

The y-axis represents the ________________ of the ball in feet.

The starting height of the ball is _______ foot.

The ball reaches a maximum height of approximately _______ feet after about _______ seconds in the air.

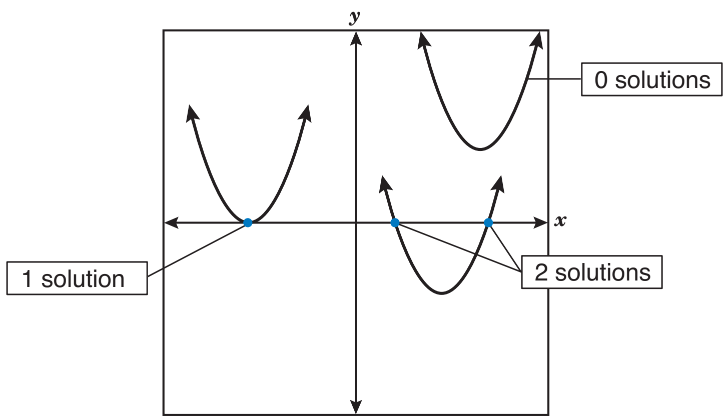
The ball hits the ground after about _______ seconds in the air.

The *x*-axis represents the number of seconds that have passed since Jason kicked the ball. The *y*-axis represents the height of the ball in feet. The starting height of the ball is 1 foot. The ball reaches a maximum height of approximately 37 feet after about 1.5 seconds in the air. The ball hits the ground after about 3 seconds in the air.

How Can You Solve a Quadratic Equation Graphically?

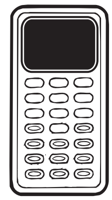
To find solutions to the quadratic equation $ax^2 + bx + c = 0$, you can look at the graph of the related quadratic function, $y = ax^2 + bx + c$.

A quadratic equation can have 0, 1, or 2 unique solutions. The number of solutions is shown by the graph of the related quadratic function.



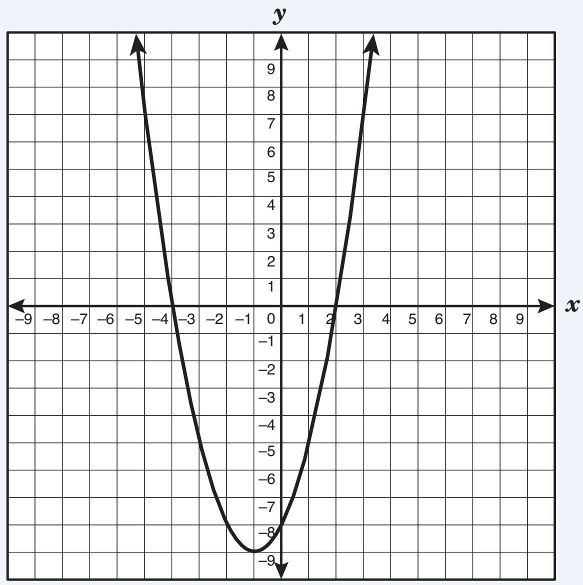
The solutions are called:

- the **roots** of the quadratic equation
- the **zeros** of the quadratic function
- the ***x*-intercepts** of the graph of the function



What are the solutions to the equation $x^2 + 2x - 8 = 0$?

The graph of $y = x^2 + 2x - 8$ is shown below.



- The points where the graph crosses the x-axis are $(-4, 0)$ and $(2, 0)$. The x-coordinates of these points are -4 and 2.
- The zeros of the function $y = x^2 + 2x - 8$ are -4 and 2.
- Therefore, the roots of the equation $x^2 + 2x - 8 = 0$ are -4 and 2. The solution set is $\{-4, 2\}$.

You can verify that these numbers are solutions by replacing x with their value in the quadratic equation.

Substitute $x = -4$	Substitute $x = 2$
$x^2 + 2x - 8 = 0$	$x^2 + 2x - 8 = 0$
$(-4)^2 + 2(-4) - 8 = 0$	$(2)^2 + 2(2) - 8 = 0$
$16 - 8 - 8 = 0$	$4 + 4 - 8 = 0$
$0 = 0$	$0 = 0$

Both numbers, -4 and 2, make the equation true. Both are solutions.

Objective 5

How Can You Solve a Quadratic Equation by Using a Table?

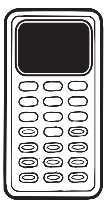
You can use the values in a table representing a quadratic function to find solutions to a quadratic equation.

- Identify the points in the table that have y-values of 0.
- The x-values of those points are the solutions to the equation.

The table below models the function $f(x) = 2x^2 - 2x - 12$. Find solutions to the quadratic equation $2x^2 - 2x - 12 = 0$.

x	y
−3	12
−2	0
−1	−8
0	−12
1	−12
2	−8
3	0
4	12

The zeros of the function are the x-coordinates of the points on the graph where the y-coordinate is 0.



Look for rows in the table where $y = 0$.

Two points in the table have a y-coordinate of 0: $(-2, 0)$ and $(3, 0)$.

The x-coordinates of these points are -2 and 3.

Substitute $x = -2$ and $x = 3$ into the function to verify that $f(x) = y = 0$.

Substitute $x = -2$

$f(x) = 2x^2 - 2x - 12$

$f(-2) = 2(-2)^2 - 2(-2) - 12$

$f(-2) = 8 + 4 - 12$

$f(-2) = 0$

Substitute $x = 3$

$f(x) = 2x^2 - 2x - 12$

$f(3) = 2(3)^2 - 2(3) - 12$

$f(3) = 18 - 6 - 12$

$f(3) = 0$

The zeros of the function, or the roots of the equation, are -2 and 3. Both -2 and 3 are solutions.

How Can You Solve a Quadratic Equation by Factoring?

A quadratic equation can be solved by factoring the quadratic expression and then setting its factors equal to zero.

Find the solutions to the quadratic equation $x^2 + x - 30 = 0$.

This equation can be solved algebraically. Because the quadratic expression $x^2 + x - 30$ can be written as the product of two factors, $(x - 5)$ and $(x + 6)$, the equation can be solved by factoring.

$$x^2 + x - 30 = (x - 5)(x + 6)$$

To verify that this equation is true, use the FOIL method to multiply the two binomials.

F L

$$(x - 5)(x + 6)$$

I

O

First	$x \cdot x = x^2$
Outer	$x \cdot 6 = 6x$
Inner	$-5 \cdot x = -5x$
Last	$-5 \cdot 6 = -30$
FOIL	$x^2 + 6x - 5x - 30$
	$x^2 + x - 30$

Write the left side of the equation as a product of these two factors.

$$x^2 + x - 30 = 0$$
$$(x - 5)(x + 6) = 0$$

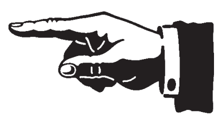
The product of two factors is 0 only if either of the factors is 0. Set each factor in the above equation equal to 0 and solve for x.

$$x - 5 = 0 \qquad x + 6 = 0$$
$$x = 5 \qquad x = -6$$

The solutions of the quadratic equation are 5 and -6.

Check both values of x to verify that the equation is true.

Substitute $x = 5$	Substitute $x = -6$
$x^2 + x - 30 = 0$	$x^2 + x - 30 = 0$
$(5)^2 + (5) - 30 \stackrel{?}{=} 0$	$(-6)^2 + (-6) - 30 \stackrel{?}{=} 0$
$25 + 5 - 30 \stackrel{?}{=} 0$	$36 - 6 - 30 \stackrel{?}{=} 0$
$0 = 0$	$0 = 0$



Do you see that . . .

Objective 5

In real-life problems modeled by quadratic equations, not all the solutions of the equation may make sense in the problem.

The floor plan of a house shows the living room as a rectangle that is 2 feet longer than it is wide. The area of the living room is 80 square feet. Find the length and the width of the living room.

- Let w represent the width of the living room.

  The length is 2 feet more than the width. The expression $w + 2$ represents the length of the room.
- Substitute these expressions for length and width into the formula for the area of a rectangle.

$$A = lw$$
$$A = (w + 2)(w)$$

- Substitute the given value for the area of the room into the equation.

$$80 = (w + 2)(w)$$

- To find the width, solve the equation for w.

  Write the equation in standard quadratic form, $ax^2 + bx + c = 0$.

$$(w + 2)(w) = 80$$
$$w^2 + 2w = 80$$
$$w^2 + 2w - 80 = 0$$

- The expression $w^2 + 2w - 80$ can be factored.

$$w^2 + 2w - 80 = (w + 10)(w - 8)$$

- Rewrite the equation with the expression factored.

$$(w + 10)(w - 8) = 0$$

- Set each factor equal to 0 and solve for w.

$$w + 10 = 0 \qquad w - 8 = 0$$
$$w = -10 \qquad w = 8$$

Do you see that . . .

- The variable w represents width. Width cannot be a negative value. The solution $w = -10$ is not used. The solution to the equation is $w = 8$.

  The width of the living room is 8 feet.
- The length of the living room is $w + 2$ feet.

  The length is $8 + 2 = 10$ feet.

The living room has a length of 10 feet and a width of 8 feet.

How Can You Solve a Quadratic Equation by Using the Quadratic Formula?

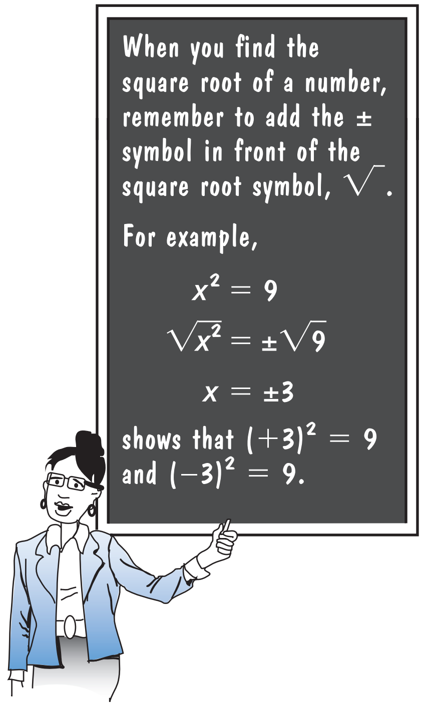
Another method used to solve quadratic equations is the quadratic formula. This method can be used to solve all quadratic equations.

The Quadratic Formula

The solutions to a quadratic equation in the standard form, $ax^2 + bx + c = 0$, are given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a, b, and c are the parameters of the quadratic equation.



Find the solution set to the equation $5x^2 - 2 = 9x$.

- First write the equation in standard form by subtracting $9x$ from both sides of the equation.

$$5x^2 - 2 = 9x$$
$$5x^2 - 9x - 2 = 0$$

- Identify the values of the constants a, b, and c.

$$a = 5$$
$$b = -9$$
$$c = -2$$

- Substitute these values for a, b, and c into the quadratic formula and simplify the expression.

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(5)(-2)}}{2(5)}$$

$$x = \frac{9 \pm \sqrt{81 + 40}}{10}$$

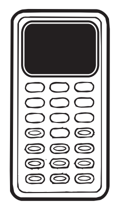
$$x = \frac{9 \pm \sqrt{121}}{10}$$

$$x = \frac{9 \pm 11}{10}$$

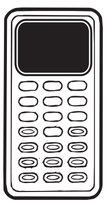
$$x = \frac{9 + 11}{10} \text{ and } x = \frac{9 - 11}{10}$$

$$x = \frac{20}{10} = 2 \text{ and } x = \frac{-2}{10} = -\frac{1}{5}$$

The solution set to the equation is $\{2, -\frac{1}{5}\}$.



Objective 5



What are the solutions of the equation $4x^2 - 6x + 1 = 0$?

- The quadratic equation is written in standard form. Identify the values of a, b, and c.

$$a = 4$$
$$b = -6$$
$$c = 1$$

- Substitute these values for a, b, and c into the quadratic formula and simplify the expression.

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(1)}}{2(4)}$$

$$x = \frac{6 \pm \sqrt{36 - 16}}{8}$$

$$x = \frac{6 \pm \sqrt{20}}{8}$$

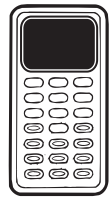
$x = \dfrac{6 + \sqrt{20}}{8}$ and $x = \dfrac{6 - \sqrt{20}}{8}$ are the roots of the equation.

- To approximate the roots of the equation, evaluate the final expressions using $\sqrt{20} \approx 4.47$.

$$x = \frac{6 + \sqrt{20}}{8} \approx \frac{6 + 4.47}{8} \approx 1.31$$

$$x = \frac{6 - \sqrt{20}}{8} \approx \frac{6 - 4.47}{8} \approx 0.19$$

The roots of the equation are $x \approx 1.31$ and $x \approx 0.19$.



Try It

Find the approximate roots of the equation $3x^2 - 5 = 2x$.

Write the quadratic equation in standard form.

$$\_\_\_\_\_\_ x^2 - \_\_\_\_\_\_ x - \_\_\_\_\_\_ = 0$$

In the equation above,

$a =$ ______ $b =$ ______ $c =$ ______

Substitute these values for a, b, and c into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-\square \pm \sqrt{\square^2 - 4 \cdot \square \cdot \square}}{2 \cdot \square}$$

$$x = \frac{\square \pm \sqrt{\square + \square}}{6}$$

$$x = \frac{\square \pm \sqrt{\square}}{6}$$

$$x = \frac{\square \pm \square}{6}$$

$$x = \frac{2 + \square}{6} \approx \_\_\_\_\_\_ \text{ and } x = \frac{2 - \square}{6} = \_\_\_\_\_\_$$

The approximate roots of the equation are ______ and ______.

Write the quadratic equation in standard form.

$$3x^2 - 2x - 5 = 0$$

In the equation above,

$a = 3$ $b = -2$ $c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 3 \cdot -5}}{2 \cdot 3}$$

$$x = \frac{2 \pm \sqrt{4 + 60}}{6}$$

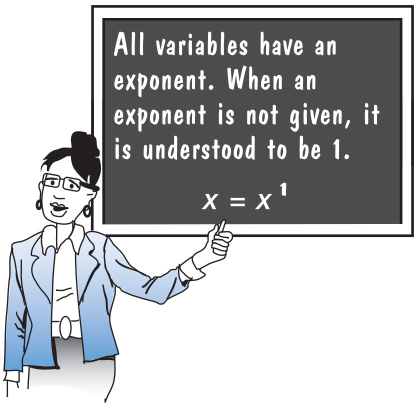
$$x = \frac{2 \pm \sqrt{64}}{6}$$

$$x = \frac{2 \pm 8}{6}$$

$$x = \frac{2 + 8}{6} \approx 1.67 \text{ and } x = \frac{2 - 8}{6} = -1$$

The approximate roots of the equation are 1.67 and -1.

How Do You Apply the Laws of Exponents in Problem-Solving Situations?



When simplifying an expression with exponents, there are several rules, known as the **laws of exponents**, which must be followed.

- When multiplying terms with like bases, add the exponents.

$$x^a \cdot x^b = x^{(a+b)}$$

Example:

$$x^4 \cdot x^2 = x^{(4+2)} = x^6$$

$$(x \cdot x \cdot x \cdot x) \cdot (x \cdot x) = xxxxxx = x^6$$

- When dividing terms with like bases, subtract the exponents.

$$\frac{x^a}{x^b} = x^{(a-b)}$$

Example:

$$\frac{x^8}{x^3} = x^{(8-3)} = x^5$$

$$\frac{\cancel{x}\cancel{x}\cancel{x}xxxxx}{\cancel{x}\cancel{x}\cancel{x}} = x^5$$

Sometimes dividing variables with exponents produces negative exponents.

Example:

$$\frac{x^3}{x^5} = x^{(3-5)} = x^{-2}$$

$$\frac{\cancel{x}\cancel{x}\cancel{x}}{\cancel{x}\cancel{x}\cancel{x}xx} = \frac{1}{x^2} = x^{-2}$$

- A term with a negative exponent is equal to the reciprocal of that term with a positive exponent.

$$x^{-a} = \frac{1}{x^a}$$

Example:

$$x^{-5} = \frac{1}{x^5}$$

- When raising a term with an exponent to a power, multiply the exponents.

$$(x^a)^b = x^{ab}$$

Example: $(x^2)^7 = x^{2 \cdot 7} = x^{14}$

$$(x^2)^7 = (xx) \cdot (xx) \cdot (xx) \cdot (xx) \cdot (xx) \cdot (xx) \cdot (xx)$$

$$(x^2)^7 = (xxxxxxxxxxxxxx)$$

$$(x^2)^7 = x^{14}$$

- Any base other than zero raised to the zero power equals one.

$$x^0 = 1$$

Example: $8^0 = 1$

Simplify the expression $-6x^4 \cdot 3x^5$.

$$\begin{aligned} -6x^4 \cdot 3x^5 &= (-6 \cdot 3) \cdot (x^4 \cdot x^5) \\ &= -18 \cdot x^{(4+5)} \\ &= -18 \cdot x^9 \\ &= -18x^9 \end{aligned}$$

Simplify the expression $(4a^{-5}b^2)^3$. Write your answer using only positive exponents.

One way to solve this problem is to first change negative exponents to positive exponents.

Since $a^{-5} = \frac{1}{a^5}$, it follows that $4a^{-5}b^2 = \frac{4b^2}{a^5}$.

The expression can be simplified as follows.

$$\begin{aligned} (4a^{-5}b^2)^3 &= \left(\frac{4b^2}{a^5}\right)^3 \\ &= \frac{4^3 \cdot b^{2 \cdot 3}}{a^{5 \cdot 3}} \\ &= \frac{64 \cdot b^6}{a^{15}} \\ &= \frac{64b^6}{a^{15}} \end{aligned}$$

Another way to solve this problem is to raise the expression to a power and then change negative exponents to positive exponents.

$$\begin{aligned} (4a^{-5}b^2)^3 &= (4^3a^{-5 \cdot 3}b^{2 \cdot 3}) \\ &= 64a^{-15}b^6 \end{aligned}$$

Since $a^{-15} = \frac{1}{a^{15}}$, the expression $64a^{-15}b^6$ is written with all positive exponents as follows.

$$\begin{aligned} 64a^{-15}b^6 &= \frac{64 \cdot b^6}{a^{15}} \\ &= \frac{64b^6}{a^{15}} \end{aligned}$$

Objective 5

Simplify the following expression.

$$\frac{\left(-3xy^2\right)^3\left(2y^2z^9\right)}{\left(xyz^4\right)^2}$$

First simplify the expressions that are raised to a power.

$$\begin{aligned}\frac{\left(-3xy^2\right)^3\left(2y^2z^9\right)}{\left(xyz^4\right)^2} &= \frac{(-3)^3(x)^3(y^2)^3(2y^2z^9)}{(x)^2(y)^2(z^4)^2}\\ &= \frac{-27x^3y^{2\cdot 3}\cdot 2y^2z^9}{x^2y^2z^{4\cdot 2}}\\ &= \frac{-27x^3y^6\cdot 2y^2z^9}{x^2y^2z^8}\end{aligned}$$

Use the exponent multiplication rule to combine like variables with exponents in the numerator.

$$\begin{aligned}\frac{-27x^3y^6\cdot 2y^2z^9}{x^2y^2z^8} &= \frac{-27\cdot 2\cdot x^3\cdot y^{6+2}\cdot z^9}{x^2y^2z^8}\\ &= \frac{-54x^3y^8z^9}{x^2y^2z^8}\end{aligned}$$

Divide like variables with exponents.

$$\begin{aligned}\frac{-54x^3y^8z^9}{x^2y^2z^8} &= \frac{-54}{1}\cdot\frac{x^3}{x^2}\cdot\frac{y^8}{y^2}\cdot\frac{z^9}{z^8}\\ &= -54x^{(3-2)}y^{(8-2)}z^{(9-8)}\\ &= -54xy^6z\end{aligned}$$

The simplified expression is $-54xy^6z$.

Try It

A rectangular prism has length $2m^{12}n^4$, width $6m^3p^2$, and height $4mn^5p^3$. Find the volume of the prism.

Substitute the given expressions for length, width, and height into the formula for the volume of a rectangular prism, $V = Bh = lwh$, and then simplify the expression.

$$V = \text{length} \cdot \text{width} \cdot \text{height}$$

$$V = \underline{\qquad\qquad} \cdot \underline{\qquad\qquad} \cdot \underline{\qquad\qquad}$$

Simplify the expression.

$$V = \underline{\qquad} \cdot \underline{\qquad} \cdot \underline{\qquad} \cdot m^{\square + \square + \square} \cdot n^{\square + \square} \cdot p^{\square + \square}$$

$$V = \underline{\qquad} \cdot m^{\square} \cdot n^{\square} \cdot p^{\square}$$

$$V = \underline{\qquad\qquad\qquad}$$

$$V = 2m^{12}n^4 \cdot 6m^3p^2 \cdot 4mn^5p^3$$

$$V = 2 \cdot 6 \cdot 4 \cdot m^{12 + 3 + 1} \cdot n^{4 + 5} \cdot p^{2 + 3}$$

$$V = 48 \cdot m^{16} \cdot n^9 \cdot p^5$$

$$V = 48m^{16}n^9p^5$$

Now practice what you've learned.

Question 42

Which of the following best describes the graph of the function $y = ax^2$ when $a < 0$?

A The graph's vertex is below the origin.

B The graph is symmetrical about the y-axis and opens upward.

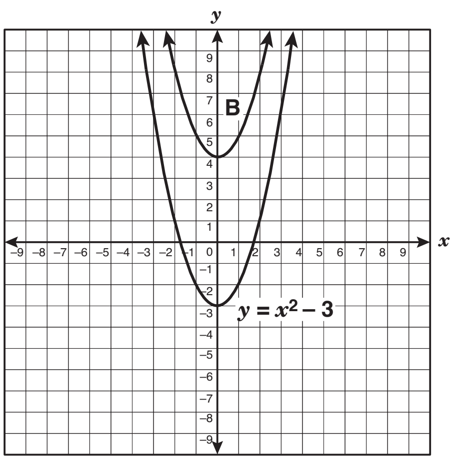
C The graph's vertex is translated above the origin.

D The graph is symmetrical about the y-axis and opens downward.

Answer Key: page 287

Question 43

The graph of $y = x^2 - 3$ and its graph translated up 7 units are shown below.



What is the equation of Graph B?

A $y = -x^2 + 4$

B $y = x^2 + 7$

C $y = x^2 + 4$

D $y = 4x^2 - 3$

Answer Key: page 287

Question 44

The graph of which function is not congruent to the graph of $y = 2x^2 + 1$?

A $y = -2x^2 + 1$

B $y = x^2 + 1$

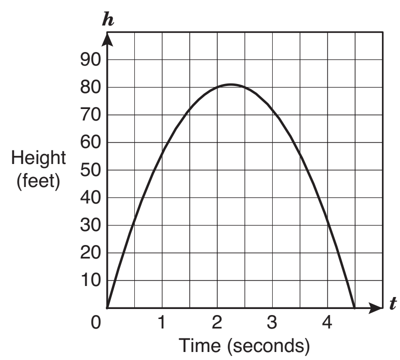
C $y = 2x^2 - 1$

D $y = -2x^2 - 1$

Answer Key: page 287

Question 45

A ball was projected into the air with an initial upward velocity of 72 feet per second. The quadratic function $h = 72t - 16t^2$ describes the height, h, of the ball t seconds after it was projected into the air.



Based on the graph, which of the following is true?

A The ball reached its maximum height in about 3 seconds.

B The ball was in the air for less than 4 seconds.

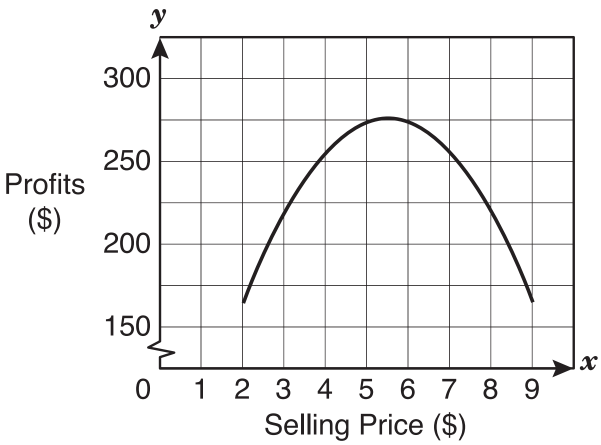
C The height 2.5 seconds after the ball was projected was 64 feet.

D The height 3.5 seconds after the ball was projected was 56 feet.

Answer Key: page 288

Question 46

A bakery determines the following relationship between the price of its cakes and its daily profits.



Which is the best conclusion that can be drawn from the graph?

A As the selling price increases, the profits increase.

B The profits range from approximately \$165 to \$275.

C An increase in the price of the cakes results in an increase in the number of cakes sold.

D The maximum number of cakes that the bakery can sell is 275.

Answer Key: page 288

Question 47

What is the solution set for the equation $x = x^2 - 42$?

A $\{-6, 7\}$

B $\{-7, 6\}$

C $\{-\sqrt{42}, \sqrt{42}\}$

D $\{-6\sqrt{7}, 6\sqrt{7}\}$

Answer Key: page 288

Question 48

A cylinder with a height of 1.5 inches has a total surface area of 4π square inches. What is its approximate radius?

A 2.35 in.

B 4.0 in.

C 0.85 in.

D 2 in.

Answer Key: page 288

Question 49

A triangle has an area of 28 square inches. The base of the triangle is 6 inches less than twice the height. What is the length of the base of the triangle?

A 5 in.

B 4 in.

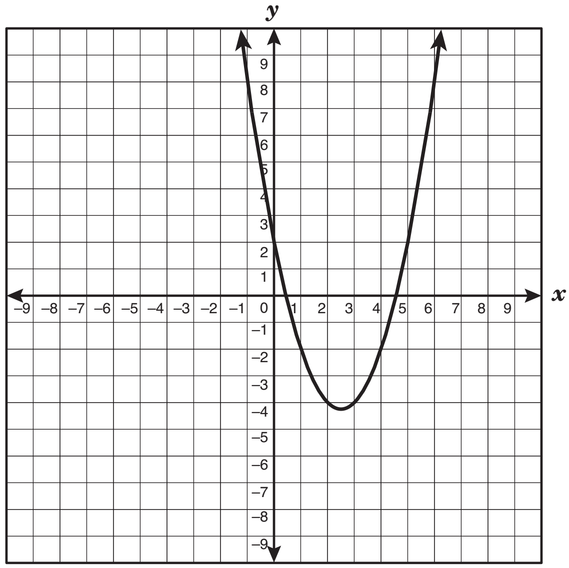
C 7 in.

D 8 in.

Answer Key: page 288

Question 50

The function $y = x^2 - 5x + 2$ is graphed below.



One of the roots of the equation $x^2 - 5x + 2 = 0$ lies between which pair of integers?

A -1 and 0

B 3 and 4

C 4 and 5

D -2 and -1

Answer Key: page 289

Question 51

The surface area of a sphere can be found using the formula $S = 4\pi r^2$.

Which expression represents the surface area of a sphere with a radius of $5x^4yz^3$ units?

A $100\pi x^8y^2z^6$

B $20\pi x^8y^2z^6$

C $100\pi x^6y^2z^5$

D $20\pi x^6y^2z^5$

Answer Key: page 289

Question 52

Which expression is equivalent to $\dfrac{(2a^3b)^4}{(3ac^4)^2\,(4a^5b)}$?

A $\dfrac{a^5b^3}{20c^8}$

B $\dfrac{4a^5b^3}{9c^8}$

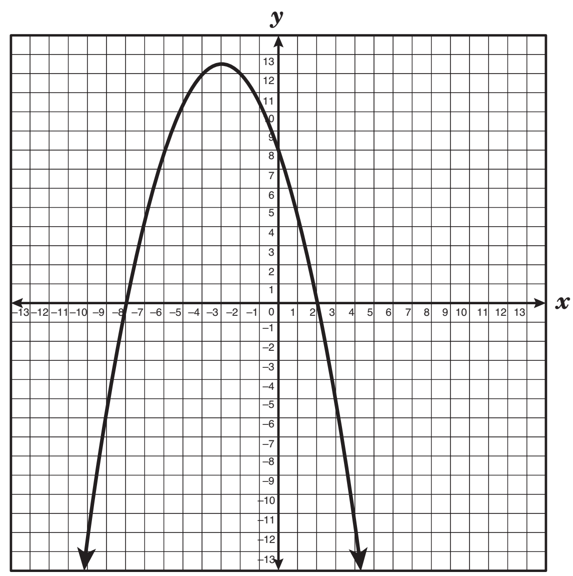
C $\dfrac{a^2b^3}{6c^6}$

D $\dfrac{4a^2b^3}{9c^6}$

Answer Key: page 289

Question 53

The graph of $f(x) = -\dfrac{1}{2}x^2 - 3x + 8$ is shown below.



Which of the following statements appears to be true?

A The vertex is at $(-3, 12)$.

B The axis of symmetry is $x = -3$.

C The zeros of the related function are -8, 2, and 8.

D The y-intercept is $(8, 0)$.

Answer Key: page 289

Objective 6

The student will demonstrate an understanding of geometric relationships and spatial reasoning.

For this objective you should be able to

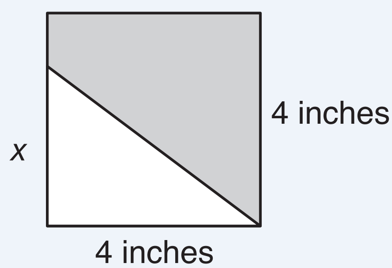
- use a variety of representations to describe geometric relationships and solve problems;
- identify, analyze, and describe patterns that emerge from two- and three-dimensional geometric figures; and
- apply the concept of congruence to justify properties of figures and solve problems.

How Do You Solve Geometric Problems?

You can use diagrams, geometric concepts, properties, definitions, and theorems to solve geometric problems. When solving a geometric problem, consider the following.

- Determine which geometric properties, definitions, or theorems apply to the problem.
- Draw a diagram to represent the problem if you are not given one.
- Identify or algebraically represent any quantities in the problem.
- Use a formula if necessary. Some formulas you need are in the Mathematics Chart.
- Use an equation to represent the relationship between the quantities in the problem.

Write an expression that represents the area of the shaded portion of the 4-inch square in the diagram below.



The area of the shaded portion is equal to the area of the square minus the area of the triangle.

- Find the area of the square.

$$A = s^2$$
$$A = 4^2$$
$$A = 16$$

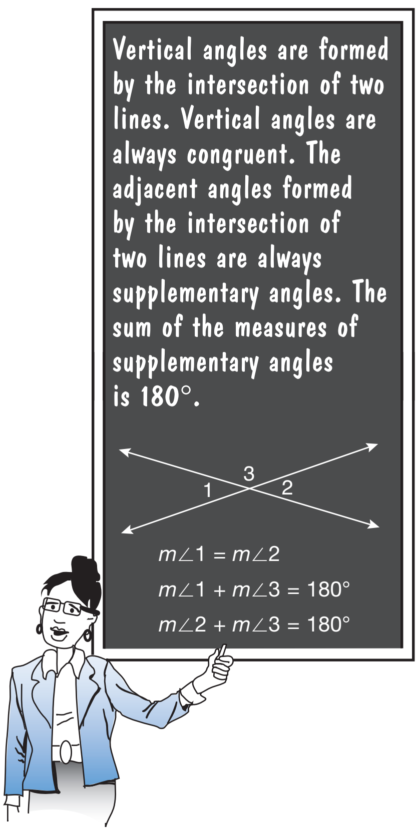
- Find the area of the triangle. Its base is one side of the square, or 4. The triangle's height is represented by x.

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2} \cdot 4 \cdot x$$
$$A = 2x$$

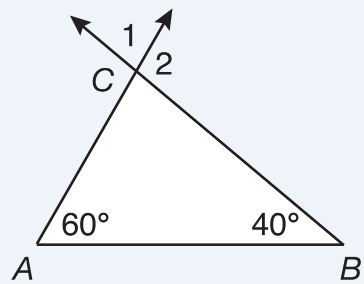
- Subtract the area of the triangle from the area of the square.

$$16 - 2x$$

The expression $16 - 2x$ represents the area in square inches of the shaded portion of the square.



In $\triangle ABC$ below, find the measures of $\angle 1$ and $\angle 2$.



The sum of the measures of the angles of a triangle is 180°.

$$60° + 40° + m\angle ACB = 180°$$

Subtract the sum $60° + 40°$ from 180° to find $m\angle ACB$.

$$m\angle ACB = 180° - (60° + 40°)$$
$$m\angle ACB = 180° - 100°$$
$$m\angle ACB = 80°$$

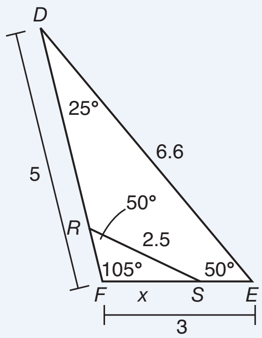
Since $\angle 1$ and $\angle ACB$ are vertical angles, their measures are equal.

Since $m\angle ACB = 80°$, $m\angle 1 = 80°$.

$\angle 1$ and $\angle 2$ are supplementary; the sum of their measures is 180°.

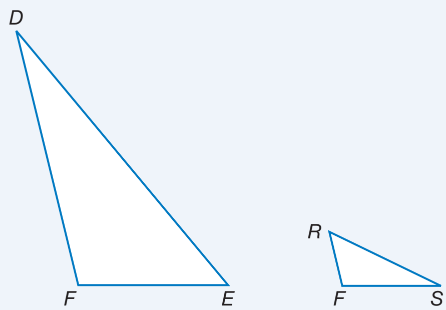
$$m\angle 1 + m\angle 2 = 180°$$
$$80° + m\angle 2 = 180°$$
$$m\angle 2 = 180° - 80°$$
$$m\angle 2 = 100°$$

In the diagram below, $\triangle FRS$ is similar to $\triangle FED$.



Write an equation that could be used to find the length of $\overline{FS}$ in terms of x.

- The lengths of corresponding sides of the triangles are proportional.
- To help you see which sides of the triangle are corresponding, you might draw the two triangles separately.



$$\frac{\triangle FRS}{\triangle FED} \qquad \frac{FR}{FE} = \frac{FS}{FD} = \frac{RS}{ED}$$

- Substitute known values.

$$\frac{2.5}{6.6} = \frac{x}{5}$$

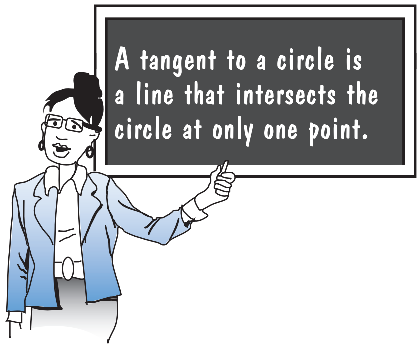
$$6.6x = 5 \cdot 2.5$$

$$6.6x = 12.5$$

The equation $\frac{2.5}{6.6} = \frac{x}{5}$, or the equation $6.6x = 12.5$, could be used to find the length of $\overline{FS}$.

See Objective 8, page 212, for more information about similar figures.

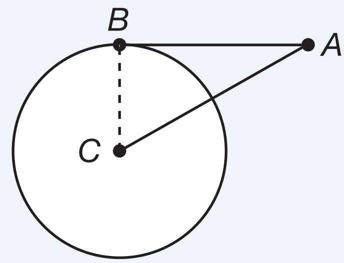
Objective 6



In the diagram below, $\overline{AB}$ is tangent to circle C at point B.

$AB = 15$, and $AC = 17$.

Find the area of circle C in terms of π.



Do you see that . . .

A tangent to a circle always forms a right angle with the radius drawn to that point. Therefore, $\triangle ABC$ is a right triangle with its right angle at vertex B.

Since $\triangle ABC$ is a right triangle, you can use the Pythagorean Theorem to find the length of $\overline{BC}$. The hypotenuse is 17, and one of the legs is 15. Let r represent the length of the radius.

$$r^2 + 15^2 = 17^2$$

$$r^2 + 225 = 289$$

$$r^2 = 64$$

$$r = 8$$

Substitute $r = 8$ into the formula for the area of a circle.

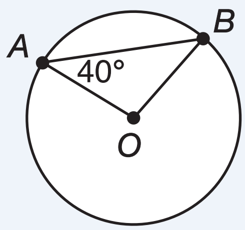
$$A = \pi r^2$$

$$A = \pi \cdot 8^2$$

$$A = 64\pi$$

The area of the circle is 64π square units.

Point O is the center of the circle below. If $m\angle OAB = 40°$, find the measure of $\widehat{AB}$.



The measure of $\widehat{AB}$ is the same as the measure of central angle $\angle AOB$.

Use the fact that the sum of the angles of a triangle is always 180° to find the measure of $\angle AOB$.

- Since $\overline{OA}$ and $\overline{OB}$ are both radii of the circle, $OA = OB$. Therefore, $\triangle AOB$ is an isosceles triangle with $m\angle OAB = m\angle OBA$.
  Since $m\angle OAB = 40°$, $m\angle OBA = 40°$.
- Let $m\angle AOB = x$.

$$\begin{aligned} m\angle OAB + m\angle OBA + m\angle AOB &= 180° \\ 40 + 40 + x &= 180 \\ 80 + x &= 180 \\ x &= 100 \end{aligned}$$

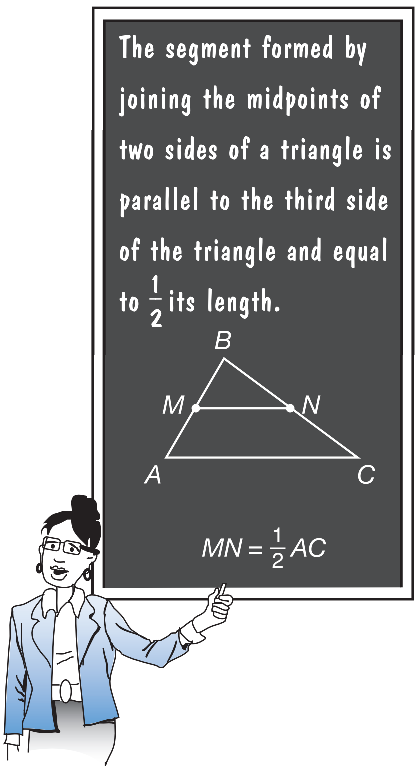
Since $m\angle AOB = 100°$, the measure of $\widehat{AB}$ also equals 100°.

See Objective 8, page 195, for more information about arcs.

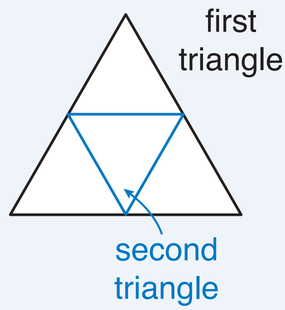
How Can You Analyze and Describe Patterns in Geometric Figures?

Geometric figures often have patterns that can be identified and described. To analyze and describe a pattern in a set of geometric figures, look for a property they have in common. The patterns may involve the figures' dimensions, their areas, their volumes, or the measures of their angles.

When solving problems that involve patterns in figures, you may need to represent a quantity algebraically and then look for the pattern in the algebraic expressions.



In the diagram shown below, the smaller triangles are formed by connecting the midpoints of the sides of the larger triangle.



The perimeter of the first triangle is 48 centimeters. If the process of forming smaller triangles in this way continues, what is the perimeter of the fifth triangle in this series?

Look for the pattern in the perimeters of the successive triangles. Even though you do not know the side lengths in the first triangle, you know that their sum is 48 cm.

Each side in the next smaller triangle is $\frac{1}{2}$ the length of the corresponding side in the larger triangle. So the perimeter of the second triangle is $\frac{1}{2}$ the perimeter of the first triangle.

$$P_{\text{second}} = \frac{1}{2} \cdot 48$$

$$P_{\text{second}} = 24$$

Continue to follow this pattern.

Triangle	Perimeter
1	48
2	$\frac{1}{2} \cdot 48 = 24$
3	$\frac{1}{2} \cdot 24 = 12$
4	$\frac{1}{2} \cdot 12 = 6$
5	$\frac{1}{2} \cdot 6 = 3$

The perimeter of the fifth triangle in the series is 3 centimeters.

If $\angle A$ is the supplement of $\angle B$ and $\angle B$ is the supplement of $\angle C$, what is the relationship between the measures of $\angle A$ and $\angle C$?

One way to approach this problem is algebraically.

- If $\angle A$ is the supplement of $\angle B$, then the sum of their measures is 180°.

$$m\angle A + m\angle B = 180°$$

- If $\angle B$ is the supplement of $\angle C$, then the sum of their measures is 180°.

$$m\angle B + m\angle C = 180°$$

- Since both sums equal 180°, the two quantities are equal to each other.

$$m\angle A + m\angle B = m\angle B + m\angle C$$

- Subtract the measure of $\angle B$ from both sides.

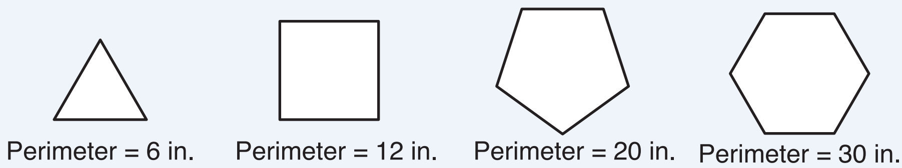
$$\begin{array}{rcl} m\angle A + m\angle B & = & m\angle B + m\angle C \\ -m\angle B & = & -m\angle B \\ \hline m\angle A & = & m\angle C \end{array}$$

If $\angle A$ is the supplement of $\angle B$ and $\angle B$ is the supplement of $\angle C$, then $m\angle A = m\angle C$.

Objective 6

Do you see that . . .

The regular polygons below form a pattern.



What is the perimeter of the seventh figure in the pattern?

The perimeter of a figure is the sum of the lengths of the figure's sides. Since the figures above are regular polygons, all the side lengths in any one figure are equal. The perimeter of the figure is equal to the length of one side multiplied by the number of sides.

Let s represent the length of one side of a polygon.

- The perimeter of the triangle is 6 inches.

  $$3s = 6$$
  $$s = 2$$

  Each side of the triangle has a length of 2 inches.

- The perimeter of the square is 12 inches.

  $$4s = 12$$
  $$s = 3$$

  Each side of the square has a length of 3 inches.

- The perimeter of the pentagon is 20 inches.

  $$5s = 20$$
  $$s = 4$$

  Each side of the pentagon has a length of 4 inches.

- The perimeter of the hexagon is 30 inches.

  $$6s = 30$$
  $$s = 5$$

  Each side of the hexagon has a length of 5 inches.

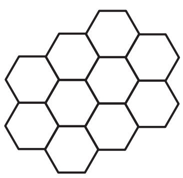
Make a table to show the pattern in these figures.

Figure	Name	Number of Sides	Side Length	Perimeter
1	Regular triangle	3	2	$3 \cdot 2 = 6$
2	Square	4	3	$4 \cdot 3 = 12$
3	Regular pentagon	5	4	$5 \cdot 4 = 20$
4	Regular hexagon	6	5	$6 \cdot 5 = 30$
5	Regular heptagon	7	6	$7 \cdot 6 = 42$
6	Regular octagon	8	7	$8 \cdot 7 = 56$
7	Regular nonagon	9	8	$9 \cdot 8 = 72$

The perimeter of the seventh figure in the pattern is 72 inches.

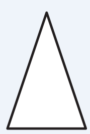
What Is a Tessellation?

A **tessellation** is a repeating pattern of figures that fills a plane completely. The figures do not overlap, and there are no gaps between them. Imagine tiles that fit together on an infinitely large floor. Many shapes tessellate. For example, squares tessellate—you can fill a plane completely with just square tiles. Regular hexagons and many other polygons also tessellate.

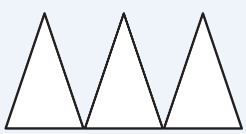


Not all regular shapes tessellate. Regular pentagons and octagons, if used alone, will not tessellate. Not all shapes that tessellate are regular, either. For example, rectangles are not regular polygons, but they tessellate—you can fill a plane with rectangular tiles.

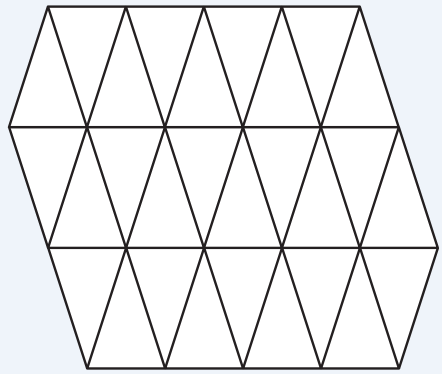
Can the isosceles triangle shown below be used alone to form a tessellation?



If the triangles are simply placed side by side, they will not form a tessellation, because there will be gaps.

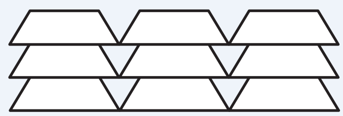


But you can create a tessellation by using the isosceles triangle together with its rotated image.

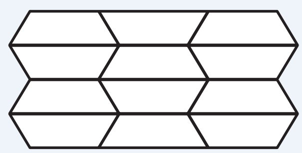


Can the isosceles trapezoid shown below be used to generate a tessellation?

Placing such trapezoids side by side does not form a tessellation, because there are gaps between the figures.



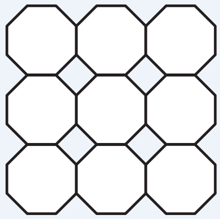
Combining the isosceles trapezoid with a 180° rotation of the trapezoid will form a tessellation.



Sometimes we can combine figures to tessellate a plane.

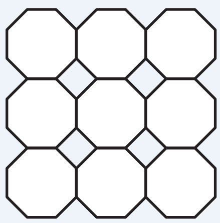
Use a regular octagon and a square to create a tessellation.

Regular octagons alone cannot be used to form a tessellation.



When the octagons are placed next to each other, there are gaps between the figures. The gaps are formed by the sides of four octagons. Since the octagons are regular, each side of the gap is the same length. The gap is in the shape of a square.

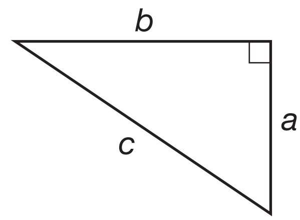
Combining a regular octagon with a square that has sides the same length as the octagon will create a tessellation.



Objective 6

What Patterns in Right Triangles Can You Use to Solve Problems?

The **Pythagorean Theorem**, $a^2 + b^2 = c^2$, can be used to solve problems involving the lengths of sides of right triangles.



$$a^2 + b^2 = c^2$$

See Objective 8, page 200, for more information about the Pythagorean Theorem.

A set of three whole numbers that satisfy the Pythagorean Theorem is called a **Pythagorean triple**. Pythagorean triples can be used to represent the side lengths of a right triangle.

Multiply each of the numbers in a Pythagorean triple by any whole number to find another Pythagorean triple.

The set {3, 4, 5} is a Pythagorean triple.

$$3^2 + 4^2 = 5^2$$
$$9 + 16 = 25$$
$$25 = 25$$

A triangle with side lengths of 3 units, 4 units, and 5 units would form a right triangle.

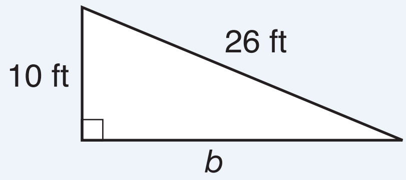
Multiply each integer in the Pythagorean triple {3, 4, 5} by 2 to obtain the set {6, 8, 10}.

This set of integers is also a Pythagorean triple.

$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = 10^2$$
$$36 + 64 = 100$$
$$100 = 100$$

A triangle with side lengths of 6 units, 8 units, and 10 units would also form a right triangle, with 10, the longest side, being its hypotenuse.

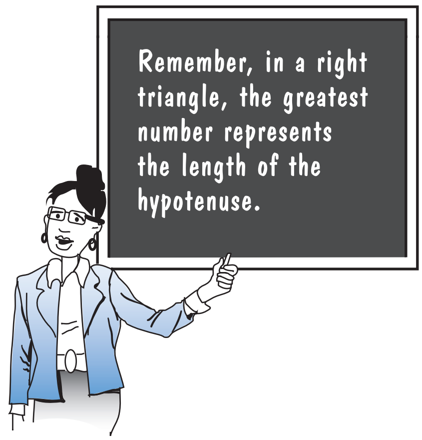
Find the length of side *b* in the right triangle below.



One way to find the length of side *b* is to use a Pythagorean triple.

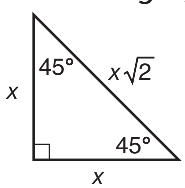
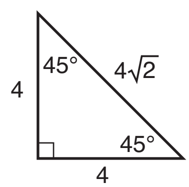
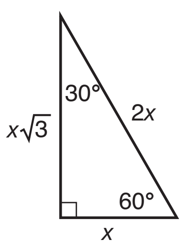
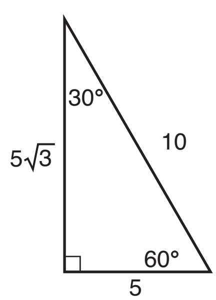
- Multiply the Pythagorean triple {5, 12, 13} by 2 to obtain the set {10, 24, 26}.
- Since {5, 12, 13} is a Pythagorean triple, the set {10, 24, 26} is also a Pythagorean triple.
- The side lengths of the right triangle in the diagram are equal to two of the values in the set {$\underline{10}$, 24, $\underline{26}$}.
- The third side length of the right triangle is equal to the remaining value in the set {10, $\underline{24}$, 26}.

Side *b* has a length of 24 feet.



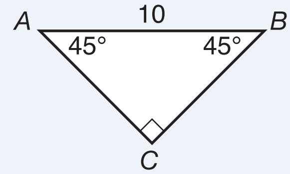
What Are the Side Relationships in Special Right Triangles?

There are two types of triangles known as special right triangles. These special right triangles occur frequently in everyday problems, so you should know the relationships of their sides.

Special Right Triangle	Side Relationships	Example
45°-45°-90° Triangle (Isosceles Right) 	The lengths of the two legs are equal. The length of the hypotenuse is the length of a leg times $\sqrt{2}$.	
30°-60°-90° Triangle 	The length of the hypotenuse is twice the length of the shorter leg. The length of the longer leg is equal to the length of the shorter leg times $\sqrt{3}$.	

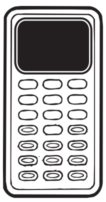
Objective 6

Find the length of side AC in the 45°-45°-90° triangle below.

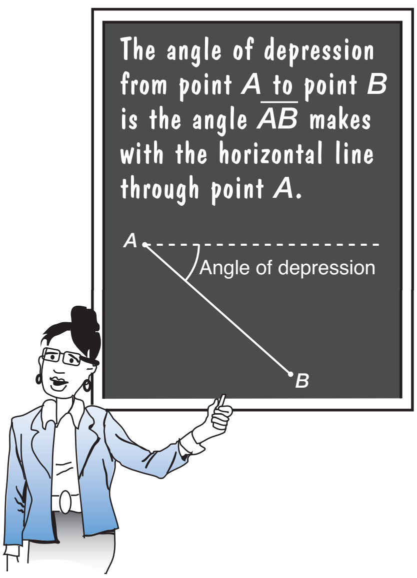


Triangle ABC is a 45°-45°-90° triangle. Identify its parts and their relationships.

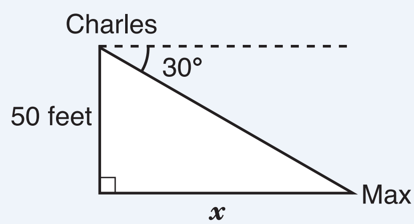
- In this special triangle the hypotenuse is equal to the length of a leg times $\sqrt{2}$.
- $\overline{AB}$ is the hypotenuse. $\overline{AC}$ is a leg.



$$AB = AC \cdot \sqrt{2}$$
$$10 = AC \cdot \sqrt{2}$$
$$\frac{10}{\sqrt{2}} = AC$$
$$7.07 \approx AC$$



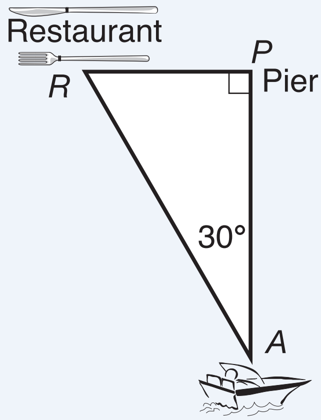
Charles is looking out a window from a point 50 feet above the ground. When Charles looks down at an angle of depression of 30°, he sees his dog Max. To the nearest foot, how far is Max from the base of the building?



- The triangle formed is a 30°-60°-90° triangle.
- The shorter leg is 50 feet long.
- The length of the longer leg is the distance Max is from the base of the building.
- The length of the longer leg is equal to the length of the shorter leg times $\sqrt{3}$.
- Max is $50 \cdot \sqrt{3} \approx 86.6$ feet from the building.

Max is approximately 87 feet from the base of the building.

Consuelo is on a boat 3 miles due south of the pier. She wants to go to a restaurant that is due west of the pier, as shown below. Find the approximate distance Consuelo must travel to reach the restaurant.



- The boat, restaurant, and pier form a right triangle. The given angles measure 30° and 90°. The other angle of the triangle measures $180° - (90° + 30°) = 60°$.
- Use the properties of 30°-60°-90° triangles.
- You need to find the length of $\overline{AR}$, the segment connecting the boat to the restaurant. $\overline{AR}$ is the hypotenuse of the triangle.
- The distance from the boat to the pier is given. $AP = 3$. This is the longer leg of the triangle.

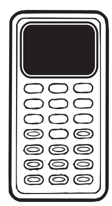
  The length of the longer leg of a 30°-60°-90° triangle is equal to the length of the shorter leg times $\sqrt{3}$.
  Let x equal the length of the shorter leg, $\overline{RP}$. Solve for x.

$$x\sqrt{3} = 3$$
$$x = \frac{3}{\sqrt{3}} \approx 1.73$$
$$RP \approx 1.73$$

- The length of the hypotenuse, $\overline{AR}$, is 2 times the length of the shorter leg. Substitute $RP \approx 1.73$.

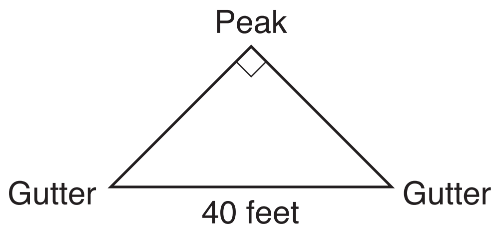
$$AR \approx 2(1.73)$$
$$AR \approx 3.46$$

Consuelo must travel approximately 3.46 miles to reach the restaurant.



Try It

A peaked roof on a house has a 90° angle at the top. The roof is the same length on each side of the peak. The distance from gutter to gutter is 40 feet. Find the approximate distance from the gutter to the peak of the roof.



The edges of the roof form a ________-________-90° triangle. The distance from gutter to gutter is ________ feet. This side is the ________________ of the triangle. The length of the hypotenuse of a 45°-45°-90° triangle is the length of a leg times ________.

Let x = the length of the leg. Write an equation and solve for x.

$$\underline{\qquad\qquad} = \underline{\qquad\qquad}$$

$$x = \frac{\square}{\square} \approx \underline{\qquad\qquad}$$

The distance from the gutter to the peak of the roof is approximately ________ feet.

The edges of the roof form a 45°-45°-90° triangle. The distance from gutter to gutter is 40 feet. This side is the hypotenuse of the triangle. The length of the hypotenuse of a 45°-45°-90° triangle is the length of a leg times $\sqrt{2}$.

$$x\sqrt{2} = 40$$

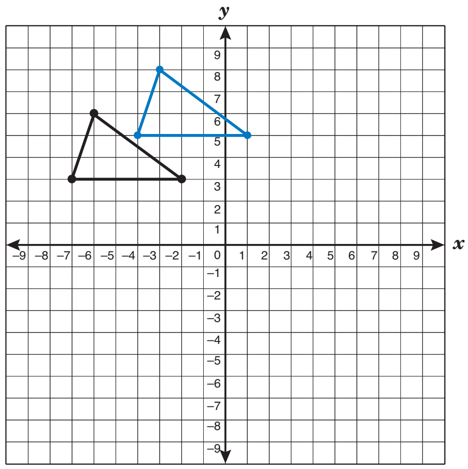
$$x = \frac{40}{\sqrt{2}} \approx 28.28$$

The distance from the gutter to the peak of the roof is approximately 28.28 feet.

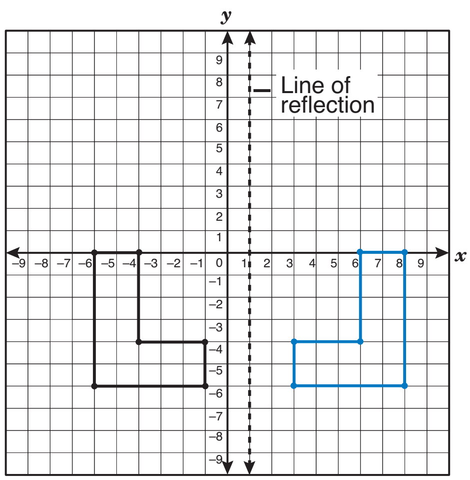
How Do You Use Transformations to Solve Problems?

Translations, reflections, and rotations are transformations of geometric figures that do not change the lengths of the segments of the figures. The original figure and its transformed image are congruent.

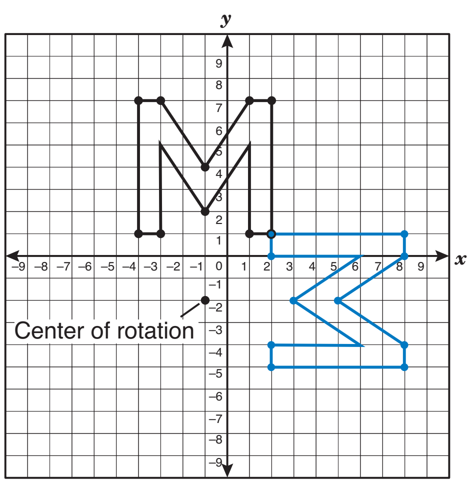
A **translation** is a movement of a figure up or down and to the left or to the right on a coordinate plane. Each point in the translated image moves the same number of units.



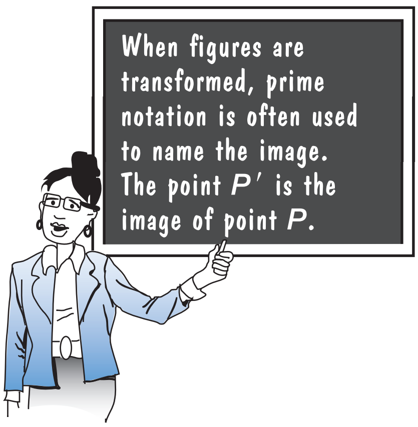
A **reflection** creates a mirror image of a figure across a line. Each point in the reflected image is the same distance from the line of reflection as its corresponding point in the original figure.



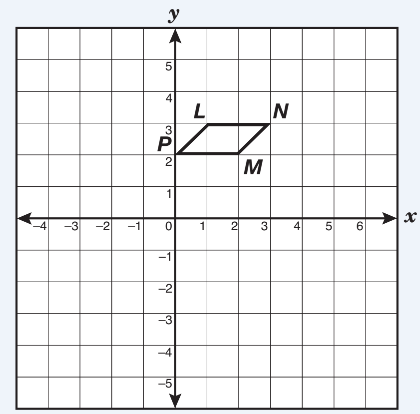
A **rotation** is the turning of a figure about a point called the center of rotation. Each point in the rotated image is moved the same number of degrees in a circle around that center of rotation.



Objective 6



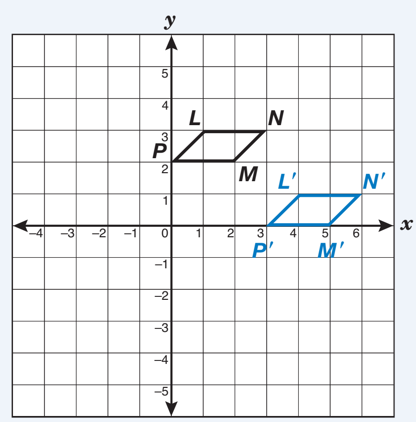
The graph of quadrilateral *MNLP* is shown below.



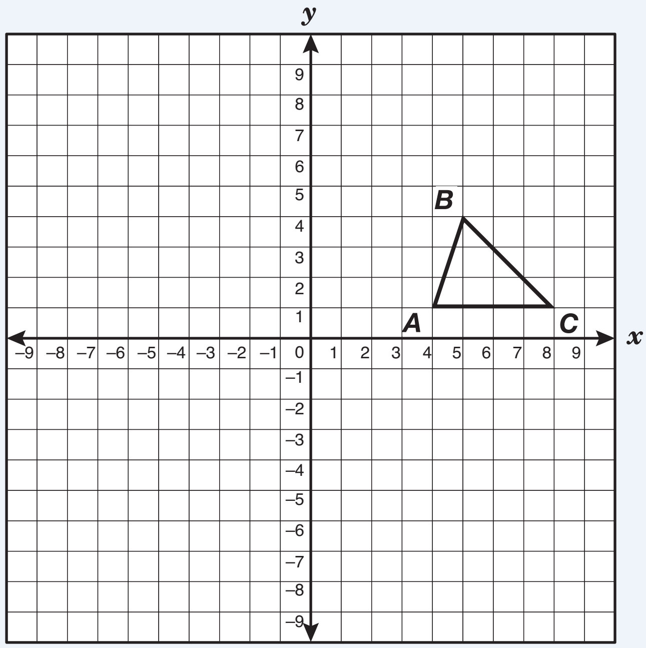
Find the image of quadrilateral *MNLP* translated 3 units right and 2 units down.

- Point *M* has coordinates (2, 2). Use the graph to find the location of M'. Start at *M* and count 3 units to the right and 2 units down. The coordinates of M' are (5, 0).
- Point *N* has coordinates (3, 3). Use the graph to find the location of N'. Start at *N* and count 3 units to the right and 2 units down. The coordinates of N' are (6, 1).
- Point *L* has coordinates (1, 3). Use the graph to find the location of L'. Start at *L* and count 3 units to the right and 2 units down. The coordinates of L' are (4, 1).
- Point *P* has coordinates (0, 2). Use the graph to find the location of P'. Start at *P* and count 3 units to the right and 2 units down. The coordinates of P' are (3, 0).

The graph of quadrilateral $M'N'L'P'$ is shown below.



The graph of $\triangle ABC$ is shown below.



Find the coordinates of $\triangle A'B'C'$, the image of $\triangle ABC$ reflected across the y-axis and translated 2 units up.

- Reflect A (4, 1) across the y-axis. The x-coordinate of A' is the negative of the x-coordinate of A. The x-coordinate of A' is -4.

  If the point is then translated 2 units up, the y-coordinate of A' will be 2 more than the y-coordinate of A. $1 + 2 = 3$.

  A' has coordinates $(-4, 3)$.

- Reflect B (5, 4) across the y-axis. The x-coordinate of B' is the negative of the x-coordinate of B. The x-coordinate of B' is -5.

  If the point is then translated 2 units up, the y-coordinate of B' will be 2 more than the y-coordinate of B. $4 + 2 = 6$.

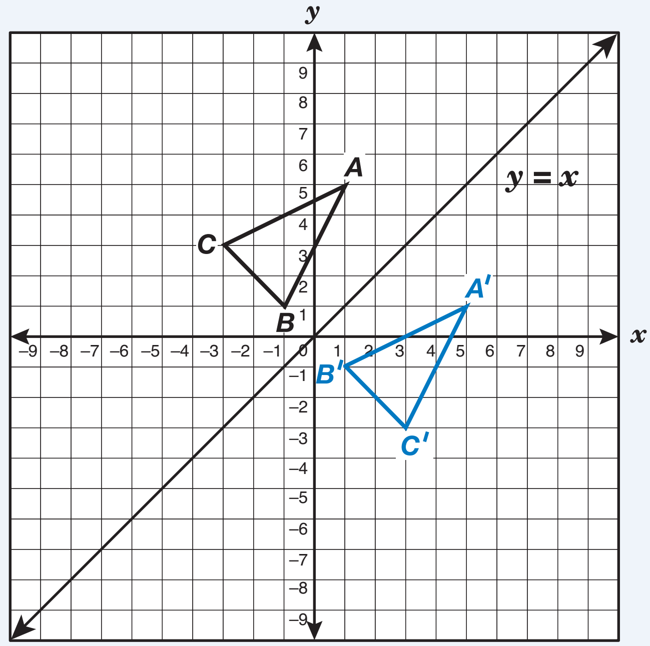
  B' has coordinates $(-5, 6)$.

- Reflect C (8, 1) across the y-axis. The x-coordinate of C' is the negative of the x-coordinate of C. The x-coordinate of C' is -8.

  If the point is then translated 2 units up, the y-coordinate of C' will be 2 more than the y-coordinate of C. $1 + 2 = 3$.

  C' has coordinates $(-8, 3)$.

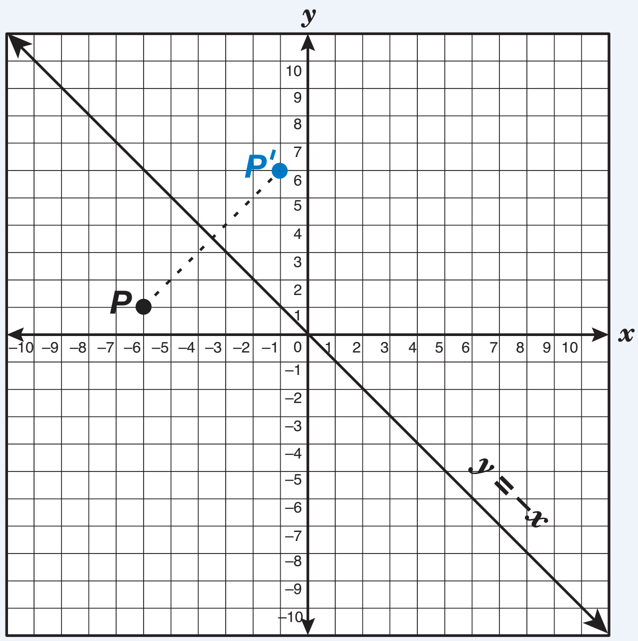
$\triangle ABC$ is shown on the graph below. If $\triangle ABC$ is reflected across the line $y = x$, what are the coordinates of A', B', and C'?



- Notice that for reflections across $y = x$, both coordinates will change. For any point with coordinates (x, y), the image after a reflection across $y = x$ will have the coordinates (y, x). That is, the order of the coordinates will be reversed.
- When A (1, 5) is reflected across $y = x$, we reverse the coordinate values to get A'. The coordinates of A' are (5, 1).
- When B (−1, 1) is reflected across $y = x$, we reverse the coordinate values to get B'. The coordinates of B' are (1, −1).
- When C (−3, 3) is reflected across $y = x$, we reverse the coordinate values to get C'. The coordinates of C' are (3, −3).

When $\triangle ABC$ is reflected across $y = x$, the coordinates of A' become (5, 1), B' (1, −1), and C' (3, −3).

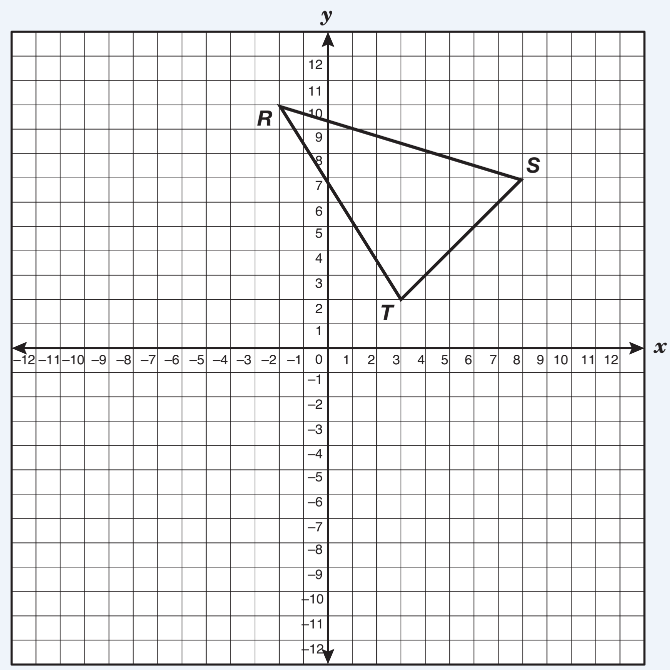
If point P $(-6, 1)$ is reflected across the line $y = -x$, what are the coordinates of P'?



- Notice that for reflections across $y = -x$, both coordinates will change. For any point with coordinates (x, y), the image after a reflection across $y = -x$ will have the coordinates $(-y, -x)$. That is, the order of the coordinates will be reversed, and the signs will change.
- When P $(-6, 1)$ is reflected across $y = -x$, we reverse the coordinate values and change the signs to get P'. The coordinates of P' are $(-1, 6)$.

When P $(-6, 1)$ is reflected across $y = -x$, the coordinates of P' are $(-1, 6)$.

The graph of $\triangle RST$ is shown below. Find the coordinates of S' if $\triangle RST$ is rotated 90° clockwise about point T.



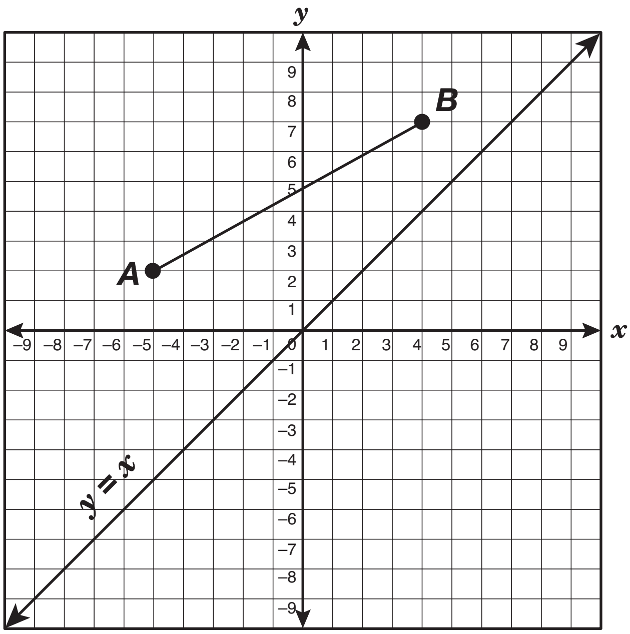
- If $\triangle RST$ is rotated 90° clockwise about point T, $\overline{ST}$ will be perpendicular to $\overline{S'T}$.
- The slopes of perpendicular lines are negative reciprocals. If the slope of $\overline{ST}$ is $\frac{5}{5}$, then the slope of $\overline{S'T}$ should be $-\frac{5}{5}$.
- Use the slope of $\overline{S'T}$, $-\frac{5}{5}$, to find point S'. Starting at point T, count 5 units down and 5 units to the right.

The point S' will have coordinates $(8, -3)$.

See Objective 3, page 69, for more information about slope.

Try It

The graph below shows the line $y = x$ and $\overline{AB}$ with endpoints at $(-5, 2)$ and $(4, 7)$. What are the coordinates of the endpoints of $\overline{A'B'}$ after $\overline{AB}$ is reflected across $y = x$?



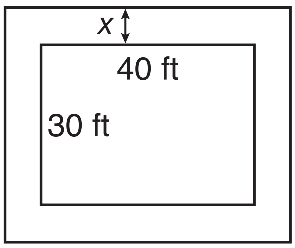
When reflecting across $y = x$, the order of the coordinates for any point is ________. When A $(-5, 2)$ is reflected across $y = x$, this gives us A' ________. When B $(4, 7)$ is reflected across $y = x$, this gives us B' ________.

When reflecting across $y = x$, the order of the coordinates for any point is reversed. When A (–5, 2) is reflected across $y = x$, this gives us A' (2, –5). When B (4, 7) is reflected across $y = x$, this gives us B' (7, 4).

Now practice what you've learned.

Question 54

Mike is building a sidewalk of uniform width around his 30-by-40-foot rectangular swimming pool. Which of the following expressions represents the total area of the sidewalk in terms of x, the width of the sidewalk?



A $(30 + 2x)(40 + 2x)$

B $(30 - 2x)(40 - 2x) + 30(40)x$

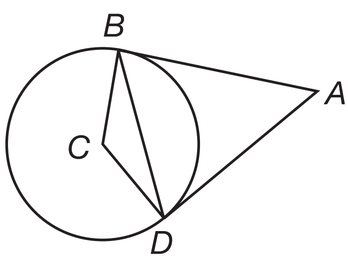
C $30(40)x$

D $(30 + 2x)(40 + 2x) - 30(40)$

Answer Key: page 289

Question 55

Segments AB and AD are tangents to circle C. The measure of $\angle A$ is 52°.



What is $m\angle CBD$?

A 52°

B 64°

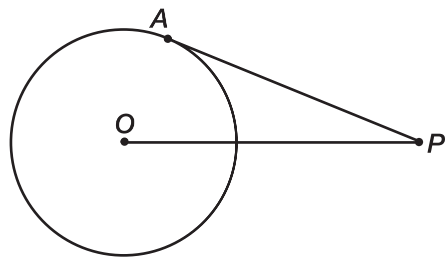
C 26°

D 128°

Answer Key: page 289

Question 56

Point P is 26 centimeters from the center of a circle with a radius of 10 cm.



Find the length of the tangent AP drawn to circle O from point P.

A 12 cm

B 24 cm

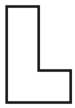
C 16 cm

D 28 cm

Answer Key: page 290

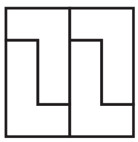
Question 57

Look at the shape below.

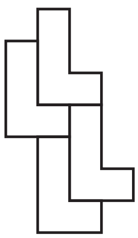


Which of these patterns can be made using only translations, rotations, or a combination of translations and rotations?

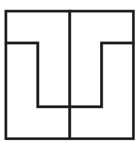
I



II



III



A I and II only

B I and III only

C II and III only

D I, II, and III

Answer Key: page 290

Question 58

Ms. Clay asked her students which of these polygons tessellate a plane.

I. An equilateral triangle

II. A square

III. A regular hexagon

IV. An octagon

Which of the following student responses is correct?

A I and II only

B I, II, and III only

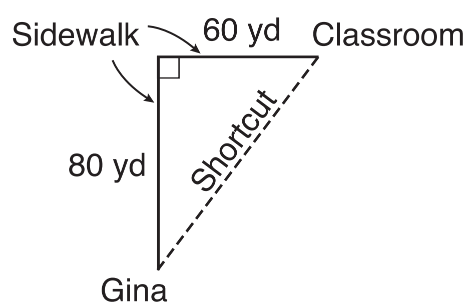
C I, II, and IV only

D II, III, and IV only

Answer Key: page 290

Question 59

Gina is in a hurry to get to her next class. If she takes a shortcut across the grass instead of following the sidewalk, how many yards does she save?



A 40 yd

B 50 yd

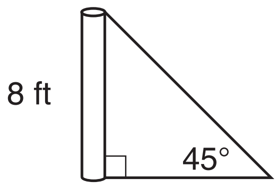
C 70 yd

D 30 yd

Answer Key: page 290

Question 60

Antoinette attaches a wire from the top of an 8-foot pole to the ground. The wire makes an angle of elevation of 45° with the ground.



What is the approximate length of the wire?

A 5.7 ft

B 11.3 ft

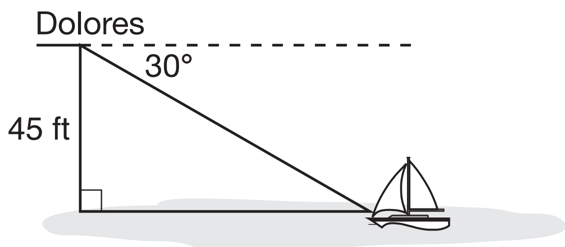
C 13.8 ft

D 9.7 ft

Answer Key: page 290

Question 61

Dolores is on a bridge that is 45 feet above a lake. She sees a boat at a 30° angle of depression. What is Dolores's approximate horizontal distance from the boat?



A 90 ft

B 26 ft

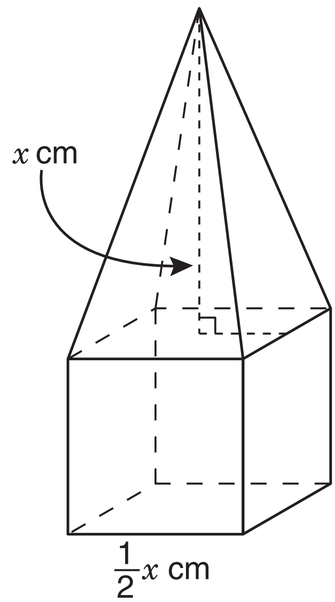
C 32 ft

D 78 ft

Answer Key: page 290

Question 62

A square pyramid sits on top of a cube, as shown below.



Which equation best represents V, the volume of this composite solid in cubic centimeters?

A $V = \frac{3}{8}x^3$

B $V = \frac{5}{24}x^3$

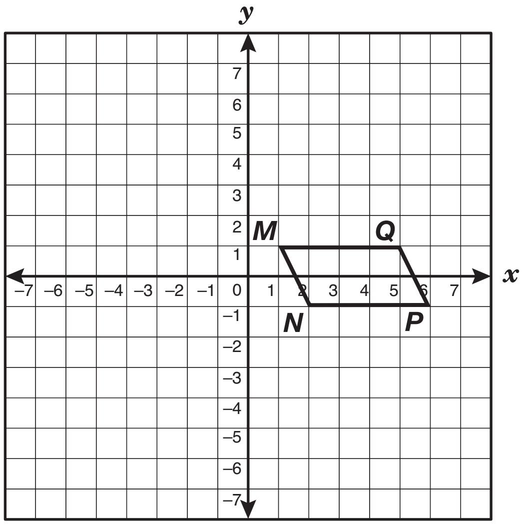
C $V = \frac{5}{6}x^3$

D $V = \frac{1}{4}x^3$

Answer Key: page 290

Question 63

Which set of coordinates represents the vertices of the image of quadrilateral *MNPQ* after a translation of 4 units left and 2 units up?



A (−3, −1), (−2, −3), (2, −3), (1, −1)

B (−3, 3), (−2, 2), (2, 2), (1, 3)

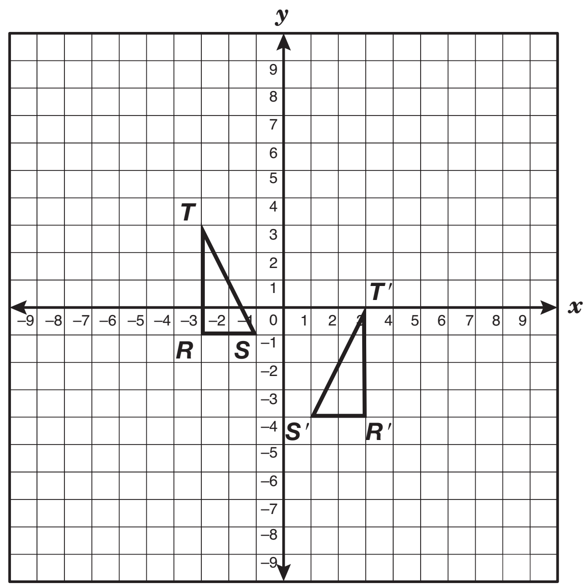
C (−3, 3), (−2, 1), (2, 1), (1, 3)

D (3, −3), (4, −5), (8, −5), (7, −5)

Answer Key: page 291

Question 64

Which transformation of $\triangle TRS$ creates $\triangle T'R'S'$ shown below?



A Reflect $\triangle TRS$ across the *y*-axis and then translate it 3 units down

B Translate $\triangle TRS$ 4 units right and 3 units down

C Reflect $\triangle TRS$ across the *x*-axis and then translate it 6 units down

D Translate $\triangle TRS$ 6 units right and 3 units down

Answer Key: page 291

Objective 7

The student will demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.

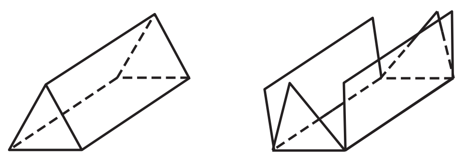
For this objective you should be able to

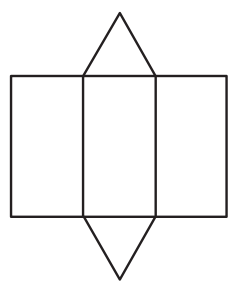
- analyze the relationship between three-dimensional objects and related two-dimensional representations and use these to solve problems;
- understand that coordinate systems provide convenient and efficient ways of representing geometric figures and use them accordingly; and
- analyze properties and describe relationships in geometric figures.

How Can a Net Be Used to Represent a 3-Dimensional Object?

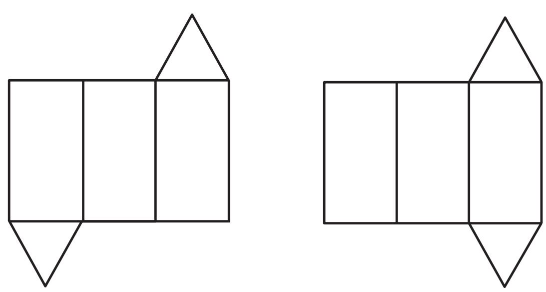
A **net** for a 3-dimensional object is a 2-dimensional representation of that object. If you unfold a 3-dimensional figure, such as a cube or a prism, the resulting 2-dimensional drawing is called a net.

If you unfold a triangular prism, you get a 2-dimensional net such as the one shown below.

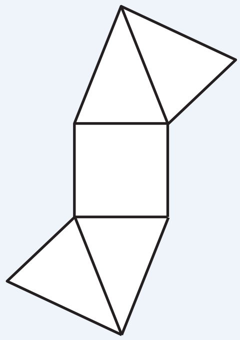




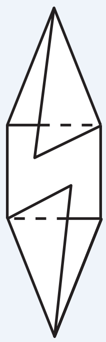
Since there is more than one way to unfold the prism, the nets below could also result from unfolding this triangular prism.



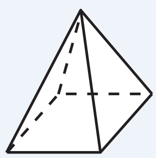
What 3-dimensional figure does this net represent?



- Fold the two outside triangles upward.



- Now fold the two inside triangles upward. The four triangles form the faces of the figure.
- The square forms the base of the figure.

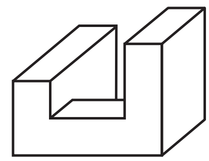


The figure is a pyramid. Its base is a square. This net represents a square pyramid.

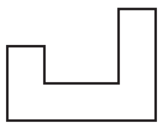
How Do You Recognize a Three-Dimensional Figure from Different Perspectives?

Another way to represent a three-dimensional figure is to make drawings of the figure from different views: from the front, the top, or a side. To recognize the figure from different perspectives, visualize what the figure would look like if you were seeing it from those points of view.

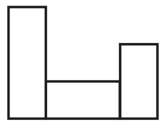
The drawings below show a three-dimensional figure and what it looks like when viewed from the top, the front, and the sides.



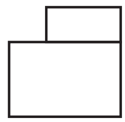
Front



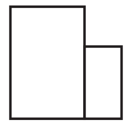
Front view



Top view



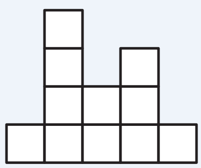
Left-side view



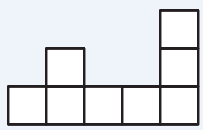
Right-side view

When given three different views of the same figure, visualize what that figure looks like in three dimensions. To recognize the figure in a three-dimensional view, look at each two-dimensional view and think about what could be hidden behind or below what is shown.

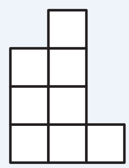
The front view, the top view, and a side view of a three-dimensional figure are shown.



Front view

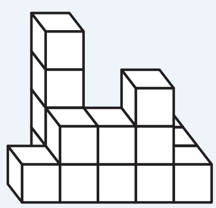


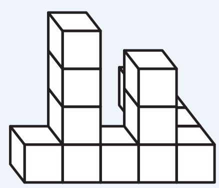
Top view



Side view

Which three-dimensional figure is represented by these views?



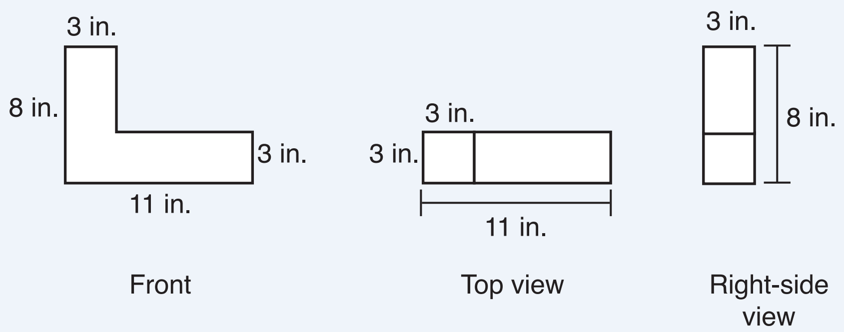


Compare the two-dimensional views to the three-dimensional figures. Only the figure on the left has the same shape when viewed from each of the three perspectives.

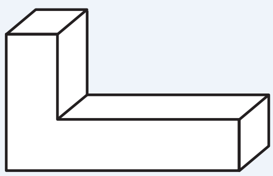
Perspective views of a figure can be used to calculate its volume or surface area.

- Use the views provided to determine the shape of the figure.
- Choose the appropriate formula for the volume or surface area of the figure.
- Then use the dimensions in the views provided to determine the dimensions of the figure.

What is the surface area of the three-dimensional figure with the dimensions shown in the three views below?



Use the three views to determine the shape of the figure.



To find the surface area, calculate the sum of the areas of all the faces.

- Calculate the area of the face shown in the front view by dividing it into 2 rectangles and adding the areas of the rectangles.

  The horizontal rectangle has a length of 11 inches and a width of 3 inches.

  $$A = 11 \cdot 3 = 33 \text{ in.}^2$$

  The vertical rectangle has a length of 3 inches and a height of $8 - 3 = 5$ inches.

  $$A = 3 \cdot 5 = 15 \text{ in.}^2$$

  The sum of their areas is $33 + 15 = 48$ square inches.

- The back face, which is not shown, has the same dimensions as the face in the front view. The back face has an area of 48 in.^{2}

- Calculate the area of the face shown in the top view. The face has a length of 11 inches and a width of 3 inches.

$$A = 11 \cdot 3 = 33 \text{ in.}^2$$

- The bottom face, which is not shown, has the same dimensions as the face shown in the top view. The bottom face has an area of 33 in.^{2}
- Calculate the area of the face shown in the right-side view. The height of the face is 8 inches; the width is 3 inches.

$$A = 8 \cdot 3 = 24 \text{ in.}^2$$

- The left face, which is not shown, has the same dimensions as the face shown in the right side view. The left face has an area of 24 in.^{2}
- The surface area of the figure is the sum of the areas of all the faces.

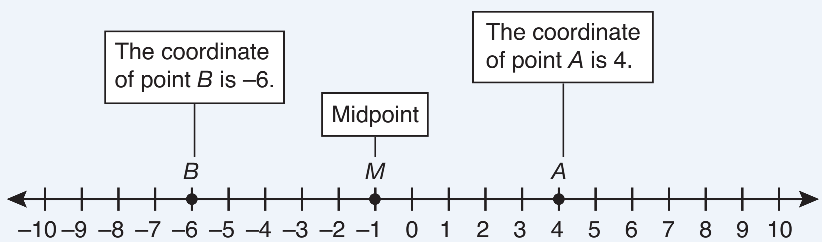
$$A = 48 + 48 + 33 + 33 + 24 + 24$$
$$A = 210 \text{ in.}^2$$

The surface area of the three-dimensional figure with the views shown is 210 square inches.

How Do You Locate and Name Points on a Line or Plane?

A number line is used to locate and name points on a line.

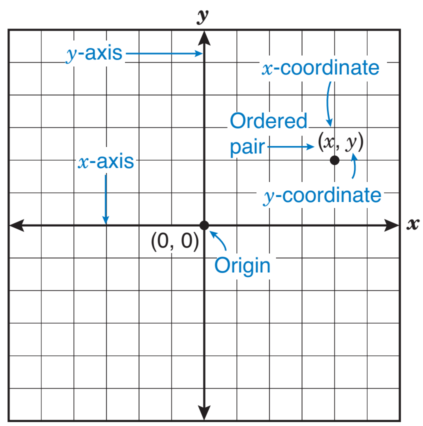
Find the coordinate of the midpoint of $\overline{AB}$.



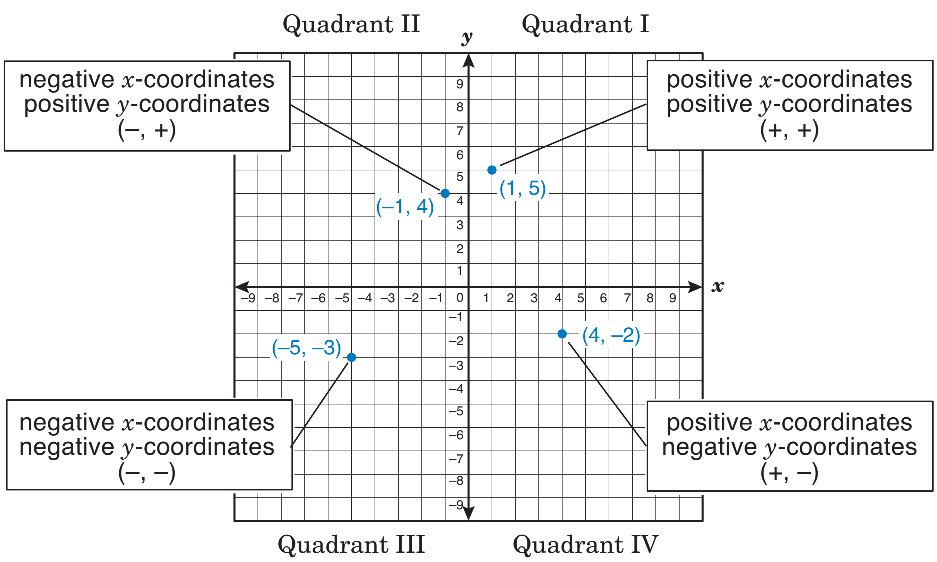
Do you see that . . .

The coordinate of the midpoint of a segment is the average of the coordinates of its endpoints. The midpoint of $\overline{AB}$ has the coordinate $\frac{(4 + -6)}{2} = \frac{-2}{2} = -1$.

A **coordinate grid** is used to locate and name points on a plane. A coordinate grid is formed by two perpendicular number lines.



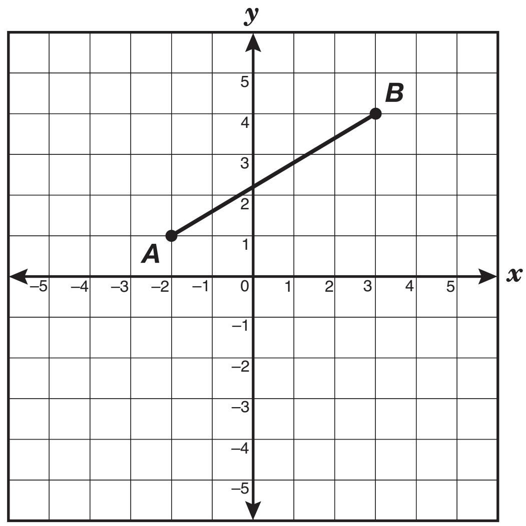
The *x*-axis and *y*-axis divide the coordinate plane into four regions called **quadrants**. The quadrants are usually referred to by the Roman numerals I, II, III, and IV.



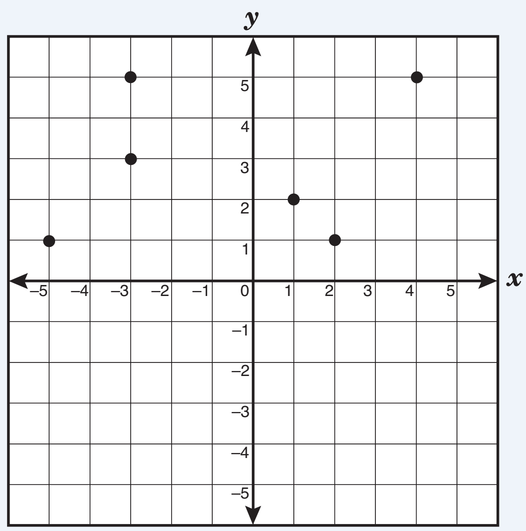
Objective 7

Two ordered pairs can be used to name the endpoints of a line segment.

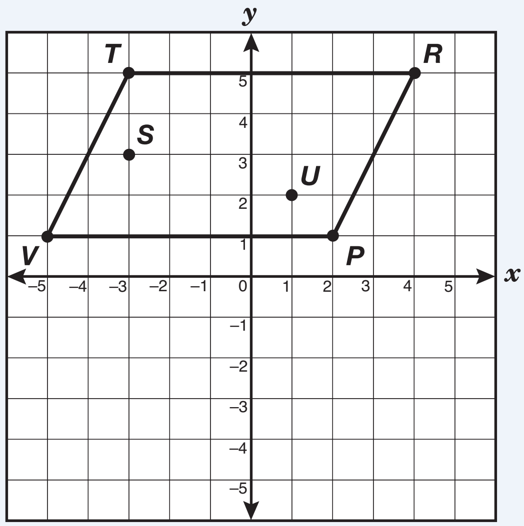
For example, $\overline{AB}$ has the ordered pairs $(-2, 1)$ and $(3, 4)$ as its endpoints.



The points graphed below are as follows: $P\ (2, 1)$, $R\ (4, 5)$, $S\ (-3, 3)$, $T\ (-3, 5)$, $U\ (1, 2)$, and $V\ (-5, 1)$. Name the figure formed by drawing $\overline{PR}$, $\overline{RT}$, $\overline{TV}$, and $\overline{VP}$.



Label these points: $P\ (2, 1)$, $R\ (4, 5)$, $T\ (-3, 5)$, and $V\ (-5, 1)$. Draw the line segments that connect them.



$\overline{TV}$ and $\overline{RP}$ are congruent and parallel. $\overline{TR}$ and $\overline{VP}$ are also congruent and parallel. A quadrilateral in which both pairs of opposite sides are congruent and parallel is a parallelogram.

Figure *PRTV* is a parallelogram.

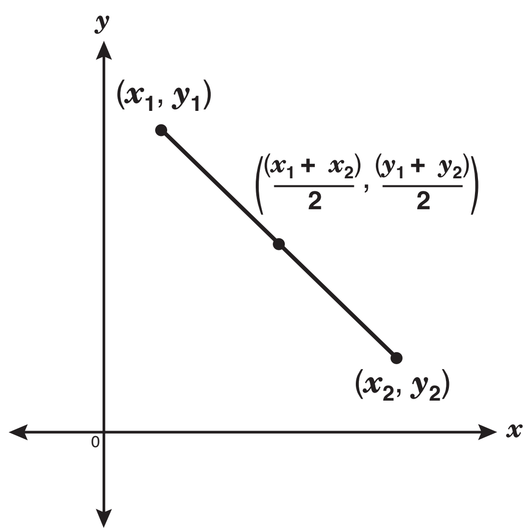
The **midpoint** of a line segment is the point that lies halfway between the segment's endpoints and divides the segment into two congruent parts. The coordinates of the midpoint are the average of the coordinates of the endpoints.

One way to find the midpoint of a line segment is to use the following formula.

Midpoint Formula

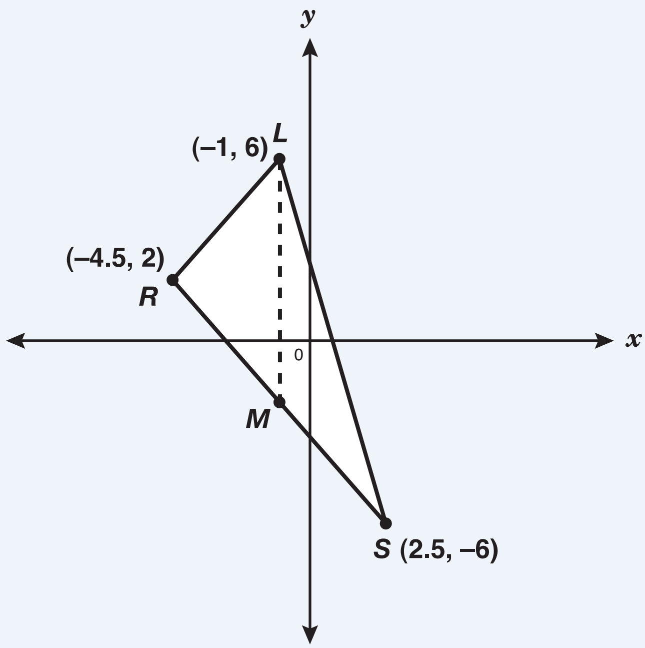
For any two points (x_1, y_1) and (x_2, y_2), the coordinates of the midpoint of the line segment they determine are given by the formula below.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



A **median** of a triangle is a line segment drawn from a vertex to the midpoint of the opposite side.

Triangle *LRS* has the following vertices: *L* (−1, 6), *R* (−4.5, 2), and *S* (2.5, −6). Find the coordinates of point *M*, the endpoint of median *LM* drawn to $\overline{RS}$.



Median *LM* is drawn from vertex *L* to point *M*, the midpoint of $\overline{RS}$.

Find the coordinates of point *M*.

- The *x*-coordinates of the endpoints of $\overline{RS}$ are −4.5 and 2.5.
- The *y*-coordinates of the endpoints of $\overline{RS}$ are 2 and −6.
- Substitute these values into the midpoint formula.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M = \left(\frac{-4.5 + 2.5}{2}, \frac{2 + (-6)}{2}\right)$$

$$M = \left(\frac{-2}{2}, \frac{-4}{2}\right)$$

$$M = (-1, -2)$$

The coordinates of point *M* are (−1, −2).

Objective 7

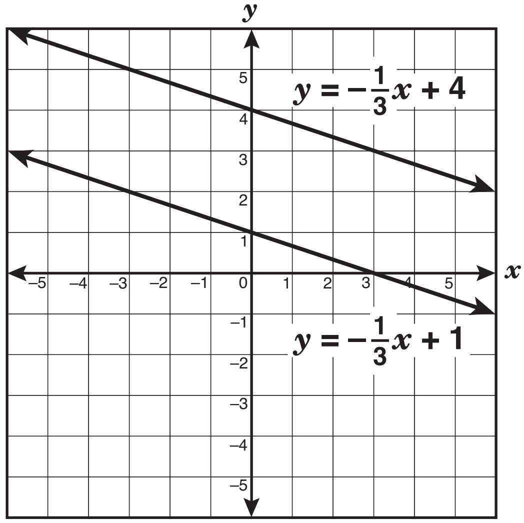
How Do You Use Slope to Investigate Geometric Relationships?

Slopes of lines can tell you whether the lines are parallel or perpendicular.

- If two lines are parallel, their equations will have the same value for the slope, m. Look at the graphs of these two equations:

$$y = -\frac{1}{3}x + 1 \text{ and } y = -\frac{1}{3}x + 4$$

Parallel lines

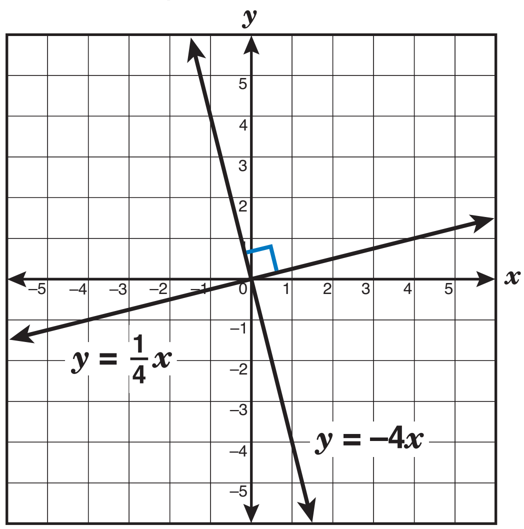


See Objective 3, page 69, for more information about slope.

- If two lines are perpendicular, then their slopes will be negative reciprocals of each other. Look at the graphs of these equations:

$$y = -4x \text{ and } y = \frac{1}{4}x$$

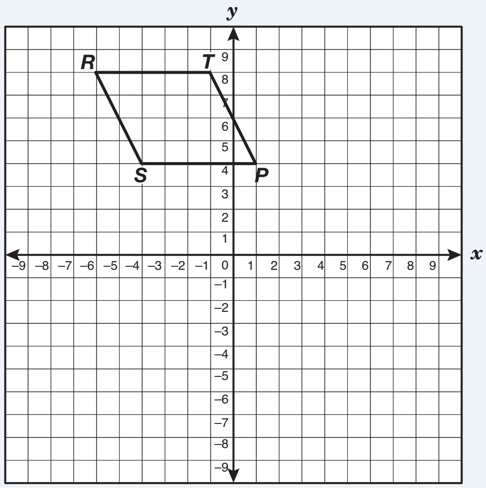
Perpendicular lines



Two numbers are negative reciprocals of each other if their product is -1. For example, $\frac{2}{3}$ and $-\frac{3}{2}$ are negative reciprocals.

- One fraction is the reciprocal of the other, and one fraction is the negative of the other.
- Since $\frac{2}{3} \cdot \frac{-3}{2} = \frac{-6}{6} = -1$, they are negative reciprocals.

Parallelogram *PTRS* is shown below.



What are the equations of the lines that contain $\overline{TP}$ and $\overline{RS}$? Since *PTRS* is a parallelogram, $\overline{TP}$ and $\overline{RS}$ are parallel. This means that the lines containing $\overline{TP}$ and $\overline{RS}$ have the same slope.

- Find the slope of the line containing $\overline{TP}$.

  The vertices T $(-1, 8)$ and P $(1, 4)$ are two points on the line.

  The slope of the line containing $\overline{TP}$ is

  $$m = \frac{4 - 8}{1 - (-1)} = \frac{-4}{2} = -2$$

- This line crosses the y-axis at the point $(0, 6)$; the y-intercept, b, is 6.

- Substitute $m = -2$ and $b = 6$ into the slope-intercept equation, $y = mx + b$. The equation of the line that contains $\overline{TP}$ is $y = -2x + 6$.

- The line that contains $\overline{RS}$ also has a slope of -2.

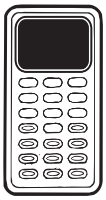
- Use the slope and a point on the line to find the y-intercept.

  The vertex R $(-6, 8)$ is a point on the line that contains $\overline{RS}$.

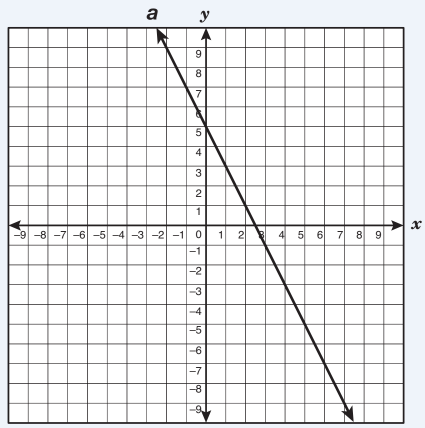
  Substitute $m = -2$, $x = -6$, and $y = 8$ into the slope-intercept equation and solve for b.

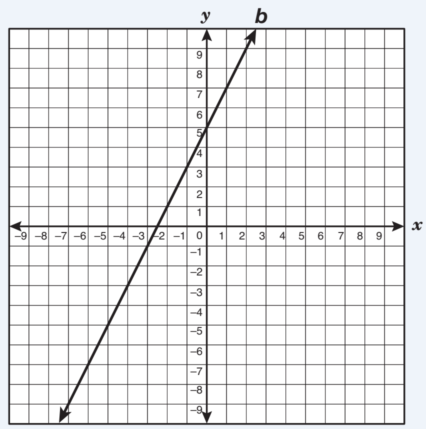
  $$\begin{aligned} y &= mx + b \\ 8 &= -2(-6) + b \\ 8 &= 12 + b \\ -4 &= b \end{aligned}$$

- Substitute $m = -2$ and $b = -4$ into the slope-intercept equation, $y = mx + b$. The equation of the line that contains $\overline{RS}$ is $y = -2x - 4$. The equations of the lines that contain $\overline{TP}$ and $\overline{RS}$ are $y = -2x + 6$ and $y = -2x - 4$, respectively.



Which line graphed below is perpendicular to the line $y = \frac{1}{2}x + 5$?





Find the slope of the line $y = \frac{1}{2}x + 5$. The equation is in the form $y = mx + b$; therefore, its slope, m, is $\frac{1}{2}$.

Which graph has a slope equal to the negative reciprocal of $\frac{1}{2}$?

The negative reciprocal of $\frac{1}{2}$ is $-\frac{2}{1}$, or -2.

	Line *a*	Line *b*
Pick any two points on the graph.	(0, 5), (4, −3)	(0, 5), (−2, 1)
Find the change in *y*-values, or the rise.	$y_2 - y_1 = -3 - 5$ $y_2 - y_1 = -8$	$y_2 - y_1 = 1 - 5$ $y_2 - y_1 = -4$
Find the change in *x*-values, or the run.	$x_2 - x_1 = 4 - 0$ $x_2 - x_1 = 4$	$x_2 - x_1 = -2 - 0$ $x_2 - x_1 = -2$
Write the slope as the ratio of rise to run.	$\frac{-8}{4} = -2$	$\frac{-4}{-2} = \frac{4}{2} = 2$

Line *a* has a slope of -2, so line *a* is perpendicular to the line $y = \frac{1}{2}x + 5$.

How Do You Find the Length of a Line Segment?

The length of a line segment is the distance between its endpoints.

Use the distance formula to find the length of a line segment.

Distance Formula

The length of a line segment with endpoints (x_1, y_1) and (x_2, y_2) is shown below.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

What is the length of a line segment with endpoints (6, 4) and (2, 7)?

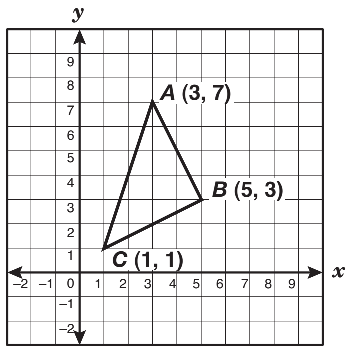
- The x-coordinates of the endpoints are $x_1 = 6$ and $x_2 = 2$.
- The y-coordinates of the endpoints are $y_1 = 4$ and $y_2 = 7$.
- Substitute these values into the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(2 - 6)^2 + (7 - 4)^2}$$
$$d = \sqrt{(-4)^2 + (3)^2}$$
$$d = \sqrt{16 + 9}$$
$$d = \sqrt{25}$$
$$d = 5$$

The length of the line segment is 5 units.

Try It

What is the approximate length of the line segment joining the midpoints of $\overline{AC}$ and $\overline{AB}$ in triangle ABC?



Find the coordinates of M, the midpoint of $\overline{AC}$.

Substitute the coordinates of points A and C into the midpoint formula.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M = \left(\frac{3 + \square}{2}, \frac{7 + \square}{2}\right)$$

$$M = \left(\frac{\square}{2}, \frac{\square}{2}\right)$$

$$M = (\underline{\qquad}, \underline{\qquad})$$

The coordinates of the midpoint of $\overline{AC}$ are (______, ______).

Find the coordinates of N, the midpoint of $\overline{AB}$.

Substitute the coordinates of points A and B into the midpoint formula.

$$N = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$N = \left(\frac{\square + 5}{2}, \frac{\square + 3}{2}\right)$$

$$N = \left(\frac{\square}{2}, \frac{\square}{2}\right)$$

$$N = (\underline{\qquad}, \underline{\qquad})$$

The coordinates of the midpoint of $\overline{AB}$ are (______, ______).

What is the length of the line segment with endpoints (2, 4) and (4, 5)?

The x-coordinates of the endpoints are $x_1 =$ ______ and $x_2 =$ ______.

The y-coordinates of the endpoints are $y_1 =$ ______ and $y_2 =$ ______.
Substitute these values into the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(\square - \square)^2 + (\square - \square)^2}$$

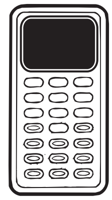
$$d = \sqrt{(\square)^2 + (\square)^2}$$

$$d = \sqrt{\square + \square}$$

$$d = \sqrt{\square}$$

$$d \approx \underline{\hspace{2cm}}$$

The length of the line segment joining the midpoints of $\overline{AC}$ and $\overline{AB}$ is approximately ______ units.



$$M = \left(\frac{3 + 1}{2}, \frac{7 + 1}{2}\right)$$

$$M = \left(\frac{4}{2}, \frac{8}{2}\right)$$

$$M = (2, 4)$$

The coordinates of the midpoint of $\overline{AC}$ are (2, 4).

$$N = \left(\frac{3 + 5}{2}, \frac{7 + 3}{2}\right)$$

$$N = \left(\frac{8}{2}, \frac{10}{2}\right)$$

$$N = (4, 5)$$

The coordinates of the midpoint of $\overline{AB}$ are (4, 5). The x-coordinates of the endpoints are $x_1 = 2$ and $x_2 = 4$. The y-coordinates of the endpoints are $y_1 = 4$ and $y_2 = 5$.

$$d = \sqrt{(4 - 2)^2 + (5 - 4)^2}$$

$$d = \sqrt{(2)^2 + (1)^2}$$

$$d = \sqrt{4 + 1}$$

$$d = \sqrt{5}$$

$$d \approx 2.2$$

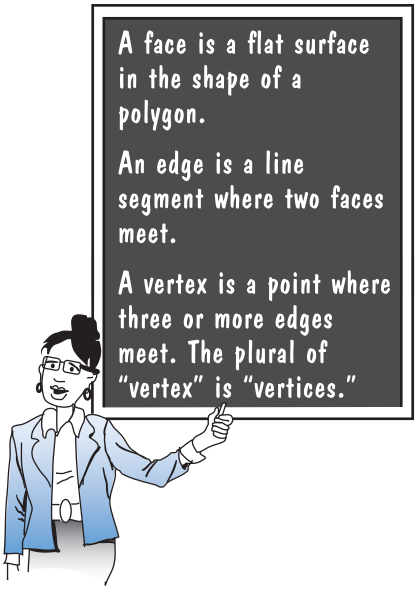
The length of the line segment joining the midpoints of $\overline{AC}$ and $\overline{AB}$ is approximately 2.2 units.

Objective 7

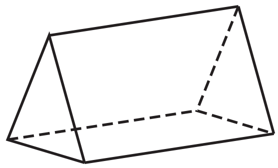
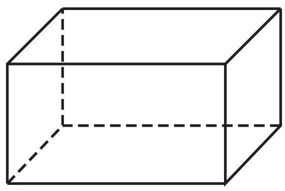
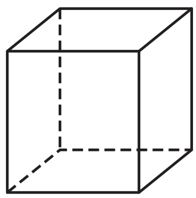
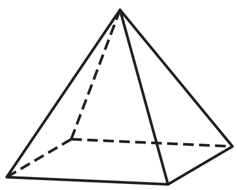
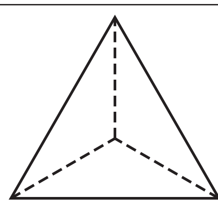
What Are Some Characteristics of Three-Dimensional Figures?

Three-dimensional figures include prisms and pyramids, as well as figures with curved surfaces.

You should be familiar with characteristics of these three-dimensional figures.

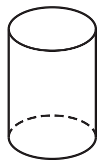
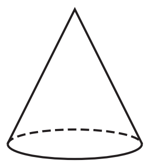
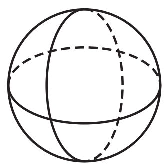


Prisms and Pyramids

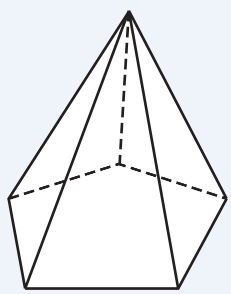
Type	Example	Properties
Triangular prism		• 5 faces 2 triangular bases 3 rectangular faces • 9 edges • 6 vertices
Rectangular prism		• 6 faces 2 rectangular bases 4 rectangular faces • 12 edges • 8 vertices
Cube		• 6 faces 2 square bases 4 square faces • 12 edges • 8 vertices
Square pyramid		• 5 faces 1 square base 4 triangular faces • 8 edges • 5 vertices
Triangular pyramid		• 4 faces 1 triangular base 3 triangular faces • 6 edges • 4 vertices

You should also be familiar with three-dimensional figures with curved surfaces. These figures include cones, cylinders, and spheres.

Three-Dimensional Figures with Curved Surfaces

Type	Example	Properties
Cylinder		• 2 circular bases • 1 curved surface
Cone		• 1 circular base • 1 curved surface • 1 vertex
Sphere		• 1 curved surface

A pentagonal pyramid is shown below.



How many faces, edges, and vertices does the solid have?

The solid has 5 triangular faces and 1 pentagonal face that forms the base. The pyramid has a total of 6 faces.

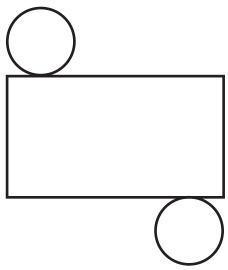
The triangular faces intersect in 5 places to form edges of the solid. The pentagonal base intersects each of the triangular faces once to form 5 more edges. The pyramid has a total of 10 edges.

The 5 triangular faces meet at 1 common vertex at the top of the pyramid. There are 5 additional vertices on the solid where the edges of the 5 triangular faces meet the pentagonal base. The pyramid has a total of 6 vertices.

Now practice what you've learned.

Question 65

The net shown below can be folded to represent which figure?



A Cone

B Sphere

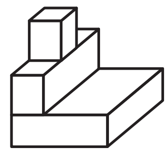
C Cylinder

D Rectangular prism

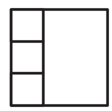
Answer Key: page 291

Question 66

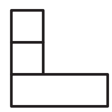
Which of the following is the top view of the 3-dimensional figure shown below?



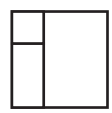
A



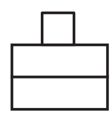
B



C



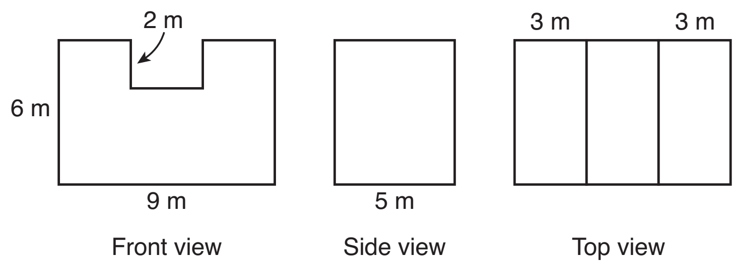
D



Answer Key: page 291

Question 67

Three views of a three-dimensional figure with the given dimensions are shown below.



What is the volume of this figure?

A 274 cubic meters

B 180 cubic meters

C 240 cubic meters

D 324 cubic meters

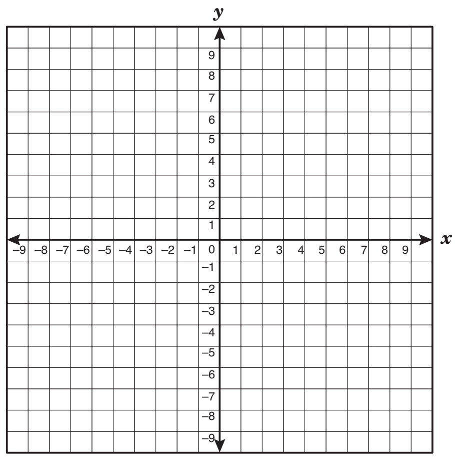
Answer Key: page 291

Question 68

The following points can be used to determine several line segments:

L (2, 5); M (4, −2); N (4, 3);

P (1, 1); R (2, −4); S (2, −1)



Which line segments could be drawn to form an isosceles trapezoid?

A $\overline{PR}$, $\overline{RM}$, $\overline{MN}$, $\overline{NP}$

B $\overline{MN}$, $\overline{NS}$, $\overline{RS}$, $\overline{MR}$

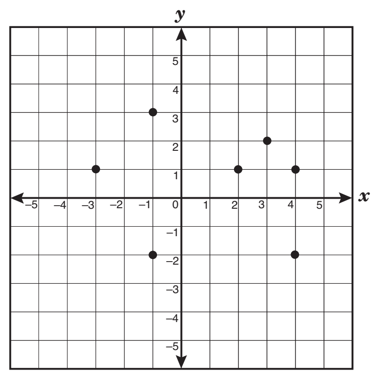
C $\overline{PS}$, $\overline{MS}$, $\overline{MN}$, $\overline{NP}$

D $\overline{MN}$, $\overline{NL}$, $\overline{LR}$, $\overline{MR}$

Answer Key: page 291

Question 69

Which set of ordered pairs forms the vertices of a right triangle?



A (4, 1), (4, −2), (3, 2)

B (−1, −2), (−1, 3), (4, −2)

C (−3, 1), (2, 1), (−1, 3)

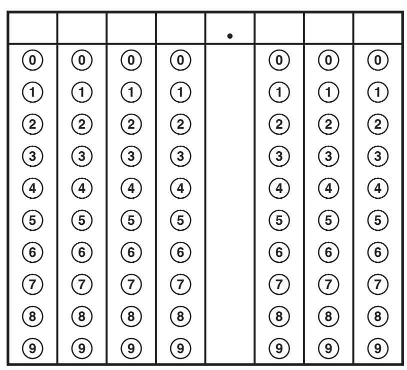
D (−3, 1), (−1, −2), (−1, 3)

Answer Key: page 292

Question 70

What is the length of the line segment with endpoints (7, −3) and (−5, 2)?

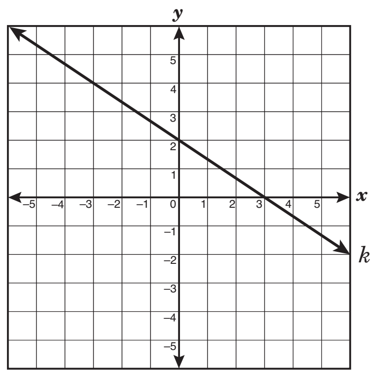
Record your answer and fill in the bubbles. Be sure to use the correct place value.



Answer Key: page 292

Question 71

Which equation represents a line perpendicular to line k graphed below?



A $y = \frac{2}{3}x - 1$

B $y = \frac{3}{2}x - 4$

C $y = -\frac{3}{2}x + 3$

D $y = -\frac{2}{3}x + 5$

Answer Key: page 292

Question 72

Line segment AB has midpoint M. The coordinates of point A are $(-5.1, 8.3)$, and the coordinates of point M are $(-1.5, -3.2)$. What are the coordinates of point B?

A $(-3.3, 2.55)$

B $(2.1, -14.7)$

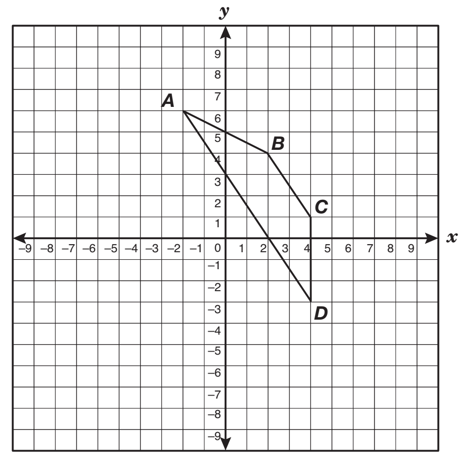
C $(-8.1, 14.7)$

D $(1.6, -2.35)$

Answer Key: page 292

Question 73

The figure shown below is a trapezoid.



Which pair of equations describes the lines that contain the bases of the trapezoid?

A $y = -\frac{3}{2}x + 3$ and $y = -\frac{3}{2}x + 7$

B $y = -\frac{1}{2}x + 5$ and $y = -\frac{1}{2}x + 4$

C $y = \frac{3}{2}x + 3$ and $y = \frac{3}{2}x + 1$

D $y = -\frac{2}{3}x + 5$ and $y = -\frac{2}{3}x + 7$

Answer Key: page 292

Question 74

A triangular prism has bases that are equilateral triangles. What shape could the lateral faces of the prism be?

A Equilateral triangles

B Rectangles

C Right triangles

D Trapezoids

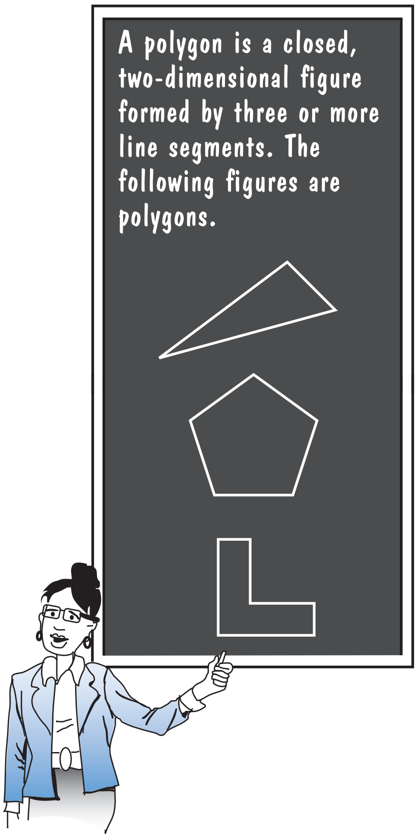
Answer Key: page 293

Objective 8

The student will demonstrate an understanding of the concepts and uses of measurement and similarity.

For this objective you should be able to

- extend measurement concepts to find area, perimeter, and volume in problem situations; and
- apply concepts of similarity to justify properties of figures and solve problems.



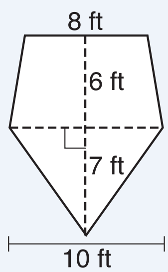
How Do You Find the Areas of Polygons and Composite Figures?

The **area** of a polygon is a measure of the number of square units that the polygon covers.

You can use the area formulas in the Mathematics Chart to help you find the areas of **composite figures**. Composite figures are made up of two or more simple shapes.

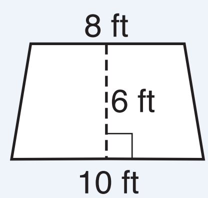
- First divide the composite figure into regions whose areas you can find.
- Next calculate the area of each of these simple shapes.
- Then add these areas to find the area of the composite figure.

Charles is building a fishpond in his backyard. The diagram shows the pond's dimensions. How many square feet will the pond occupy?



- The pond can be divided into a trapezoid and a triangle.
- Find the area of the trapezoid. One base, b_1, measures 8 feet. The other base, b_2, measures 10 feet, and the height, h, measures 6 feet.

$$A = \frac{1}{2}(b_1 + b_2)h$$
$$A = \frac{1}{2}(8 + 10)(6)$$
$$A = \frac{1}{2}(18)(6)$$
$$A = 54 \text{ ft}^2$$



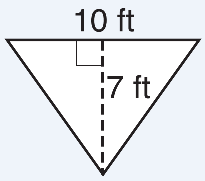
The area of the trapezoid is 54 square feet.

- Find the area of the triangle. The base, b, measures 10 feet, and the height, h, measures 7 feet.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(10)(7)$$

$$A = 35 \text{ ft}^2$$



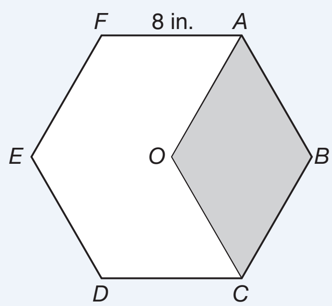
The area of the triangle is 35 square feet.

- To find the total area, add the area of the trapezoid and the area of the triangle.

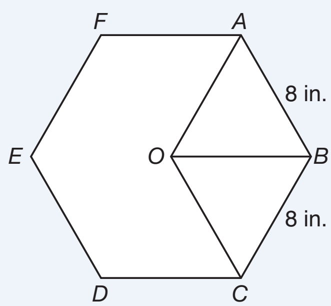
$$54 + 35 = 89 \text{ ft}^2$$

The pond will occupy 89 square feet.

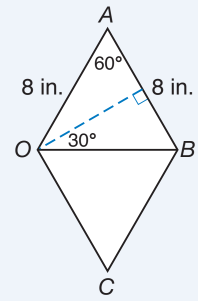
Figure $ABCDEF$ is a regular hexagon with a side length of 8 inches. What is the area of rhombus $OABC$, formed by drawing segments OA and OC from point O, the center of the hexagon?



- To find the area of the figure, divide the rhombus into two simple regions by drawing segment OB. Two congruent triangles are formed.



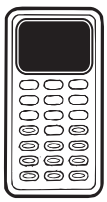
- The triangles formed are equilateral triangles. Each angle in the triangles is 60°. Draw the height of $\triangle AOB$ from point O to $\overline{AB}$.



- Find the height using the special properties of a 30°-60°-90° triangle. The length of the hypotenuse of the small triangle, $\overline{OA}$, is 8 inches. The length of the shorter leg is half the length of the hypotenuse, $\frac{1}{2}(8) = 4$. The height of the triangle is the longer leg. Its length is the length of the shorter leg times $\sqrt{3}$, or $4\sqrt{3}$.
- Find the area of $\triangle AOB$.

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(8)(4\sqrt{3})$$
$$A = 16\sqrt{3}$$

- There are 2 such triangles, so their combined area is $2(16\sqrt{3}) = 32\sqrt{3}$.

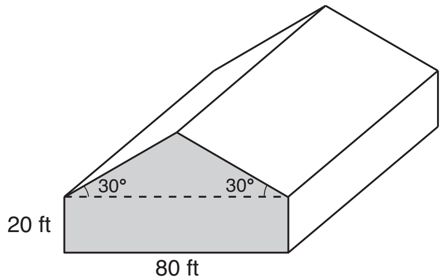


$$32\sqrt{3} \approx 55.43$$

The area of the rhombus is approximately 55 square inches.

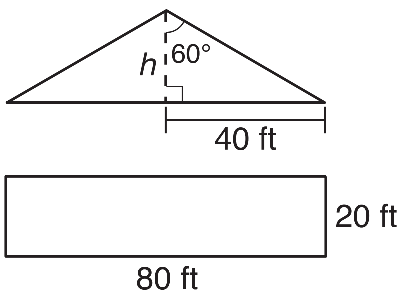
Try It

Henry is building a barn like the one drawn below. How many square feet of sheet metal will he need to completely cover the side of the barn shown shaded below?



The side of the barn can be divided into a rectangle and an

__________________ triangle.



Find the area of the rectangle.

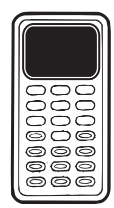
The formula for the area of a rectangle is $A = lw$.

The length of the rectangle is ________ feet, and its width is

________ feet.

$$A = \underline{\qquad\quad} \cdot \underline{\qquad\quad}$$

$$A = \underline{\qquad\quad}\ \text{ft}^2$$



Find the base and height of the 30°-60°-90° triangle.

The shorter leg is the height of the triangle.

Let the height be represented by h.

The length of the longer leg is ______________ the length of

the base of the barn, or ________ feet.

The length of the longer leg is also equal to the length of the shorter

leg, h, times ________.

$$h\sqrt{3} = \_\_\_\_\_\_$$

$$\frac{h\sqrt{3}}{\square} = \frac{40}{\square}$$

$$h = \frac{\square}{\square}$$

$$h \approx \_\_\_\_\_\_$$

Use the formula $A = \frac{1}{2}bh$ to find the area of the triangle.

$$A = \frac{1}{2}bh$$

$$A \approx \frac{1}{2} \cdot \_\_\_\_\_\_ \cdot \_\_\_\_\_\_$$

$$A \approx \_\_\_\_\_\_ \text{ ft}^2$$

Find the total area by adding the area of the ______________

to the area of the ______________.

$$\text{Total area} \approx \_\_\_\_\_\_ + \_\_\_\_\_\_ \approx \_\_\_\_\_\_ \text{ ft}^2$$

Henry will need approximately _________ square feet of sheet metal to completely cover the side of the building.

The side of the barn can be divided into a rectangle and an isosceles triangle. The length of the rectangle is 80 feet, and its width is 20 feet.

$$A = 80 \cdot 20$$
$$A = 1600 \text{ ft}^2$$

The length of the longer leg is half the length of the base of the barn, or 40 feet. The length of the longer leg is also equal to the length of the shorter leg, h, times $\sqrt{3}$.

$$h\sqrt{3} = 40$$

$$\frac{h\sqrt{3}}{\sqrt{3}} = \frac{40}{\sqrt{3}}$$

$$h = \frac{40}{\sqrt{3}}$$

$$h \approx 23$$

Use the formula $A = \frac{1}{2}bh$ to find the area of the triangle.

$$A \approx \frac{1}{2} \cdot 80 \cdot 23$$

$$A \approx 920 \text{ ft}^2$$

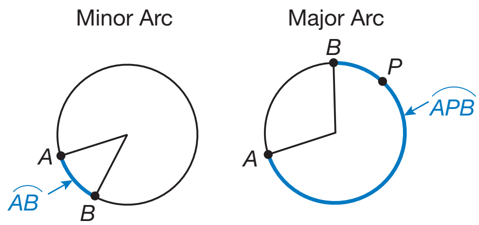
Find the total area by adding the area of the rectangle to the area of the triangle.

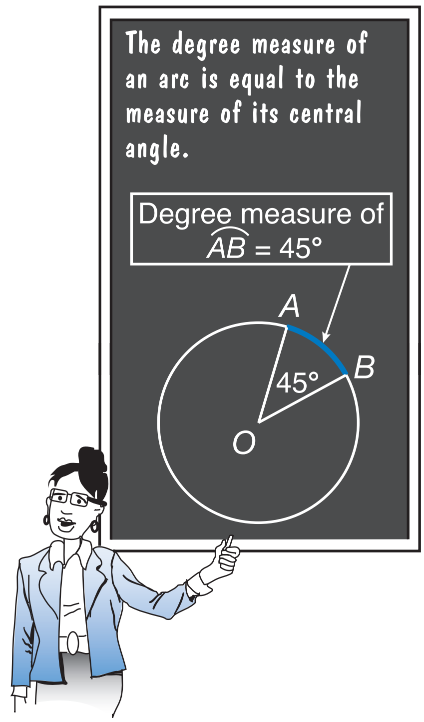
$$\text{Total area} \approx 1600 + 920 \approx 2520 \text{ ft}^2$$

Henry will need approximately 2520 square feet of sheet metal to completely cover the side of the building.

How Do You Find the Length of an Arc of a Circle?

An **arc** of a circle is the part of a circle determined by any two points on the circle. A minor arc is named using its two endpoints. A major arc is named using its two endpoints and one other point on the arc.

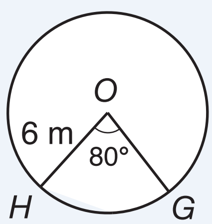




The length of an arc of a circle is proportional to the circumference of the circle. You can find the length of an arc of a circle by setting up and solving a proportion. There are 360 degrees in a circle.

$$\frac{\text{length of arc}}{\text{circumference of circle}} = \frac{\text{degree measure of arc}}{360°}$$

For the circle below, what is the length of $\overset{\frown}{GH}$?



Use a proportion to find the length of $\overset{\frown}{GH}$.

$$\frac{\text{length of arc}}{\text{circumference of circle}} = \frac{\text{degree measure of arc}}{360°}$$

- Let the length of the arc be represented by x.
- The circumference of the circle can be calculated using the formula $C = 2\pi r$ from the Mathematics Chart. The diagram shows that the radius, r, of the circle is 6 meters.

$$C = 2\pi r$$
$$C = 2\pi(6)$$
$$C = 12\pi$$
$$C \approx 37.7$$

  The circumference of the circle is about 37.7 meters.

- The measure of the central angle of $\overset{\frown}{GH}$ is 80°. Therefore, the degree measure of the arc is also 80°.
- Substitute the known values into the proportion.

$$\frac{\text{length of } \overset{\frown}{GH}}{\text{circumference of circle}} = \frac{\text{degree measure of } \overset{\frown}{GH}}{360°}$$

$$\frac{x}{37.7} = \frac{80}{360}$$

- Use cross products to solve for x.

$$360x = 80 \cdot 37.7$$

$$360x = 3016$$

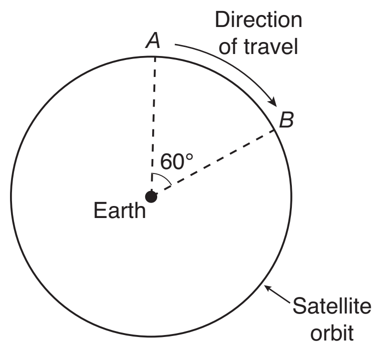
$$\frac{360x}{360} = \frac{3016}{360}$$

$$x \approx 8.4$$

The length of $\overset{\frown}{GH}$ is about 8.4 meters.

Try It

A satellite moves in a nearly circular orbit around Earth. The radius of the orbit is about 42,000 kilometers. At 1 P.M. the satellite is at point A. At 5 P.M. the satellite is at point B. Approximately how many kilometers does the satellite travel between 1 P.M. and 5 P.M.?

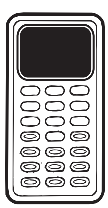


The distance the satellite travels is equal to the length of $\overset{\frown}{AB}$.

Write a proportion that can be used to solve the problem.

$$\frac{\text{length of arc}}{\text{circumference of orbit}} = \frac{\text{degree measure of arc}}{360°}$$

Let x represent the length of $\overset{\frown}{AB}$.



Use the formula $C = 2\pi r$ to find the circumference of the orbit.

The radius of the orbit is ____________ kilometers.

$$C = 2 \cdot \_\_\_\_\_\_\_\_\_ \cdot \_\_\_\_\_\_\_\_\_$$

$$C \approx \_\_\_\_\_\_\_\_\_ \text{km}$$

The degree measure of $\overset{\frown}{AB}$ is _______ .

Substitute the known values into the proportion.

$$\frac{x}{\square} = \frac{\square}{360}$$

Use cross products to solve for x.

$$360x = \underline{\hspace{3cm}} \cdot \underline{\hspace{2cm}}$$

$$360x = \underline{\hspace{3cm}}$$

$$\frac{360x}{\square} = \frac{15{,}833{,}640}{\square}$$

$$x \approx \underline{\hspace{3cm}}$$

The length of $\overset{\frown}{AB}$ is about ______________ kilometers. The satellite travels approximately ______________ kilometers between 1 P.M. and 5 P.M.

The radius of the orbit is 42,000 kilometers.

$$C = 2 \cdot \pi \cdot 42{,}000$$

$$C \approx 263{,}894 \text{ km}$$

The degree measure of $\overset{\frown}{AB}$ is 60°.

$$\frac{x}{263{,}894} = \frac{60}{360}$$

$$360x = 263{,}894 \cdot 60$$

$$360x = 15{,}833{,}640$$

$$\frac{360x}{360} = \frac{15{,}833{,}640}{360}$$

$$x \approx 43{,}982$$

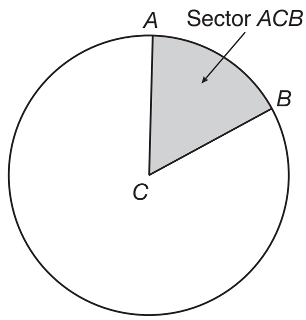
The length of $\overset{\frown}{AB}$ is about 43,982 kilometers. The satellite travels approximately 43,982 kilometers between 1 P.M. and 5 P.M.

How Do You Find the Area of a Sector of a Circle?

A **sector** of a circle is a part of a circle bounded by two radii and an arc. A sector is shaped like a piece of pie.

A sector can be named using the two endpoints on the arc and the center of the circle.

In the diagram below, sector *ACB* is the region bounded by radius *AC*, radius *BC*, and arc *AB*.

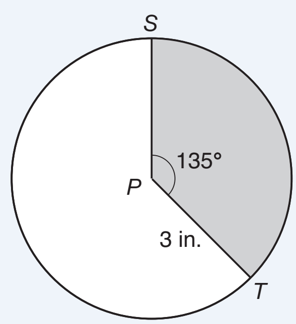


The area of a sector of a circle is proportional to the area of the circle.

You can calculate the area of a sector of a circle by setting up and solving a proportion.

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{measure of central angle}}{360°}$$

What is the area of the shaded sector in the diagram below?



Use a proportion to find the area of the sector.

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{measure of central angle}}{360°}$$

- Let the area of the sector be represented by x.
- Use the formula $A = \pi r^2$ to find the area of the circle. The radius, r, of the circle is 3 inches.

$$A = \pi r^2$$
$$A = \pi(3)^2$$
$$A = 9\pi$$
$$A \approx 28.27 \text{ in.}^2$$

The area of the circle is about 28.27 square inches.

- The measure of the central angle is 135°.
- Substitute the known values into the proportion.

$$\frac{\text{area of sector } SPT}{\text{area of circle}} = \frac{m\angle SPT}{360°}$$

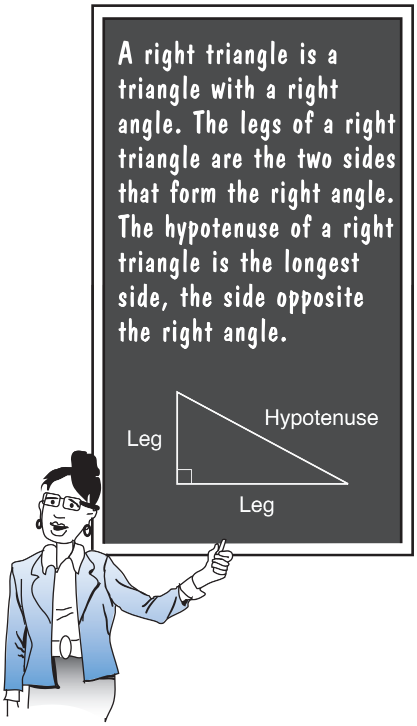
$$\frac{x}{28.27} = \frac{135}{360}$$

- Use cross products to solve for x.

$$360x = 135 \cdot 28.27$$
$$360x = 3816.45$$
$$\frac{360x}{360} = \frac{3816.45}{360}$$
$$x \approx 10.6 \text{ in.}^2$$

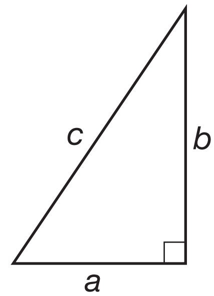
The area of the sector is about 10.6 square inches.

Objective 8



How Can You Solve Problems Using the Pythagorean Theorem?

The **Pythagorean Theorem** is a relationship among the lengths of the three sides of a right triangle. The Pythagorean Theorem applies only to right triangles.



- In any right triangle with leg lengths a and b and hypotenuse length c, $a^2 + b^2 = c^2$.
- If the side lengths of any triangle satisfy the equation $a^2 + b^2 = c^2$, then the triangle is a right triangle, and c is its hypotenuse.

Any set of three whole numbers that satisfy the Pythagorean Theorem is called a **Pythagorean triple**. The set of numbers {5, 12, 13} forms a Pythagorean triple because these numbers satisfy the Pythagorean Theorem. To show this, substitute 13 for c in the formula—since 13 is the greatest number—and substitute 5 and 12 for a and b.

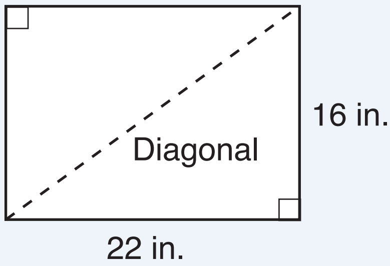
$$a^2 + b^2 = c^2$$
$$5^2 + 12^2 = 13^2$$
$$25 + 144 = 169$$
$$169 = 169$$

A triangle with side lengths 5, 12, and 13 units is a right triangle.

Any multiple of a Pythagorean triple is also a Pythagorean triple. Since the set of numbers {5, 12, 13} is a Pythagorean triple, the triple formed by multiplying each number in the set by 2, {10, 24, 26}, is also a Pythagorean triple. A triangle with side lengths 10, 24, and 26 units is also a right triangle.

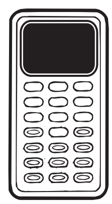
Yolanda's television screen is 22 inches long and 16 inches wide. She plans to buy a television with a larger screen. When she gets to the store, she finds that television sizes are based on the approximate length of the screen's diagonal. What size is the television Yolanda currently owns?

Draw a picture of Yolanda's television screen.



The diagonal divides the rectangular screen into two right triangles. For each triangle the diagonal is the hypotenuse, and the length and width of the rectangle are the legs.

Use the Pythagorean Theorem to find the length of the diagonal.

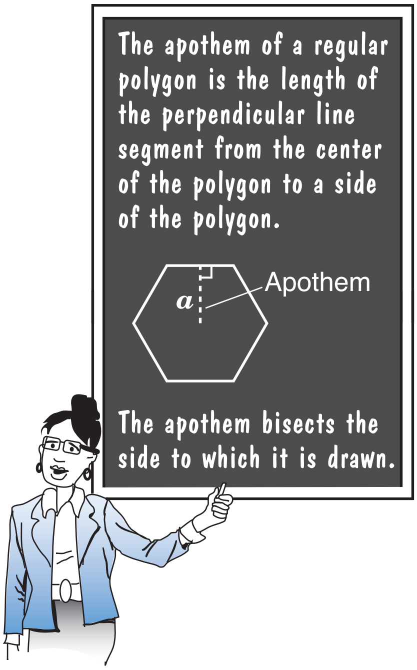


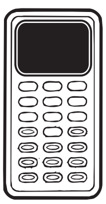
Let leg $a = 22$ inches and leg $b = 16$ inches. Solve for c, the length of the hypotenuse.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 22^2 + 16^2 &= c^2 \\ 484 + 256 &= c^2 \\ 740 &= c^2 \\ \sqrt{740} &= c \\ 27.2029 &\approx c \end{aligned}$$

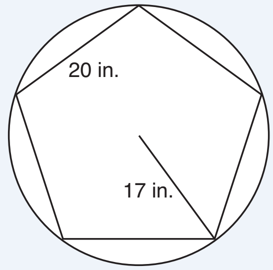
The diagonal of the screen is about 27 inches long. Yolanda owns a 27-inch television.

Objective 8



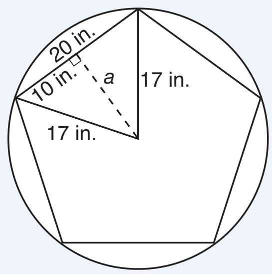


Find the area of a regular pentagon with a side length of 20 inches inscribed in a circle with a radius of 17 inches.



To find the area of the pentagon, use the formula for the area of a regular polygon, $A = \frac{1}{2}aP$. In this formula, a is the apothem and P is the perimeter.

Use the Pythagorean Theorem to find a.



The hypotenuse of the small triangle is the radius of the circle, 17 inches. One leg of the small triangle is $\frac{1}{2}$ the side length of the pentagon, or 10 inches.

$$\begin{aligned} a^2 + 10^2 &= 17^2 \\ a^2 + 100 &= 289 \\ a^2 &= 189 \\ a &= \sqrt{189} \\ a &\approx 13.75 \text{ in.} \end{aligned}$$

The apothem is approximately 13.75 inches.

Find the perimeter of the pentagon.

Since this is a regular pentagon with a side length of 20 inches, the perimeter is 5 times 20, or 100 inches.

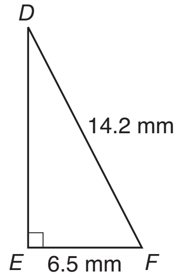
Substitute $a \approx 13.75$ and $P = 100$ into the formula.

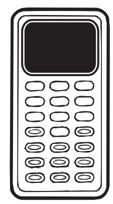
$$\begin{aligned} A &= \frac{1}{2}aP \\ A &\approx \frac{1}{2}(13.75)(100) \\ A &\approx 687.5 \text{ in.}^2 \end{aligned}$$

The area of the regular pentagon is approximately 687.5 square inches.

Try It

What is the perimeter of $\triangle DEF$?





The triangle is a right triangle. Side _______ is its hypotenuse, and sides _______ and _______ are its legs.

Use the _______________ Theorem to find the length of $\overline{DE}$.

Let a equal the length of $\overline{DE}$, let b equal _______ millimeters, and let c equal _______ millimeters.

$$a^2 + b^2 = c^2$$
$$a^2 + \underline{\qquad}^2 = \underline{\qquad}^2$$
$$a^2 + \underline{\qquad} = \underline{\qquad}$$
$$a^2 = \underline{\qquad}$$
$$a = \sqrt{\boxed{}}$$
$$a \approx \underline{\qquad} \text{ mm}$$

The length of $\overline{DE}$ is approximately _______ millimeters.

Find the perimeter of the triangle.

$$P \approx \underline{\qquad} + \underline{\qquad} + \underline{\qquad}$$
$$P \approx \underline{\qquad} \text{ mm}$$

The perimeter of the triangle is approximately _______ millimeters.

Side DF is its hypotenuse, and sides DE and EF are its legs. Use the Pythagorean Theorem to find the length of $\overline{DE}$. Let a equal the length of $\overline{DE}$, let b equal 6.5 millimeters, and let c equal 14.2 millimeters.

$$a^2 + b^2 = c^2$$
$$a^2 + 6.5^2 = 14.2^2$$
$$a^2 + 42.25 = 201.64$$
$$a^2 = 159.39$$
$$a = \sqrt{159.39}$$
$$a \approx 12.625 \text{ mm}$$

The length of $\overline{DE}$ is approximately 12.6 millimeters.

$$P \approx 12.6 + 6.5 + 14.2$$
$$P \approx 33.3 \text{ mm}$$

The perimeter of the triangle is approximately 33.3 millimeters.

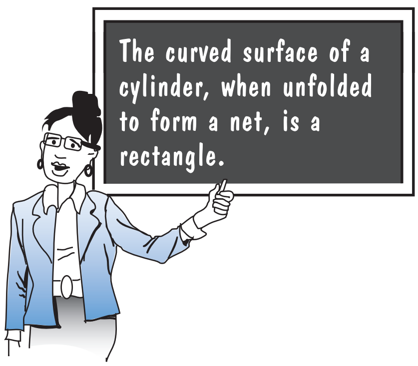
How Do You Find Surface Area?

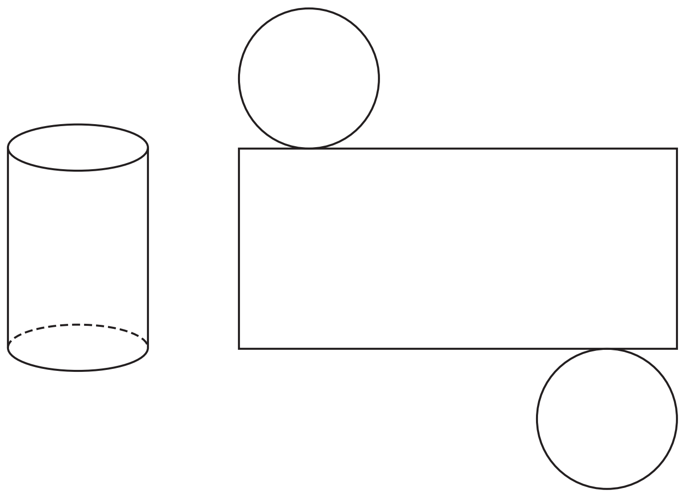
You can use models or formulas to find the surface area of prisms, cylinders, and other three-dimensional figures.

- The **total surface area** of a three-dimensional figure is equal to the sum of the areas of all its surfaces.
- The **lateral surface area** of a three-dimensional figure is equal to the sum of the areas of all its faces and curved surfaces. Lateral surface area does not include the area of the figure's bases.

One way to find the surface area of a three-dimensional figure is to use a net of the figure. A **net** of a three-dimensional figure is a two-dimensional drawing that shows what the figure would look like when opened up and unfolded with all its surfaces laid out flat. Use the net to find the area of each surface.

For example, the net of a cylinder is shown below. It is composed of two circles for the bases and a rectangle for the curved surface.

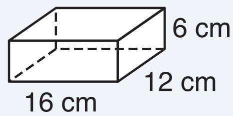




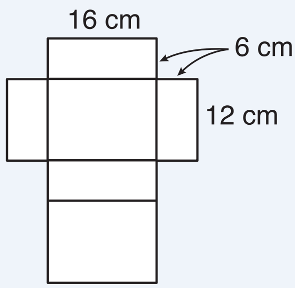
- The **total surface area** of a cylinder is equal to the sum of the areas of the two circular bases and the area of the rectangle that forms the curved surface of the cylinder.
- The **lateral surface area** of a cylinder is equal to the area of the rectangle that forms the curved surface of the cylinder.

You can also find the surface area of a solid figure by using a formula. Substitute the appropriate dimensions of the figure into the formula and calculate its surface area. The formulas for the total surface area and lateral surface area of several solid figures are included in the Mathematics Chart.

What is the minimum area of cardboard needed to make a box that is 16 centimeters long, 12 centimeters wide, and 6 centimeters tall?



One way to find the total surface area of the box is to draw a net showing all six of its faces.



The box is a rectangular prism. Each face is a rectangle. The area of a rectangle is equal to its length times its width.

- Two faces measure 16 centimeters by 12 centimeters. Find the area of each of these faces.

$$A = 16 \cdot 12 = 192 \text{ cm}^2$$

- Two faces measure 16 centimeters by 6 centimeters. Find the area of each of these faces.

$$A = 16 \cdot 6 = 96 \text{ cm}^2$$

- Two faces measure 12 centimeters by 6 centimeters. Find the area of each of these faces.

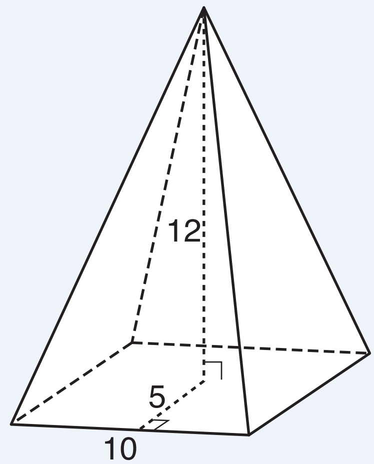
$$A = 12 \cdot 6 = 72 \text{ cm}^2$$

- Add the areas of the six faces to find the total surface area of the box.

$$S = 192 + 192 + 96 + 96 + 72 + 72 = 720 \text{ cm}^2$$

The total surface area of the box is 720 square centimeters. It will take at least 720 square centimeters of cardboard to make the box.

What is the total surface area of the square pyramid shown below?



Use the formula for the total surface area of a pyramid, $S = \frac{1}{2}Pl + B$, where l is the slant height and B is the area of the square base of the pyramid.

The perimeter of the base of the pyramid is $4 \cdot 10 = 40$, and the area of the square base is $10 \cdot 10 = 100$.

$$S = \frac{1}{2}Pl + B$$

$$S = \frac{1}{2}(40)(l) + 100$$

Notice that the slant height of the pyramid is not given. However, the slant height can be found because the height of the pyramid and the apothem of the base of the pyramid are given. These three lengths form a right triangle, with the height and apothem as the legs and the slant height as the hypotenuse. This is a Pythagorean triple:

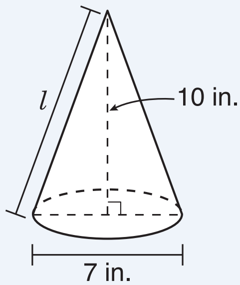
$$\{5, 12, 13\}$$

Therefore, the slant height is 13.

$$S = \frac{1}{2}(40)(13) + 100$$
$$S = (20)(13) + 100$$
$$S = (260) + 100$$
$$S = 360$$

The total surface area of the pyramid is 360 square units.

What is the lateral surface area of the cone shown below?



Use the formula for the lateral surface area of a cone, $S = \pi r l$, where l is its slant height.

- The height, slant height, and radius of the cone form a right triangle. Use the Pythagorean Theorem to find the slant height, l, of the cone.

  The radius of the cone is equal to its diameter divided by 2.

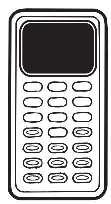
  $$7 \div 2 = 3.5 \text{ in.}$$

  Let $a = 3.5$, the radius of the cone, and let $b = 10$, the height of the cone.

  $$l^2 = a^2 + b^2$$
  $$l^2 = (3.5)^2 + (10)^2$$
  $$l^2 = 12.25 + 100$$
  $$l = \sqrt{112.25}$$
  $$l \approx 10.59 \text{ in.}$$

- Substitute the cone's dimensions into the formula $S = \pi r l$.

  $$S = \pi r l$$
  $$S \approx \pi(3.5)(10.59)$$
  $$S \approx 116.44 \text{ in.}^2$$



The lateral surface area of the cone is about 116.44 square inches.

What Is the Volume of a Three-Dimensional Figure?

The **volume** of a three-dimensional figure is a measure of the space it occupies. Volume is measured in cubic units.

You can use formulas or models to find the volume of three-dimensional figures. The formulas for calculating the volume of several three-dimensional figures are in the Mathematics Chart.

Objective 8

When using a formula to find the volume of a three-dimensional figure, follow these guidelines:

- Identify the three-dimensional figure you are working with. This will help you select the correct volume formula.
- Use models to help visualize the three-dimensional figure and to assign the variables in the volume formula. A model can also be used to find the dimensions of a figure.
- Substitute the appropriate dimensions of the figure for the corresponding variables in the volume formula.
- Calculate the volume. State your answer in cubic units.

Find the volume of the globe shown below to the nearest cubic inch.

Use the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3$$

In this formula r represents the radius of the sphere. Since the diameter of the sphere is given, first divide by 2 to get the radius.

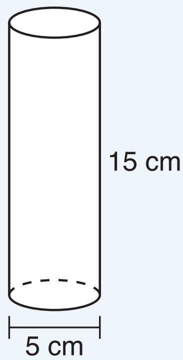
$$\begin{aligned} d &= 2r \\ 20 &= 2r \\ \frac{20}{2} &= \frac{2r}{2} \\ r &= 10 \end{aligned}$$

Substitute 10 for r into the formula.

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi(10^3) \\ &= \frac{4}{3}\pi(1000) \\ &\approx 4188.79 \text{ in.}^3 \end{aligned}$$

Rounding this value gives the volume of the sphere to be approximately 4189 cubic inches.

How many cubic centimeters of metal are needed to make the solid metal rod shown below?



- The metal rod is shaped like a cylinder. Use the formula for the volume of a cylinder, $V = Bh$, where B is the area of the base.
- The base of a cylinder is a circle, so the area of the base equals πr^2.

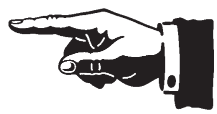
  The cylinder's radius, r, is equal to its diameter divided by 2. The cylinder's diameter is 5 centimeters, so its radius is $5 \div 2 = 2.5$ centimeters.

$$B = \pi(2.5)^2$$
$$B = \pi(6.25)$$
$$B \approx 19.635 \text{ cm}^2$$

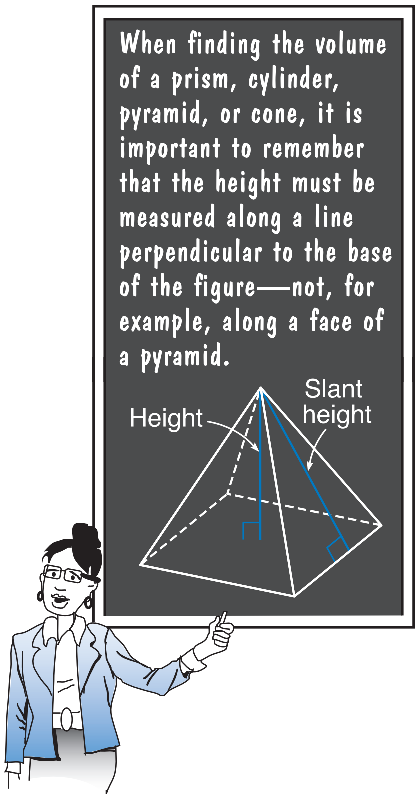
- Substitute the values for B and h into the volume formula, $V = Bh$. The height, h, of the cylinder is 15 centimeters.

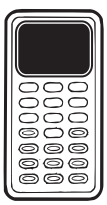
$$V \approx (19.635)(15)$$
$$V \approx 294.525 \text{ cm}^3$$

It will take about 294.5 cubic centimeters of metal to make the metal rod.

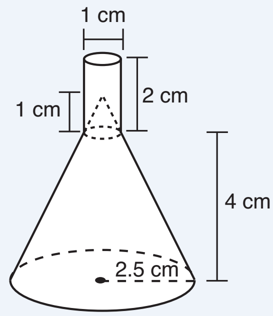


Do you see that . . .

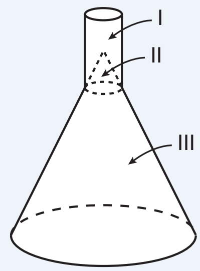




At maximum, how many cubic centimeters of liquid can the laboratory flask below hold?



To find the volume of the flask, divide it into sections and find the volume of each section.



$V = \text{I} + \text{III} - \text{II}$

The volume of the flask is equal to the volume of its cylindrical neck, plus the volume of the large cone that forms its base, minus the volume of the small cone at its top (so that the volume of the cone's tip is not counted twice).

Volume of a Laboratory Flask

Volume of neck I	Volume of small cone II	Volume of large cone III
Cylinder	Cone	Cone
$V = Bh$	$V = \frac{1}{3}Bh$	$V = \frac{1}{3}Bh$
Base: use πr^2 $B = \pi(0.5)^2$ $B = 0.25\pi$ $B \approx 0.79$ cm^2	Base: use πr^2 $B = \pi(0.5)^2$ $B = 0.25\pi$ $B \approx 0.79$ cm^2	Base: use πr^2 $B = \pi(2.5)^2$ $B = 6.25\pi$ $B \approx 19.63$ cm^2
Height: 2 cm	Height: 1 cm	Height: $4 + 1 = 5$ cm
Volume: $V = Bh$ $V \approx (0.79)(2)$ $V \approx 1.58$ cm^3	Volume: $V = \frac{1}{3}Bh$ $V \approx \frac{1}{3}(0.79)(1)$ $V \approx 0.26$ cm^3	Volume: $V = \frac{1}{3}Bh$ $V \approx \frac{1}{3}(19.63)(5)$ $V \approx 32.72$ cm^3

$$\text{Flask's volume} \approx \text{Volume I} + \text{Volume III} - \text{Volume II}$$
$$\approx 1.58 + 32.72 - 0.26$$
$$\approx 34.04 \text{ cm}^3$$

The volume of the flask is approximately 34 cubic centimeters.

Try It

How many cubic yards of rock are needed to build a square pyramid with a height of 150 yards and a base with a side length of 250 yards?

The formula for the volume of a pyramid is $V = \frac{1}{3}Bh$.

Find the area of the base, B.

The base of a square pyramid is a ______________.

The length of each side of the base is _______ yards.

$$B = \_\_\_\_\_\_\_ \cdot \_\_\_\_\_\_\_$$
$$B = \_\_\_\_\_\_\_\_\_\_\_\_ \text{ yd}^2$$

Find the volume of the pyramid.

The height, h, of the pyramid is _______ yards.

$$V = \frac{1}{3} \cdot \_\_\_\_\_\_\_\_\_\_\_\_ \cdot \_\_\_\_\_\_\_\_\_\_\_\_$$
$$V = \_\_\_\_\_\_\_\_\_\_\_\_ \text{ yd}^3$$

It will take ____________ cubic yards of rock to build the pyramid.

The base of a square pyramid is a square. The length of each side of the base is 250 yards.

$$B = 250 \cdot 250$$
$$B = 62{,}500 \text{ yd}^2$$

The height, h, of the pyramid is 150 yards.

$$V = \frac{1}{3} \cdot 62{,}500 \cdot 150$$
$$V = 3{,}125{,}000 \text{ yd}^3$$

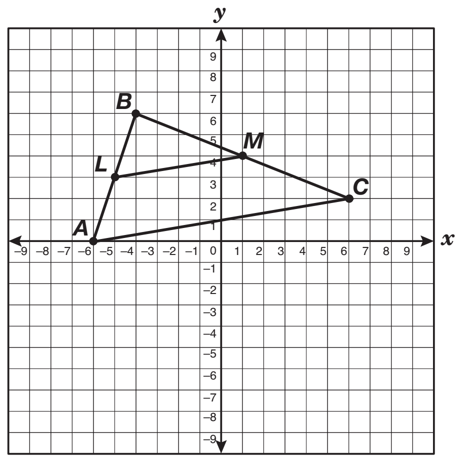
It will take 3,125,000 cubic yards of rock to build the pyramid.

When Are Figures Similar?

Similar figures have the same shape. You can use transformations to show that figures are similar. The corresponding angles of similar figures are congruent. The lengths of the corresponding sides are proportional.

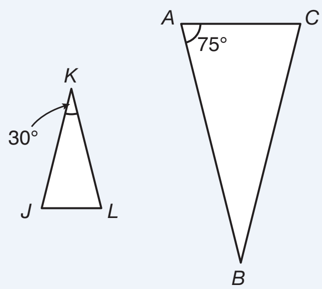
$\triangle ABC$ is similar to $\triangle LBM$.

$\triangle ABC \sim \triangle LBM$



$$m\angle A = m\angle MLB$$
$$m\angle B = m\angle B$$
$$m\angle C = m\angle BML$$
$$\frac{AB}{LB} = \frac{BC}{BM} = \frac{AC}{LM}$$

$\triangle JKL$ and $\triangle ABC$ are both isosceles triangles. Is $\triangle JKL$ similar to $\triangle ABC$?



In $\triangle JKL$, $m\angle K = 30°$. Since the triangle is isosceles, $m\angle J = m\angle L$. Since the sum of the angles in a triangle is equal to 180°, $m\angle J + m\angle L = 180° - 30°$, or 150°. Each angle must be $\frac{1}{2}$ that sum, or 75°.

In $\triangle ABC$, $m\angle A = 75°$. Since the triangle is isosceles, $m\angle A = m\angle C$. Therefore, $m\angle C = 75°$. Since the sum of the angles in a triangle is equal to 180°, $m\angle B = 180° - (75° + 75°)$, or $m\angle B = 30°$.

The corresponding angles of the two triangles are all congruent.

The two triangles are similar: $\triangle JKL \sim \triangle ABC$.

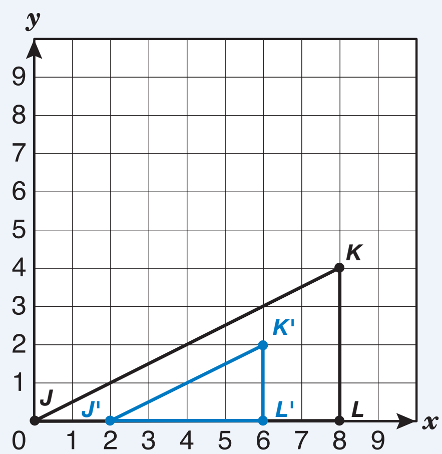
Dilations

A **dilation** is a proportional enlargement or reduction of a figure through a point called the center of dilation. The size of the enlargement or reduction is called the **scale factor** of the dilation.

- If the dilated image is larger than the original figure, then the scale factor > 1. This is called an **enlargement**.
- If the dilated image is smaller than the original figure, then the scale factor < 1. This is called a **reduction**.

A figure and its dilated image are always similar.

What scale factor was used to transform $\triangle JKL$ into $\triangle J'K'L'$?



To find the scale factor, compare the lengths of a pair of corresponding sides.

- Of the line segments that make up the triangles, it is easiest to find the lengths of $\overline{JL}$ and $\overline{J'L'}$ because they lie along the x-axis.
- The length of $\overline{JL}$ is the difference between the x-coordinates of points J and L.

$$JL = 8 - 0 = 8$$

- The length of $\overline{J'L'}$ is the difference between the x-coordinates of J' and L'.

$$J'L' = 6 - 2 = 4$$

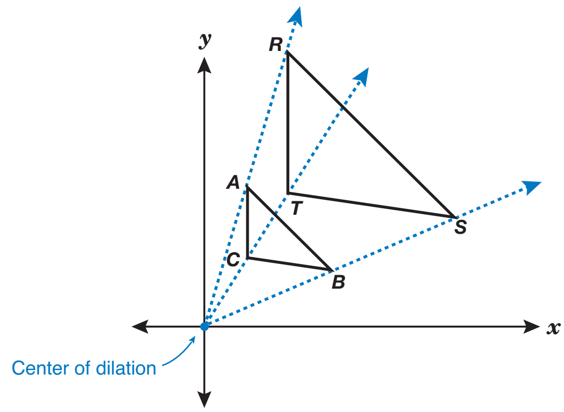
- The scale factor is the ratio of these lengths. Since the dilated image is smaller than the original figure, the scale factor is < 1.

$$\frac{J'L'}{JL} = \frac{4}{8} = \frac{1}{2}$$

The scale factor used to dilate $\triangle JKL$ to form $\triangle J'K'L'$ is $\frac{1}{2}$.

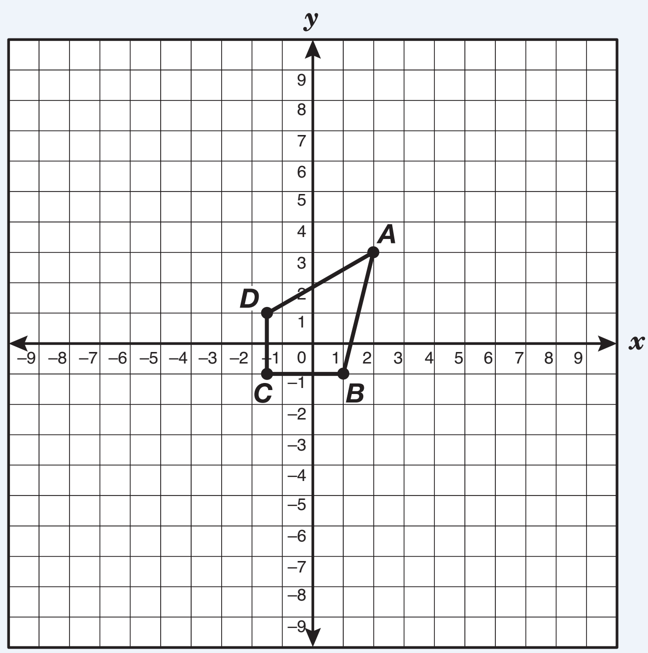
Each side of the dilated triangle is $\frac{1}{2}$ the length of the corresponding side of the original triangle.

Another way to view a dilation is as a projection through a **center of dilation**.



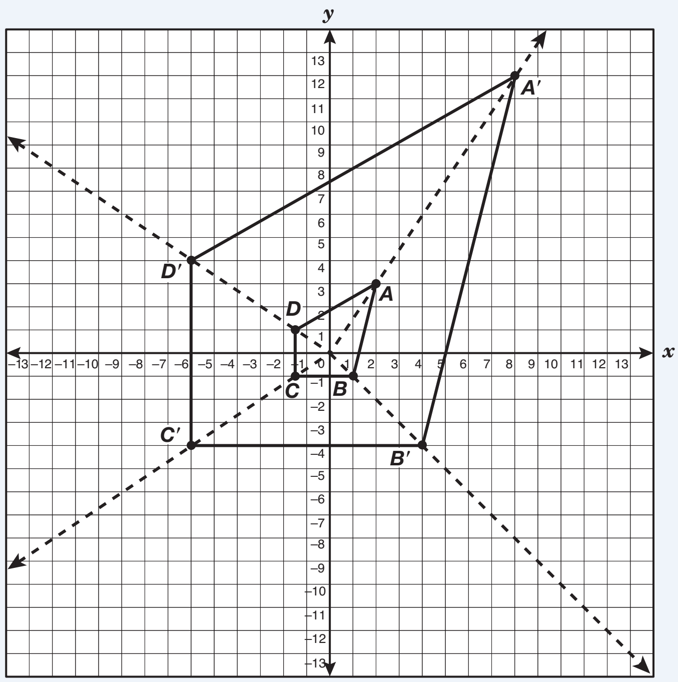
Looking at a dilation in this way can help you see it as an enlargement or a reduction. $\triangle ABC$ has been dilated to form $\triangle RST$.

A quadrilateral is graphed below. What will be the coordinates of its vertices if it is dilated by a scale factor of 4 using (0, 0) as the center of dilation?



To find the coordinates of the enlarged quadrilateral, draw rays from the origin through the points A, B, C, and D.

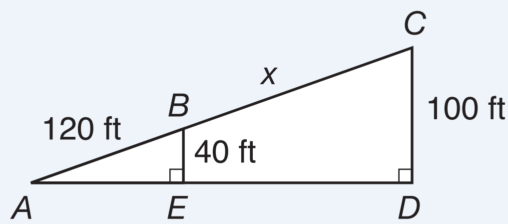
The scale factor is 4, so the corresponding vertices of the enlarged quadrilateral should be four times the distance from the origin of the original quadrilateral. Draw the rays and determine the coordinates of the corresponding points.



The coordinates of $A'B'C'D'$ are: A' (8, 12), B' (4, −4), C' (−6, −4), and D' (−6, 4).

If you know that two figures are similar, you can set up and solve a proportion to find a missing measurement. Use the missing measurement to solve the problem.

The diagram below shows a portion of a bridge support. What is the distance from point *B* to point *C*?



$$m\angle AEB = m\angle ADC, \text{ and } m\angle BAE = m\angle CAD.$$

Triangle *ABE* is similar to triangle *ACD* because their angles are equal in measure.

Use a proportion to find *AC*. Then subtract *AB* from *AC* to find *BC*.

- Set up a proportion that relates *AC* to three known measurements. The known measurements are *BE* (40 feet), *CD* (100 feet), and *AB* (120 feet). $\overline{BE}$ corresponds to $\overline{CD}$. They are both opposite $\angle A$. $\overline{AB}$ corresponds to $\overline{AC}$. They are opposite the right angles, $\angle BEA$ and $\angle D$.

$$\frac{BE}{CD} = \frac{AB}{AC}$$

$$\frac{40}{100} = \frac{120}{AC}$$

Use cross products to solve for *AC*.

$$40 \cdot AC = 100 \cdot 120$$

$$40 \cdot AC = 12{,}000$$

$$AC = 300 \text{ ft}$$

The length of $\overline{AC}$ is 300 feet.

- Subtract to find the length of $\overline{BC}$.

$$AC - AB = BC$$

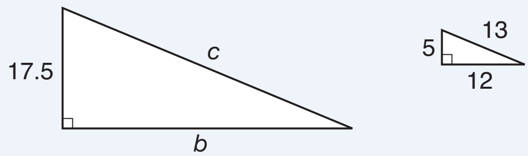
$$300 - 120 = BC$$

$$180 = BC$$

The distance from point *B* to point *C* is 180 feet.

The lengths of the sides of a right triangle are in the ratio 5:12:13. If the length of the shortest side of the triangle is 17.5 centimeters, what is the perimeter of the triangle?

One way to solve this problem is to use proportions to find the lengths of the other two sides of the triangle. Since the lengths of the sides of the triangle are in the ratio 5:12:13, the triangle is similar to a triangle with side lengths of 5, 12, and 13 units. Draw a diagram and identify corresponding sides.



Set up and solve two proportions to find the lengths of the missing sides.

$$\frac{5}{17.5} = \frac{12}{b}$$

Solve using cross products.

$$5b = 12 \cdot 17.5$$
$$5b = 210$$
$$b = 42 \text{ cm}$$

$$\frac{5}{17.5} = \frac{13}{c}$$

Solve using cross products.

$$5c = 13 \cdot 17.5$$
$$5c = 227.5$$
$$c = 45.5 \text{ cm}$$

Add the lengths of the sides to find the perimeter of the triangle.

$$P = 17.5 + 42 + 45.5 = 105 \text{ cm}$$

You could also solve this problem by representing the sides of the larger triangle with the expressions $5x$, $12x$, and $13x$.

Write and solve an equation showing that the shortest side is 17.5 cm.

$$5x = 17.5$$
$$\frac{5x}{5} = \frac{17.5}{5}$$
$$x = 3.5 \text{ cm}$$

Since $x = 3.5$, the sides of the triangle are:

$$5x = 5 \cdot 3.5 = 17.5 \text{ cm}$$
$$12x = 12 \cdot 3.5 = 42 \text{ cm}$$
$$13x = 13 \cdot 3.5 = 45.5 \text{ cm}$$

The perimeter of the triangle is the sum of the sides.

$$P = 17.5 + 42 + 45.5 = 105 \text{ cm}$$

Both methods result in a perimeter of 105 centimeters.

Try It

On a blueprint of a house, the rectangular floor of a bedroom is 4 inches in length and 3 inches in width. The actual length of the bedroom is 20 feet. How many square feet of carpet will be needed to cover the bedroom floor?

The rectangle representing the bedroom on the blueprint is similar to the actual rectangular floor of the bedroom.

Use a proportion to find the actual width of the bedroom floor. Then multiply the length by the width to find the actual area of the floor.

Let x represent the actual width. The known measurements are the blueprint length (_______ inches), the actual length (_______ feet), and the blueprint width (_______ inches). Write a proportion using these measurements.

$$\frac{\text{blueprint length}}{\text{blueprint width}} = \frac{\text{actual length}}{\text{actual width}}$$

$$\frac{\square}{3} = \frac{\square}{x}$$

Use cross products to solve for x.

$$\_\_\_\_\_\_\_ x = 20 \cdot \_\_\_\_\_\_\_$$
$$\_\_\_\_\_\_\_ x = \_\_\_\_\_\_\_$$
$$x = \_\_\_\_\_\_\_ \text{ ft}$$

The actual width of the bedroom is _______ feet.

Find the area of the bedroom.

$$A = lw$$
$$A = \_\_\_\_\_\_\_ \cdot \_\_\_\_\_\_\_$$
$$A = \_\_\_\_\_\_\_ \text{ ft}^2$$

It will take _______ square feet of carpet to cover the bedroom floor.

The known measurements are the blueprint length (4 inches), the actual length (20 feet), and the blueprint width (3 inches).

$$\frac{4}{3} = \frac{20}{x}$$
$$4x = 20 \cdot 3$$
$$4x = 60$$
$$x = 15 \text{ ft}$$

The actual width of the bedroom is 15 feet.

$$A = 20 \cdot 15$$
$$A = 300 \text{ ft}^2$$

It will take 300 square feet of carpet to cover the bedroom floor.

How Does a Dilation Affect the Perimeter, Area, and Volume of a Figure?

When the dimensions of a figure are dilated by a scale factor:

- The perimeter of the dilated figure is changed by the same scale factor.

  For example, if the dimensions were increased by a factor of 3, then the perimeter would increase by a factor of 3.

  If the dimensions of two similar figures are in the ratio $\frac{a}{b}$, then their perimeters are also in the ratio $\frac{a}{b}$.
- The area of the dilated figure is changed by the square of the scale factor.

  For example, if the dimensions were reduced by a factor of $\frac{1}{2}$, then the area would be reduced by a factor of $\left(\frac{1}{2}\right)^2$, or $\frac{1}{4}$.

  If the dimensions of two similar figures are in the ratio $\frac{a}{b}$, then their areas are in the ratio $\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$.
- The volume of the dilated figure is changed by the cube of the scale factor.

  For example, if the dimensions were increased by a factor of 2, then the volume would increase by a factor of 2^3, or 8.

  If the dimensions of two similar solid figures are in the ratio $\frac{a}{b}$, then their volumes are in the ratio $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$.

A hexagon with a perimeter of 54 centimeters is dilated by a scale factor of $\frac{1}{4}$. What is the perimeter of the dilated hexagon?

The perimeter of the dilated figure will change by a scale factor of $\frac{1}{4}$.

To find the perimeter of the dilated hexagon, multiply the perimeter of the original hexagon by the scale factor, $\frac{1}{4}$.

$$54 \cdot \frac{1}{4} = 13.5$$

The perimeter of the dilated hexagon is 13.5 centimeters.

A statue has a volume of 54 cubic feet and a height of 6 feet. A similar statue is 2 feet tall. What is the volume of the smaller statue?

- Because the statues are similar, the smaller statue is a dilation (reduction) of the larger statue. Determine the scale factor of the dilation.

$$\text{(scale factor)} \cdot \text{(height of original statue)} = \text{height of dilated statue}$$

$$\text{(scale factor)} \cdot (6) = 2 \text{ ft}$$

$$\text{scale factor} = \frac{2}{6} = \frac{1}{3}$$

The scale factor of the dilation is $\frac{1}{3}$.

- The volume of the dilated statue is equal to the volume of the original statue multiplied by the cube of the scale factor.

$$\left(\frac{1}{3}\right)^3 = \frac{1^3}{3^3} = \frac{1}{27}$$

$$V = \frac{1}{27}(54)$$

$$V = 2$$

The volume of the smaller statue is 2 cubic feet.

Try It

The dimensions of a family-size can of soup are 1.5 times the dimensions of a regular can of soup. A label covers the curved surface of each cylindrical can. If the label of the regular can has an area of 45 square centimeters, what is the area of the label on the family-size can?

The family-size can is a dilation of the regular can. The scale factor of the dilation is _______.

To find the area of the family-size label, multiply the ____________ of the scale factor by the area of the regular label.

$$\text{area of family-size label} = (1.5)^{\square} \cdot \underline{\qquad}$$

$$= \underline{\qquad} \cdot \underline{\qquad}$$

$$= \underline{\qquad\qquad} \text{ cm}^2$$

The area of the label on the family-size can is ______________ square centimeters.

The scale factor of the dilation is 1.5. To find the area of the family-size label, multiply the square of the scale factor by the area of the regular label.

$$\text{area of family-size label} = (1.5)^2 \cdot 45$$

$$= 2.25 \cdot 45$$

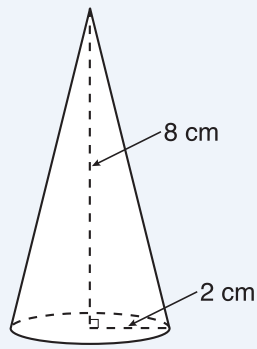
$$= 101.25 \text{ cm}^2$$

The area of the label on the family-size can is 101.25 square centimeters.

How Is the Surface Area or Volume of a Three-Dimensional Figure Affected If Only Its Height Is Changed?

If the height of a three-dimensional figure is changed by a scale factor, its surface area and volume will be changed by the same factor.

If the height of the cone shown below is increased by a scale factor of 3, what will be the effect on the volume of the new cone?



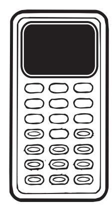
The volume of the enlarged cone is also increased by a factor of 3.

- To prove this, first find the volume of the original cone using $V = \frac{1}{3}Bh$. The base of a cone is a circle. Use πr^2 to find its area.

$$B = \pi(2)^2$$
$$B = 4\pi$$
$$B \approx 12.57 \text{ cm}^2$$

  Substitute this value into the volume formula.

$$V = \frac{1}{3}Bh$$
$$V \approx \frac{1}{3}(12.57)(8)$$
$$V \approx 33.52 \text{ cm}^3$$



- Use the scale factor to find the volume of the enlarged cone.

The volume of the original cone . . . → $33.52 \cdot 3 = 100.56$ ← . . . should be the volume of the enlarged cone.

. . . times the scale factor . . .

- Use the formula to find the volume of the enlarged cone.

  The height of the enlarged cone equals the height of the original cone multiplied by the scale factor, 3.

$$\text{Height: } 8 \cdot 3 = 24$$
$$\text{Area of Base: } \pi(2)^2 = 4\pi \approx 12.57$$
$$V = \frac{1}{3}Bh$$

$$V \approx \frac{1}{3}(12.57)(24)$$
$$V \approx 100.56 \text{ cm}^3$$

The volume of the enlarged cone will increase by a factor of 3.

The height of a pyramid with a surface area of 225 ft^2 is reduced by a scale factor of $\frac{1}{5}$. What is the surface area of the reduced pyramid? The surface area of the reduced pyramid should decrease by the same scale factor that its height is decreased by, $\frac{1}{5}$.

The surface area of the original pyramid . . .

$$225 \cdot \frac{1}{5} = 45$$

. . . times the scale factor . . .

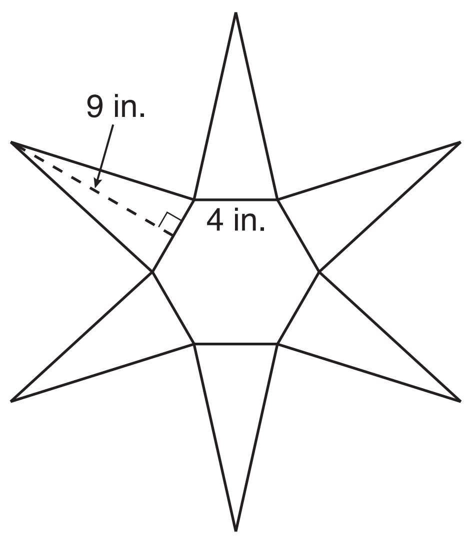
. . . should be the surface area of the reduced pyramid.

The reduced pyramid has a surface area of 45 square feet.

Now practice what you've learned.

Question 75

Elroy plans to make a design out of cardboard in the shape shown below. The center of the design is a regular hexagon with a side length of 4 inches. The hexagon is surrounded by congruent isosceles triangles. Each triangle has a height of 9 inches.



At minimum, how many square inches of cardboard does Elroy need for his design?

A 150 in.^{2}

B 108 in.^{2}

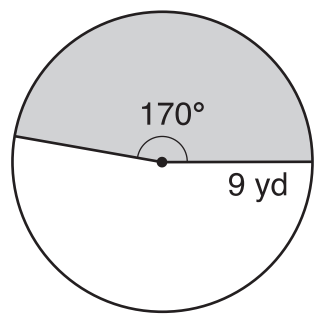
C 42 in.^{2}

D 115 in.^{2}

Answer Key: page 293

Question 76

The designated fishing area of a circular pond at a park is marked with two ropes attached to a buoy at the center of the pond. Each rope is 9 yards long, and together they form an angle of 170°.



What is the approximate area of the sector that is designated for fishing?

A 120 yd^2

B 140 yd^2

C 134 yd^2

D 127 yd^2

Answer Key: page 294

Question 77

Use the ruler on the Mathematics Chart to measure radius $\overline{CN}$ of the circle shown below to the nearest tenth of a centimeter.

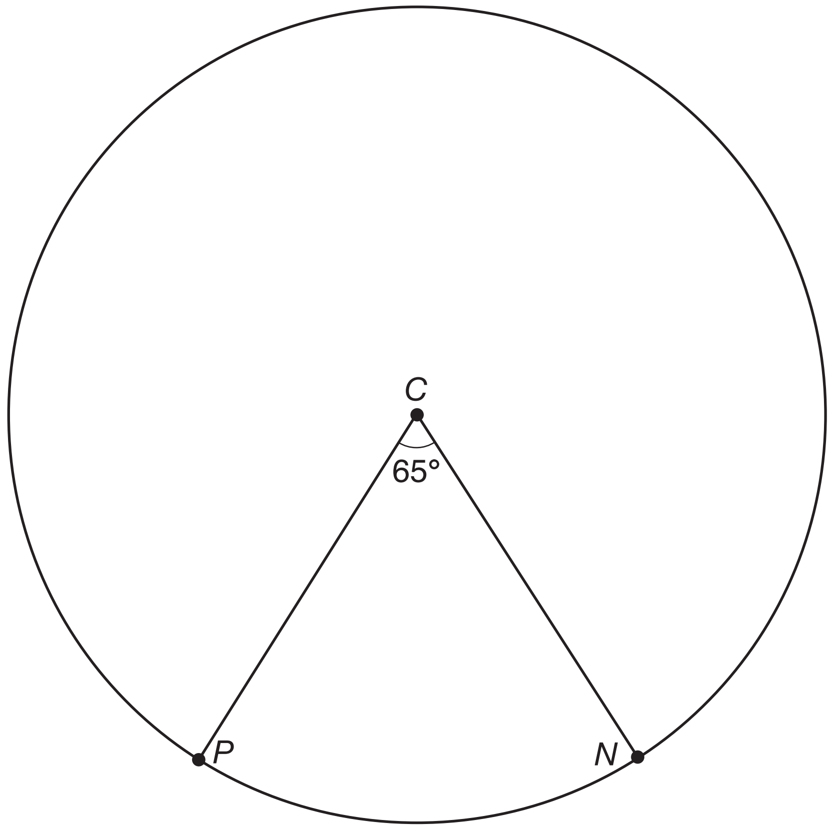
Which is the best approximation of the length of $\overset{\frown}{PN}$?

A 16.5 cm

B 6.1 cm

C 3.1 cm

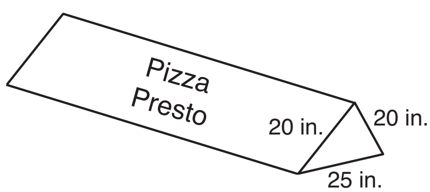
D 2.3 cm



Answer Key: page 294

Question 78

The owner of Pizza Presto wants to place a sign on the roof of his delivery truck. The sign is in the shape of a triangular prism. The owner needs to determine whether the truck will fit in the parking garage, which has 8 feet of clearance.



If the distance from the ground to the top of the truck's roof is 6 feet 10 inches, what will be the height of the truck with the sign, to the nearest inch?

A 7 feet 5 inches

B 7 feet 6 inches

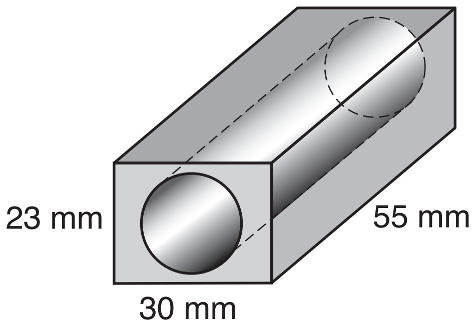
C 7 feet 11 inches

D 8 feet 2 inches

Answer Key: page 294

Question 79

Odin is casting metal machine parts. Each part is in the shape of a rectangular prism with a cylindrical hole extending entirely through its length. The prism has the dimensions shown in the diagram. The diameter of the cylindrical hole is 20 millimeters.



What is the approximate volume of metal used in each machine part?

A 10,222 mm^3

B 20,671 mm^3

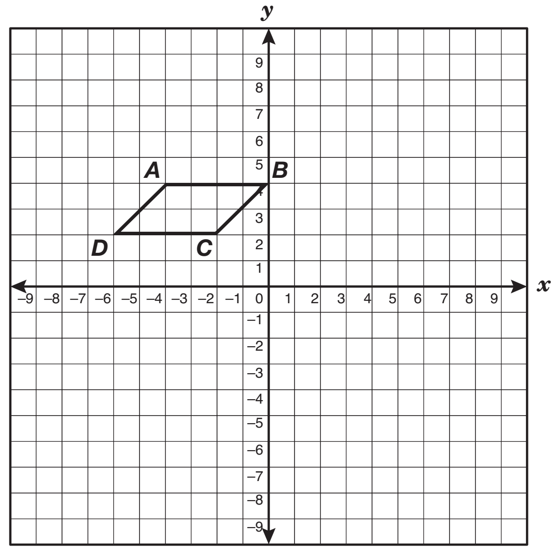
C 9,062 mm^3

D 37,636 mm^3

Answer Key: page 294

Question 80

The graph of parallelogram *ABCD* is shown below.



Which set of coordinates identifies the vertices of a parallelogram that is similar to *ABCD*?

A (0, 2), (2, 2), (1, 1), (−1, 1)

B (−2, 2), (0, 2), (1, 0), (−1, 0)

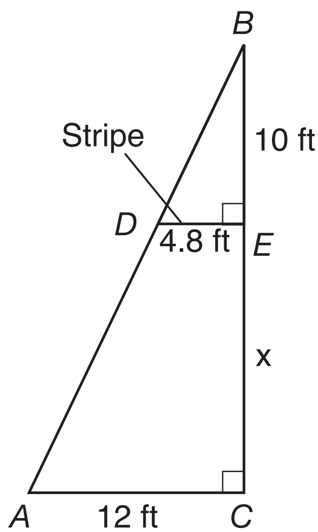
C (−8, 8), (−3, 8), (−5, 4), (−10, 4)

D (−3, 1), (1, 1), (−1 , 5), (−5, 5)

Answer Key: page 295

Question 81

The sail shown below has a horizontal stripe parallel to the base of the sail. What is the distance, x, from the bottom of the sail to the stripe?



A 25 feet

B 15 feet

C 5.8 feet

D 4 feet

Answer Key: page 295

Question 82

The length, width, and height of a cube are changed to 3 times their original size. By what factor does the surface area of the cube increase?

A 3

B 6

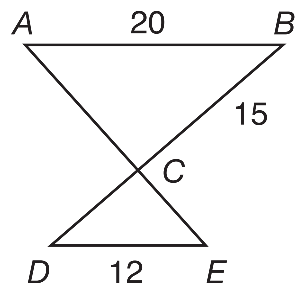
C 9

D 27

Answer Key: page 295

Question 83

In the diagram below, $\overline{AB}$ is parallel to $\overline{DE}$. $AB = 20$ inches, $DE = 12$ inches, and $BC = 15$ inches. What is the length of $\overline{DC}$?



A 25 in.

B 7 in.

C 9 in.

D 90 in.

Answer Key: page 295

Question 84

An ice-cream shop sells sugar cones and waffle cones. Each type of cone is 15 centimeters tall. The sugar cone has a radius of 2 centimeters. The radius of the waffle cone is 3 times as long as the radius of the sugar cone. By what factor is the volume of the waffle cone greater than that of the sugar cone?

A 9

B 3

C 6

D 27

Answer Key: page 295

Question 85

A spherical ball for a valve is 1.5 inches in diameter. What is the volume of the ball to the nearest tenth of an inch?

A 1.8 cubic inches

B 9.4 cubic inches

C 14.1 cubic inches

D 18.8 cubic inches

Answer Key: page 296

Question 86

Christy has a wooden square pyramid that she uses as a paperweight. The edge length of the base is 12 cm and the slant height of the pyramid is 15 cm. She wants to varnish the sides of the paperweight but not the bottom of it. How much area will she need to varnish?

A 90 cm^2

B 360 cm^2

C 504 cm^2

D 1080 cm^2

Answer Key: page 296

Objective 9

The student will demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.

For this objective you should be able to

- identify proportional relationships in problem situations and solve problems;
- apply concepts of theoretical and experimental probability to make predictions;
- use statistical procedures to describe data; and
- evaluate predictions and conclusions based on statistical data.

How Do You Solve Problems Involving Proportional Relationships?

A **ratio** is a comparison of two quantities. A **proportion** is a statement that two ratios are equal. There are many real-life problems that involve proportional relationships. For example, you use proportions when converting units of measurement. You also use proportions to solve problems involving percents and rates.

To solve problems that involve proportional relationships, follow these guidelines:

- Identify the ratios to be compared. Be certain to compare the corresponding quantities in the same order.
- Write a proportion, an equation in which the two ratios are set equal to each other.
- Solve the proportion. Use the fact that the cross products in a proportion are equal.

Vinita is entering data from a science experiment into her computer. It takes her 45 minutes to enter 20% of the data. If Vinita continues to enter data at the same rate, how many hours will it take her to enter all the data?

- Write a ratio that shows the percent of the job that Vinita has finished.

$$20\% = \frac{20}{100}$$

- Write a second ratio that compares a part to a whole. Vinita finishes part of the job in 45 minutes. Let x represent the number of minutes needed to finish the entire job. The ratio $\frac{45}{x}$ compares a part to a whole.

- Write a proportion, an equation setting the two ratios equal to each other.

$$\frac{\text{part of job}}{\text{whole job}} \quad \frac{20}{100} = \frac{45}{x}$$

- Use cross products to solve for x.

$$20x = 100(45)$$

$$20x = 4500$$

$$\frac{20x}{20} = \frac{4500}{20}$$

$$x = 225$$

It will take 225 minutes to enter all the data.

- The question asked for the number of hours it would take to enter all the data, so change 225 minutes to hours. Since there are 60 minutes in one hour, divide 225 minutes by 60 to find the number of hours.

$$225 \div 60 = 3.75$$

It will take Vinita 3.75 hours to enter all the data, or 3 hours 45 minutes.

Another way to find a percent of a number is to convert the percent to a decimal and multiply.

Find 18% of 120.

Convert 18% to a decimal.

$$18\% = 0.18$$

Multiply 120 by 0.18.

$$0.18(120) = 21.6$$

So 18% of 120 is 21.6.

Try It

Harry wants to buy two new tires for his car. The Tire Shop has the tires Harry wants. They are advertised as follows: "Buy 1 tire at full price, $45, and get the second for 25% off." Another store, Tires Unlimited, has the same tires listed for $42.50 each but is having a 10%-off sale this week. At which of these two tire stores should Harry buy his tires to save money? How much money will he save?

Find the cost of the two tires at the Tire Shop:

List price for first tire: $__________

Second tire:

$________ − 25% of $________ =

________ − 0.25(________) =

________ − ________ =

$________

Cost of two tires: $________ + $________ = $________

Find the cost of the two tires at Tires Unlimited:

List price for one tire: $________

Discounted price:

$________ − 10% of $________ =

________ − 0.10(________) =

________ − ________ =

$________

Cost of two tires: 2 • $________ = $________

Harry should buy his tires at ____________________. By doing so, he will save $________ − $________ = $ ________.

List price for first tire: $45

Second tire: $45 − 25% of $45 = 45 − 0.25(45) = 45 − 11.25 = $33.75

Cost of two tires: $45 + $33.75 = $78.75

Find the cost of the two tires at Tires Unlimited:

List price for one tire: $42.50

Discounted price: $42.50 − 10% of $42.50 = 42.50 − 0.10(42.50) = 42.50 − 4.25 = $38.25

Cost of two tires: 2 • $38.25 = $76.50

Harry should buy his tires at Tires Unlimited. By doing so, he will save $78.75 − $76.50 = $2.25.

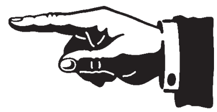
What Is Probability?

Probability is a measure of how likely an event is to occur. The probability of an event occurring is the ratio of the number of favorable outcomes to the number of all possible outcomes. In a probability experiment favorable outcomes are the outcomes that you are interested in.

The probability, P, of an event occurring must be from 0 to 1.

- If an event is impossible, its probability is 0.
- If an event is certain to occur, its probability is 1.

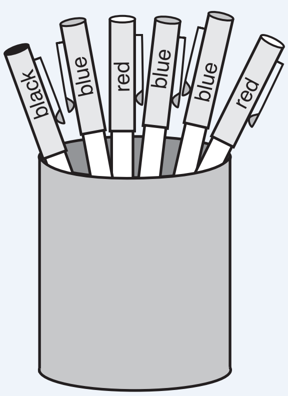
For example, if each of the letters in the word *father* is written on a card and placed in a basket, the probability of choosing a vowel from the basket is the ratio of the number of favorable outcomes, 2 vowels, to the number of all possible outcomes, 6 letters. The probability of choosing a vowel is $\frac{2}{6}$, or $\frac{1}{3}$. This is often written as $P(\text{vowel}) = \frac{1}{3}$.



Do you see that . . .

Randomly selecting a particular letter from a word is a **simple event**. A simple event in a probability experiment is determined by the outcome of one trial in the experiment. An event that is made up of a sequence of simple events is called a **compound event**. For example, randomly selecting a vowel first and a consonant second is a compound event.

A pen holder on the top of a desk contains 2 red pens, 3 blue pens, and 1 black pen. What is the probability of randomly choosing a blue pen from the holder ?



- There are a total of 6 possible outcomes for this experiment because there are 6 pens.
- A favorable outcome for this experiment is choosing a blue pen. There are 3 favorable outcomes because there are 3 blue pens.
- The probability of choosing a blue pen is the ratio of the number of favorable outcomes to the number of all possible outcomes.

$$\frac{\text{favorable outcomes}}{\text{all possible outcomes}} = \frac{3}{6}$$

This could be expressed as $\frac{3}{6}$ or $\frac{1}{2}$; it is not always necessary to simplify fractions in probability.

The probability of choosing a blue pen is $\frac{1}{2}$.

There are two kinds of compound events: those with independent events and those with dependent events.

- If the outcome of the first event does not affect the possible outcomes for the second event, the events are called **independent events**.

  Suppose you spin a spinner with the numbers 1 through 4 written on it and then you spin it again. Does the outcome of the first spin affect the likelihood of spinning a 3 on the second spin? No. The events are independent.

- If the outcome of the first event affects the possible outcomes for the second event, the events are called **dependent events**.

  Suppose you randomly draw 2 marbles, one at a time, from a bag with 4 white marbles and 1 red marble in it. You draw the second marble without replacing the first. Does the outcome of the first draw affect the likelihood of drawing a red marble on the second draw? Yes, because there are 5 marbles in the bag on the first draw, but only 4 on the second draw. The events are dependent.

How Do You Find the Probability of Compound Events?

One way to find the probability of a compound event is to multiply the probabilities of the simple events that make up the compound event.

If $P(A)$ represents the probability of event A and $P(B)$ represents the probability of event B, then the probability of the compound event (A and B) can be represented algebraically.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

For example, the probability of a coin landing tails up on one toss is $\frac{1}{2}$.

$$P(T) = \frac{1}{2}$$

The probability of a coin landing tails up on two tosses can be found as follows:

$$P(T \text{ and } T) = P(T) \cdot P(T) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

A jar contains 9 black marbles, 5 blue marbles, and 2 yellow marbles. You randomly draw a marble, put it back in the jar, and then randomly draw a second marble. What is the probability that you draw a yellow marble both times?

Because the first marble is replaced, the outcome of the first draw does not affect the outcome of the second draw. The events are independent.

- Add to find the total number of marbles in the jar.

$$9 + 5 + 2 = 16$$

  There are 16 marbles in the jar, so there are 16 possible outcomes for each draw.

- Find the probability of getting a yellow marble on the first draw. There are 2 favorable outcomes and 16 possible outcomes.

$$P(\text{yellow}_{\text{first}}) = \frac{2}{16} = \frac{1}{8}$$

- Find the probability of getting a yellow marble on the second draw. Since you replaced the marble, there are still 2 favorable outcomes and 16 possible outcomes.

$$P(\text{yellow}_{\text{second}}) = \frac{2}{16} = \frac{1}{8}$$

- Since these are independent events, the probability of drawing a yellow marble both times is the product of those probabilities.

$$\begin{aligned} P(\text{yellow}_{\text{first}} \text{ and yellow}_{\text{second}}) &= P(\text{yellow}_{\text{first}}) \cdot P(\text{yellow}_{\text{second}}) \\ &= \frac{1}{8} \cdot \frac{1}{8} \\ &= \frac{1}{64} \end{aligned}$$

The probability of drawing a yellow marble on both tries is $\frac{1}{64}$.

Another way to find the probability of some compound events is to look at the sample space for the experiment and identify the number of favorable outcomes. This method works only if the outcomes are all equally likely.

Consider the following experiment in which you spin a spinner divided into three congruent sectors on which the letters A, B, and C are written. What is the probability of spinning the letter B on two spins in a row?

Since the outcomes are equally likely, you could use a table to list all the possible outcomes. The favorable outcome is shaded.

Sample Space

First Spin	Second Spin
A	A
A	B
A	C
B	A
B	B
B	C
C	A
C	B
C	C

Do you see that . . .

There are 9 possible outcomes, but only 1 of them is favorable. The probability of getting a B on the spinner twice in a row is $\frac{1}{9}$.

This matches the result obtained using the rule for finding the probability of a compound event.

$$\begin{aligned} P(\mathrm{B}_{\text{first}} \text{ and } \mathrm{B}_{\text{second}}) &= P(\mathrm{B}_{\text{first}}) \cdot P(\mathrm{B}_{\text{second}}) \\ &= \frac{1}{3} \cdot \frac{1}{3} \\ &= \frac{1}{9} \end{aligned}$$

Finding the probability for dependent events is slightly more complicated, since the probability for the second event is affected by what happens in the first event. Let us revisit the bag of marbles example, but with some different marbles.

The bag has 7 marbles in it: 4 blue marbles and 3 green marbles. We will randomly draw one marble from the bag, note its color, and not put it back in the bag. We will then draw the second marble.

Suppose we want to determine the probability of drawing a blue marble and then on the second draw a green marble, without putting any of the marbles back. Since there are 4 blue marbles out of a total of 7 marbles, the probability of drawing a blue marble on the first draw is $\frac{4}{7}$.

$$P(B) = \frac{4}{7}$$

For the second draw, we have to realize that there is one fewer marble in the bag. There are 3 green marbles in the bag, out of a total of 6 remaining marbles. The probability of drawing a green marble on the second draw is $\frac{3}{6}$.

$$P(G) = \frac{3}{6}$$

So the probability of drawing a blue marble, followed by a green marble, without putting them back, can be found as follows:

$$P(B \text{ and } G) = P(B) \cdot P(G) = \frac{4}{7} \cdot \frac{3}{6} = \frac{12}{42}$$

A dresser drawer contains a number of unmatched socks: 6 black socks, 6 blue socks, and 4 brown socks. A sock is taken randomly from the dresser, and then, without replacing the first sock, a second sock is removed. What is the probability that both socks taken from the dresser are blue?

- First, see whether the events are dependent or independent.

  There are 16 possible outcomes for the first draw because there are a total of $6 + 6 + 4 = 16$ socks. However, there are only 15 possible outcomes for the second draw because one of the socks has already been removed. The two events are dependent.

- Find the probability of picking a blue sock first. There are 6 favorable outcomes and 16 possible outcomes.

$$P(\text{blue}_{\text{first}}) = \frac{6}{16} = \frac{3}{8}$$

- Find the probability of picking a blue sock the second time.

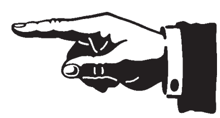
  You can assume that one blue sock is removed on the first draw, leaving only 5 blue socks in the dresser. There are now only 5 favorable outcomes. There are 15 possible outcomes.

$$P(\text{blue}_{\text{second}}) = \frac{5}{15} = \frac{1}{3}$$

- Find the compound probability.

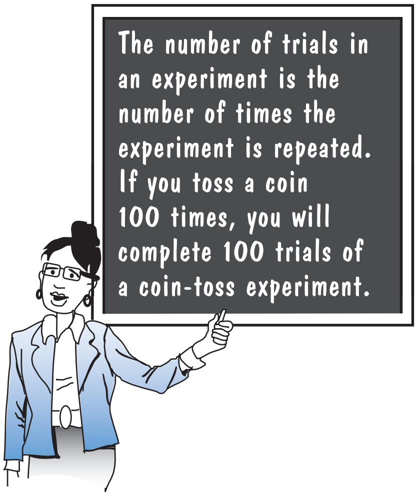
$$\begin{aligned} P(\text{blue}_{\text{first}} \text{ and } \text{blue}_{\text{second}}) &= P(\text{blue}_{\text{first}}) \cdot P(\text{blue}_{\text{second}}) \\ &= \frac{3}{8} \cdot \frac{1}{3} \\ &= \frac{3}{24} \\ &= \frac{1}{8} \end{aligned}$$

The probability that both socks taken from the drawer are blue is $\frac{1}{8}$.



Do you see that . . .

Objective 9



What Is the Difference Between Theoretical and Experimental Probability?

The **theoretical probability** of an event occurring is the ratio comparing the number of ways the favorable outcome should occur to the number of all possible outcomes. If you toss a fair coin, theoretically the coin should land on heads $\frac{1}{2}$ of the time.

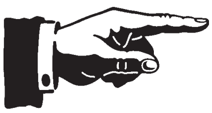
$$P(H) = \frac{1}{2} = 0.50$$

The **experimental probability** of an event occurring is the ratio comparing the actual number of times the favorable outcome occurs in a series of repeated trials of an experiment to the total number of trials. If you toss a coin 100 times, it is possible that the coin will land heads up 48 times, and tails up 52 times. The experimental probability of the coin landing heads up in this situation would be $\frac{48}{100}$.

$$P(H) = \frac{48}{100} = 0.48$$

The two types of probabilities, theoretical and experimental, are not always equal. In this case the theoretical probability is 0.50, but the experimental probability is 0.48.

Do you see that . . .



For a given situation the experimental probability is usually close to, but slightly different from, the theoretical probability. The greater the number of trials, the closer the experimental probability should be to the theoretical probability.

A spinner is divided into three equal sections labeled 1, 2, and 3. Vivian tests the spinner by spinning it 50 times and recording her results in the table below.

Section	Frequency
1	16
2	14
3	20

How does the theoretical probability of spinning a 2 compare to Vivian's experimental results?

- Theoretically the spinner is equally likely to land on any one of the three sections. The theoretical probability of spinning a 2 is $\frac{1}{3}$, or about 0.33.
- The experimental probability of spinning a 2 is the ratio of the number of times the spinner lands on 2 in the experiment to the total number of trials in the experiment. The spinner lands on 2 in 14 out of the 50 trials. The experimental probability of landing on 2 is $\frac{14}{50}$, or 0.28.

The theoretical probability of spinning a 2 is about 0.33, which is slightly greater than Vivian's experimental results, 0.28.

How Do You Use Probability to Make Predictions and Decisions?

You can use either theoretical or experimental probabilities to make predictions. If you know the probability of an event occurring and you also know the total number of trials, then you can predict the likely number of favorable outcomes.

- Write a ratio that represents the probability of an event occurring.
- Write a ratio that compares the number of favorable outcomes to the number of trials.
- Write a proportion.
- Solve the proportion.

A news crew polled 300 voters after they voted in a mayoral election. The results of the poll are shown in the table below.

Candidate	Frequency
Rogers	116
DeLuca	184

Based on these experimental results, by about how many votes should DeLuca win the election if approximately 100,000 votes are cast?

First find the experimental probability that a voter will vote for DeLuca.

- Of the total 116 + 184 = 300 voters polled, 184 voted for DeLuca.

  The experimental probability that a voter will vote for DeLuca is $\frac{184}{300}$.

Then predict the number of voters who will vote for DeLuca.

- Let d represent the total number of votes DeLuca will receive in the election. Write a ratio comparing the total number of votes for DeLuca to the total number of votes cast, 100,000.

$$\frac{d}{100{,}000}$$

- Write a proportion setting the two ratios equal to each other.

$$\frac{184}{300} = \frac{d}{100{,}000}$$

- Solve for d using cross products.

$$300d = 184(100{,}000)$$

$$300d = 18{,}400{,}000$$

$$d \approx 61{,}333$$

About 61,333 voters will vote for DeLuca.

Subtract the number of votes for DeLuca from the total number of votes to find the number of votes for Rogers.

$$100{,}000 - 61{,}333 = 38{,}667$$

About 38,667 voters will vote for Rogers.

Find the difference between the number of votes for DeLuca and the number of votes for Rogers.

$$61{,}333 - 38{,}667 = 22{,}666$$

Based on the results of the poll, DeLuca should win the election by about 22,666 votes.

Try It

A gardening store receives a shipment of rosebushes. Because they are not in bloom, the store clerk must look at the tag on each rosebush to find out what color its flowers will be. He looks at the tags on 30 bushes and records the results in the table below.

Rose Color	Frequency
Pink	10
Yellow	8
Red	12

Based on these results, if there are 180 rosebushes in the shipment, about how many of them could be expected to have yellow flowers?

First find the experimental probability of a rosebush having yellow flowers.

Out of the _______ rosebushes checked by the clerk, _______ are marked as having yellow flowers.

The experimental probability of a bush having yellow flowers is $\frac{\square}{\square}$.

Next find the number of yellow rosebushes in the shipment.

There are a total of _______ rosebushes in the shipment.
Let y represent the number of rosebushes in the shipment that have yellow flowers.

Write a proportion that can be used to solve for y.

$$\frac{\square}{30} = \frac{y}{\square}$$

Solve for y using cross products.

$$30y = \_\_\_\_ \cdot \_\_\_\_$$
$$30y = \_\_\_\_$$
$$y = \_\_\_\_$$

About _______ rosebushes in the shipment could be expected to have yellow flowers.

Out of the 30 rosebushes checked by the clerk, 8 are marked as having yellow flowers. The experimental probability of a bush having yellow flowers is $\frac{8}{30}$. There are a total of 180 rosebushes in the shipment.

$$\frac{8}{30} = \frac{y}{180}$$
$$30y = 8 \cdot 180$$
$$30y = 1440$$
$$y = 48$$

About 48 rosebushes in the shipment could be expected to have yellow flowers.

How Do You Use Mode, Median, Mean, and Range to Describe Data?

There are many ways to describe the characteristics of a set of data. The mode, median, and mean are all called **measures of central tendency**.

Mode	The **mode** of a set of data describes which value occurs most frequently. If two or more numbers occur the same number of times and more often than all the other numbers in the set, those numbers are all modes for the data set. If each of the numbers in a set occurs the same number of times, the set of data has no mode.	Use the mode to show which value or values in a set of data occur most often. For the set {1, 4, 9, 3, 1, 6} the mode is 1 because it occurs most frequently. The set {1, 4, 3, 3, 1, 6} has two modes, 1 and 3, because they both occur twice and most frequently.
Median	The **median** of a set of data describes the middle value when the set is ordered from greatest to least or from least to greatest. If there are an even number of values, the median is the average of the two middle values. Half the values are greater than the median, and half the values are less than the median. The median is a good measure of central tendency to use when a set of data has an outlier, a number that is very different in value from the other numbers in the set.	Use the median to show which number in a set of data is in the middle when the numbers are listed in order. For the set {1, 4, 9, 3, 6} the median is 4 because it is in the middle when the numbers are listed in order: {1, 3, 4, 6, 9}. For the set {1, 4, 9, 3, 1, 6} the median is $\frac{3+4}{2} = 3.5$ because 3 and 4 are in the middle when the numbers are listed in order: {1, 1, 3, 4, 6, 9}. Their values must be averaged to find the median.
Mean	The **mean** of a set of data describes the average of the numbers. To find the mean, add all the numbers and then divide by the number of items in the set. The mean of a set of data can be greatly affected if one of the numbers is an outlier, a number that is very different in value from the other numbers in the set. The mean is a good measure of central tendency to use when a set of data does not have any outliers.	Use the mean to show the numerical average of a set of data. For the set {1, 4, 9, 3, 1, 6} the mean is the sum, 24, divided by the number of items, 6. The mean is $24 \div 6 = 4$.
Range	The **range** of a set of data describes how big a spread there is from the largest value in the set to the smallest value.	Use the range to show how much the numbers vary. For the set {1, 4, 9, 3, 1, 6} the range is $9 - 1 = 8$.

To decide which of these measures to use to describe a set of data, look at the numbers and ask yourself, *What am I trying to show about the data?*

The table shows the attendance at the last six football games of the season at Vista High School.

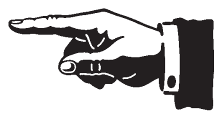
Date	Attendance
Oct. 30	225
Nov. 6	218
Nov. 13	207
Nov. 20	230
Dec. 4	242
Dec. 10	478

Which measure of central tendency best describes the number of people who typically attend football games at Vista High School?

- Each attendance number in the set occurs only once. The set of data has no mode.
- The mean attendance at the games is equal to the average of the data, or the sum of the attendance figures divided by the number of games.

  $$1600 \div 6 \approx 266.67$$

  Notice that the attendance at the last game, 478 people, is much greater than the attendance at the other games. This value is an outlier. Because the mean is affected by the outlier, it does not give a very good representation of attendance at a typical game; it is too high.



Do you see that . . .

- The median attendance is the average of the two middle values when the attendance numbers are listed from least to greatest.

  $$207 \quad 218 \quad \underline{225} \quad \underline{230} \quad 242 \quad 478$$

  The two middle values are 225 and 230, and their average is 227.5.

  The median is not affected by the outlier.

For this situation the median best describes the number of people who typically attend a football game at Vista High School.

Objective 9

Sometimes you need to describe how a change in data affects one or more measures of the data set.

The table shows Ryan's golf scores in his first five tournaments of the year.

Round	Score
1	78
2	76
3	82
4	80
5	79

Ryan's score in his sixth tournament was 103. How does Ryan's score in his sixth tournament affect the various measures of his golf score?

Range: The range is the difference between the highest score and the lowest score.
For the first 5 tournaments, it was $82 - 76 = 6$.
With the sixth tournament included, the range is $103 - 76 = 27$.
The range of values increases significantly because the new value is well outside the previous range. It is an outlier.

Median: The median is found by listing the scores in order and picking the middle value.
Scores for the first 5 tournaments: 76, 78, <u>79</u>, 80, 82

$$\text{median} = 79$$

With the sixth tournament included: 76, 78, <u>79</u>, <u>80</u>, 82, 103

$$\text{median} = (79 + 80) \div 2 = 79.5$$

The median value is not significantly affected by the outlier.

Mode: The mode of a set of numbers tells which value occurs most frequently.
For the first 5 tournaments, there is no mode. No value is repeated.
With the sixth score included, there still is no mode.
The mode is not affected by the outlier.

Mean: Find the mean of Ryan's first five scores. The mean is the average of the five scores.

$$\text{mean} = \frac{78 + 76 + 82 + 80 + 79}{5} = 79$$

Find the mean of Ryan's six scores. Since the sixth score, 103, is greater than the other data values, it raises the mean.

$$\text{mean} = \frac{78 + 76 + 82 + 80 + 79 + 103}{6} = 83$$

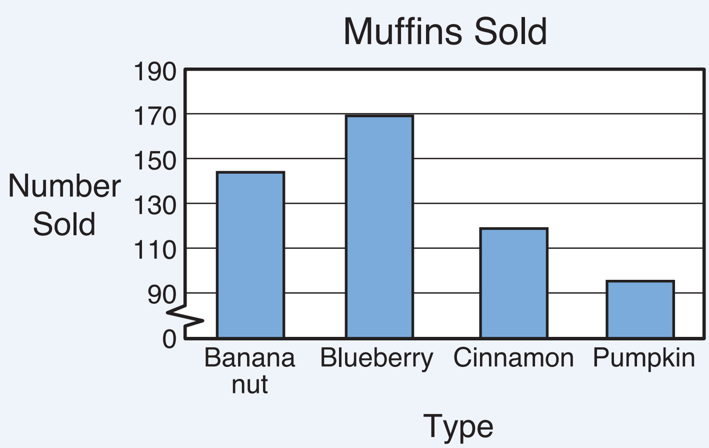
The sixth score raises Ryan's mean score by $83 - 79 = 4$ strokes.

How Do You Use Graphs to Represent Data?

There are many ways to represent data graphically. Bar graphs, histograms, and circle graphs are three types of graphs used to display data. Other types of graphs include line plots, stem and leaf plots, and box and whisker plots. Graphical representations of data often make it easier to see relationships in the data. However, if the conclusions drawn from a graph are to be valid, you must read and interpret the data from the graph accurately.

A **bar graph** uses bars of different heights or lengths to show the relationships between different groups or categories of data.

The bar graph shows the numbers of different types of muffins that were sold at a bakery one morning. What conclusions can you draw about the popularity of the different types of muffins?



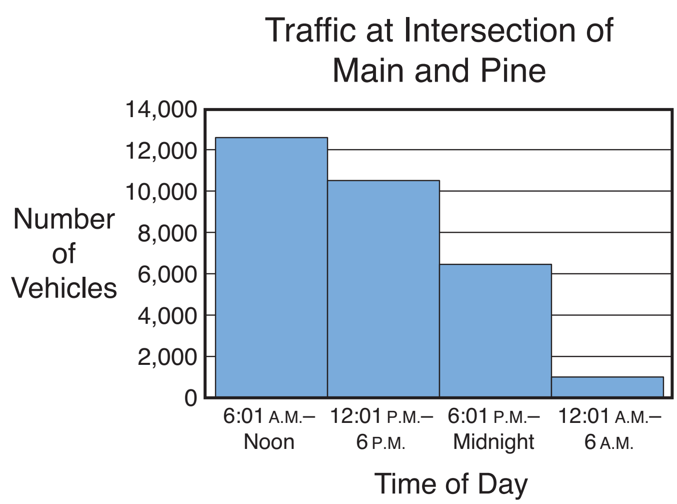
The broken line on the vertical axis indicates that the bakery sold at least 90 muffins of each type. Using the broken line allows the graph to have shorter bars.

- The graph shows that blueberry muffins were the most popular type, followed by banana nut, cinnamon, and pumpkin.

- Notice that although the length of the bar for blueberry muffins is about twice as long as the bar for cinnamon muffins, the actual number of blueberry muffins sold is not twice the number of cinnamon muffins sold. The vertical scale of the graph shows that about 170 blueberry muffins and about 120 cinnamon muffins were sold. Therefore, blueberry muffins were not twice as popular as cinnamon muffins.

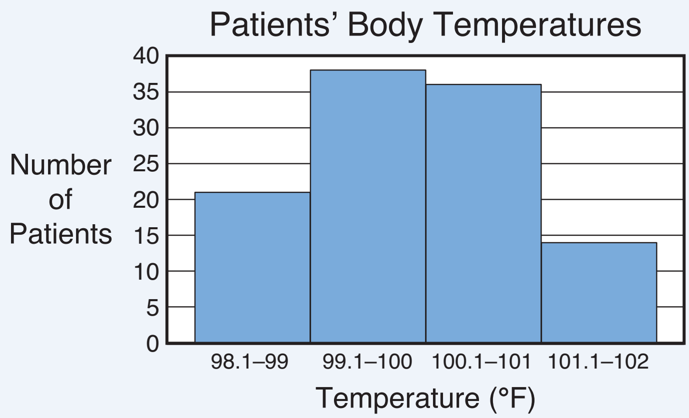
A **histogram** is a special kind of bar graph that shows the number of data points that fall within specific intervals of values. The intervals into which the data's range is divided should be equal. If the intervals are not equal, the graph could be misleading and result in invalid conclusions.

Look at the histogram below showing the number of vehicles passing through an intersection at various times of day.



About 12,400 vehicles passed through the intersection between 6:01 A.M. and noon. About 10,300 passed through between 12:01 P.M. and 6 P.M. About 6,300 passed through between 6:01 P.M. and midnight. About 1,000 passed through between 12:01 A.M. and 6 A.M.

When people go to a doctor's office feeling ill, one of the first things the nurse does is take their body temperature. Normal body temperature is around 98.6°F. The histogram shows the body temperatures of patients visiting a doctor's office during a one-week period.



What conclusions can you draw from the graph about the patients' body temperatures?

On the graph the patients' body temperatures are divided into intervals.

- The greatest number of patients had a temperature between 99.1°F and 100°F.
- The least number of patients had a temperature between 101.1°F and 102°F.
- Most of the doctor's patients had temperatures greater than 99°F. Therefore, most of the doctor's patients had above-normal body temperatures when they visited the office.

A **circle graph** represents a set of data by showing the relative size of the parts that make up the whole. The circle represents the whole, or the sum of all the data elements. Each section of the circle represents a part of the whole. The number of degrees in the central angle of the section should be proportional to the number of degrees in the circle, 360°.

Suppose the manager of a grocery store constructs a circle graph to compare the amounts of the various types of peanut butter sold at his store. He uses the data for peanut butter sales during the first week in March, as shown in the table below, to build the graph.

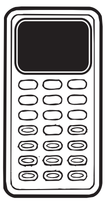
Peanut Butter Sales

Type	Jars Sold
Chunky	48
Smooth	84
Low-fat	40
Premium	28

What central angle should be used for the section of the circle representing smooth peanut butter?

- Find the fraction of peanut butter sales that were of smooth peanut butter. There were 84 jars of smooth peanut butter sold. Add to find the total number of jars sold.

$$48 + 84 + 40 + 28 = 200$$



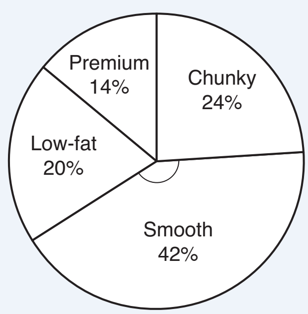
Two hundred jars were sold in all. Therefore, $\frac{84}{200} = \frac{21}{50}$ of the jars sold contained smooth peanut butter.

- A circle has 360°. Set up and solve a proportion.

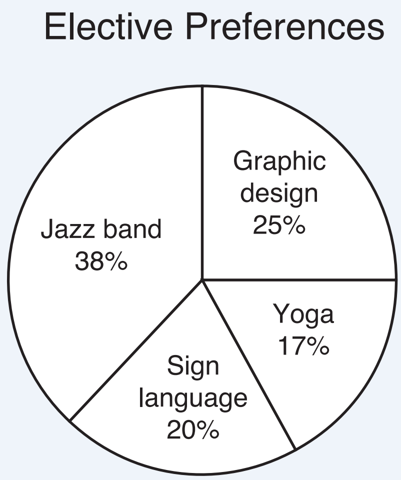
$$\frac{21}{50} = \frac{x}{360}$$
$$50x = 21 \cdot 360$$
$$50x = 7560$$
$$x = 151.2$$

The central angle for the section representing smooth peanut butter should measure 151.2°.

Peanut Butter Sales



Next year new elective classes will be offered at Winston High School. Students were surveyed about which of the four possible new electives they would prefer to see offered. The results of the survey are shown below.



What conclusions can you draw about the students' elective preferences?

- The most-preferred elective choice is jazz band.
- The graph shows that 25%, or $\frac{1}{4}$, of the students surveyed would prefer graphic design.
- The smallest fraction of students would prefer yoga.

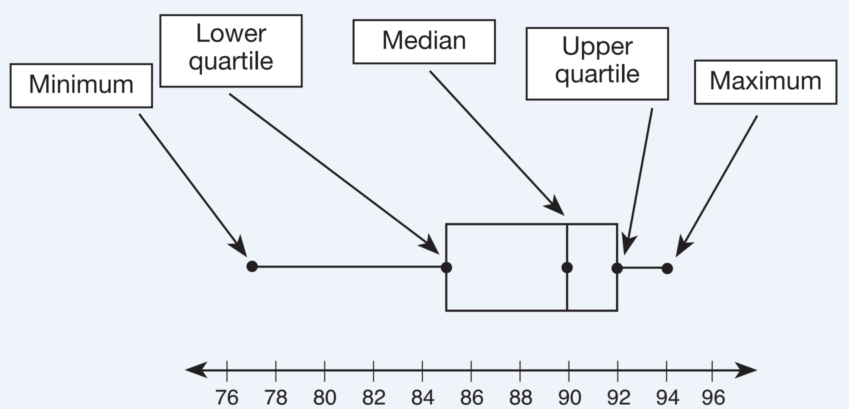
Objective 9

A **box and whisker plot** can be used to describe the distribution of a data set (that is, how near or far the data points are from one another). It also gives specific information about certain values related to the data set.

The table below represents the high temperatures in degrees Fahrenheit for the month of July.

High Temperature	Frequency
77	1
78	0
79	0
80	1
81	1
82	0
83	0
84	4
85	1
86	1
87	2
88	2
89	2
90	4
91	1
92	5
93	4
94	2

The box and whisker plot below represents the data set shown in the table above.



As you can see from the box and whisker plot above, the data appear to be closer together for the higher values and more spread out for the lower values. The table supports this; there are 16 high temperatures that are in the 90s.

The **lower quartile** can be thought of as the middle value of the first half of the data set. One-fourth (one quarter) of the data set will be at or below the lower quartile. Likewise, the **upper quartile** is the middle value of the second half of the data set. Three-fourths of the data will be at or below the upper quartile. In this sense, quartiles are very much like medians.

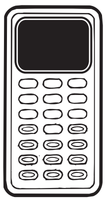
For example, in the sample data set, there are 31 values. When the data set is ordered, the median is the number in the 16th place (in this case, 90 degrees). The data set is now split into two halves. The lower half of the data has places 1 through 15, and the upper half has places 17 through 31.

As mentioned above, the lower quartile is the middle value of the lower half of the data (places 1 through 15). The middle value would then be in the 8th place: 85 degrees. Likewise, the upper quartile is the middle value of the upper half of the data (places 17 through 31). The middle value of the upper half of the data would be in the 24th place: 92 degrees.

What data can be interpreted from this graph?

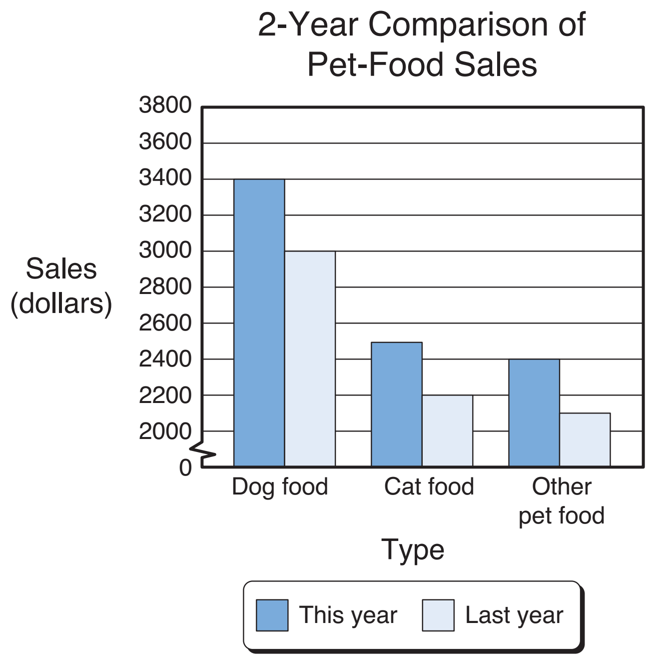
The lowest daily high temperature for July was 77. The highest daily high temperature was 94. The median high temperature was 90. The lower quartile was 85, and the upper quartile was 92.

Objective 9



Try It

Carol manages Whiskers Pet Store. She wanted to see how sales this year compared to sales at the same time last year. The graph below shows the amounts of money spent on dog food, cat food, and other pet food at Whiskers Pet Store last month, as well as during the same month one year ago.



Carol concluded that the store's sales of dog food had the greatest percent increase. Was her conclusion valid?

First read the graph to find the amount of sales for each type of food for each year. Calculate the sales increase, first in dollars and then in percent.

Pet Food	Sales (Last Year)	Sales (This Year)	$ Increase	% Increase
Dog food	$3000	$3400	$400	$\frac{400}{3000} \approx 0.133 \approx 13.3\%$
Cat food	$______	$______	$______	$\frac{\square}{2200}$ ≈ ______ ≈ ______%
Other pet food	$______	$______	$______	$\frac{\square}{\square}$ ≈ ______ ≈ ______%

The greatest percent increase in sales was for ______________. Carol's conclusion was ______________.

First read the graph to find the amount of sales for each type of food for each year. Calculate the sales increase, first in dollars and then in percent.

Pet Food	Sales (Last Year)	Sales (This Year)	$ Increase	% Increase
Dog food	$3000	$3400	$400	$\frac{400}{3000} \approx 0.133 \approx 13.3\%$
Cat food	$2200	$2500	$300	$\frac{300}{2200} \approx 0.136 \approx 13.6\%$
Other pet food	$2100	$2400	$300	$\frac{300}{2100} \approx 0.143 \approx 14.3\%$

The greatest percent increase in sales was for other pet food. Carol's conclusion was not valid.

Now practice what you've learned.

Question 87

Marty's horse Dither weighed 1200 pounds. Over a three-month period, Marty increased the horse's food intake, and Dither's weight increased by 10%. But over the next two months, Marty exercised the horse more frequently, and Dither's weight decreased by 10%. What did Dither weigh at the end of this five-month period?

A 1200 lb

B 1188 lb

C 1320 lb

D 1298 lb

Answer Key: page 296

Question 88

In how many fewer minutes will a taxi complete a 10-mile drive at an average speed of 40 miles per hour than at an average speed of 30 miles per hour?

A 5 minutes

B 10 minutes

C 8 minutes

D 13 minutes

Answer Key: page 296

Question 89

A weather forecaster states that there is a 20% chance of rain on each of the next three days. To the nearest whole percent, what is the probability that it will not rain at all over the next three-day period?

A 51%

B 80%

C 40%

D 64%

Answer Key: page 296

Question 90

Mr. Kumar needs $0.50 to pay for the newspaper. He has 6 dimes, 5 quarters, and 4 nickles in his pocket. If Mr. Kumar reaches into his pocket and randomly takes out 2 coins one at a time without putting either one back, what is the probability that he will select the 2 quarters he needs?

A $\frac{3}{5}$

B $\frac{4}{45}$

C $\frac{2}{21}$

D $\frac{13}{21}$

Answer Key: page 296

Question 91

Amelia surveyed 50 students at her school about the average number of hours of television they watch per day. Her survey results are shown in the table below.

Television Viewing Habits

Hours Watched per Day	Number of Students
Less than 1	11
At least 1 but less than 2	22
At least 2 but less than 3	9
At least 3 but less than 4	6
4 or more	2

There are 460 students at Amelia's school. Based on the results of this survey, about how many students at Amelia's school watch an average of less than 2 hours of television per day?

A 202

B 304

C 152

D 139

Answer Key: page 297

Question 92

A park ranger is analyzing a table showing the daily high temperatures of a lake during the summer months. Which measure of data could the ranger best use to show that the temperature of the lake remains almost constant over this time period?

A Mean

B Median

C Range

D Mode

Answer Key: page 297

Question 93

A car dealer has six used minivans for sale. The mileage on the minivans is shown in the table below.

Used Minivans

Color	Mileage (thousands of miles)
White	90
Tan	35
Black	110
Silver	15
Blue	32
Maroon	150

The car dealer plans to describe the minivans' mileage in a newspaper ad by using one of the following data measures. Which measure of the data would make the vehicles appear to have the lowest mileage?

A Mean

B Median

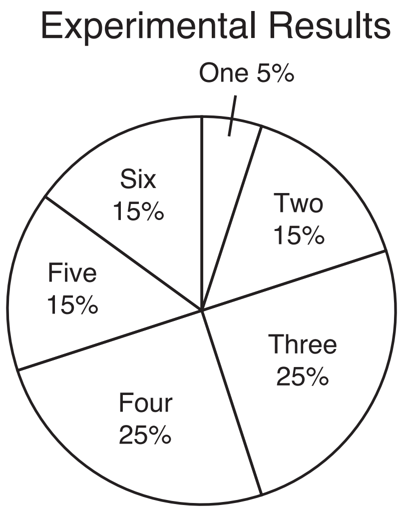
C Range

D Mode

Answer Key: page 297

Question 94

The circle graph below shows the results of a fair number cube being tossed 20 times.



Which table matches the data presented in the circle graph?

A

Number Landing Face Up	One	Two	Three	Four	Five	Six
Frequency	1	3	4	4	3	3

B

Number Landing Face Up	One	Two	Three	Four	Five	Six
Frequency	3	3	4	4	3	3

C

Number Landing Face Up	One	Two	Three	Four	Five	Six
Frequency	1	3	5	5	3	3

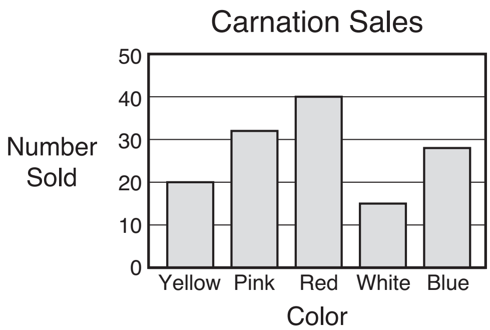
D

Number Landing Face Up	One	Two	Three	Four	Five	Six
Frequency	3	3	3	5	5	3

Answer Key: page 297

Question 95

The senior class sold carnations during a fund-raiser. The bar graph shows the number sold for each color of carnation.



Ramiro plans to make a circle graph of the data in the bar graph. Approximately what percent of the circle graph should represent red carnations?

A 40%

B 50%

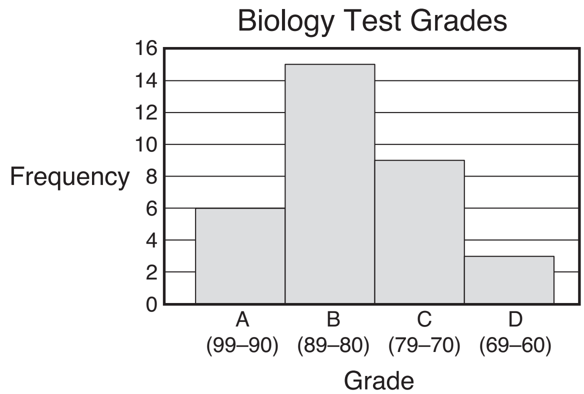
C 20%

D 30%

Answer Key: page 297

Question 96

The histogram shows the grade distribution for Mr. Holt's biology test.



Which of the following statements is best supported by the data in the histogram?

A Approximately 20% of Mr. Holt's students received a B.

B The median grade on the exam was a B.

C Approximately 15 of Mr. Holt's students took the exam.

D The range of the test scores was less than 20.

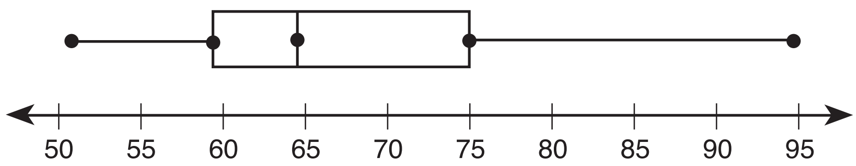
Answer Key: page 298

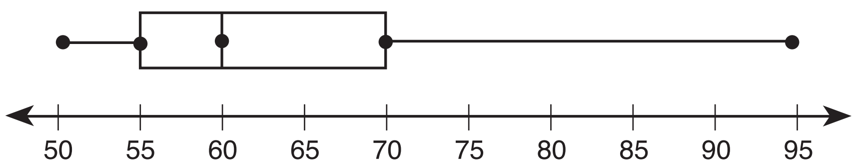
Question 97

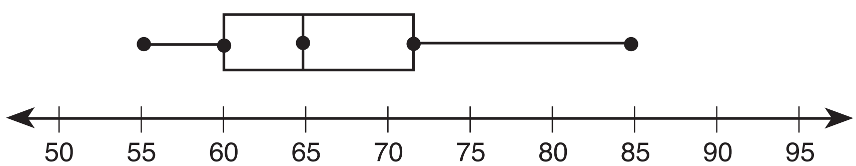
A high school basketball team kept track of the number of points they scored in the last 12 games of the season. The scores are displayed in the list below.

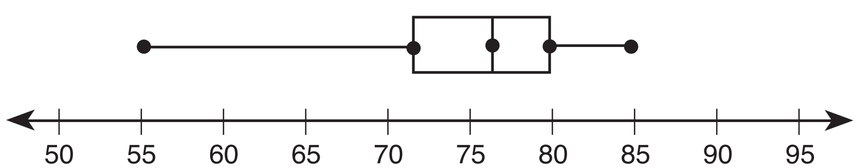
65, 72, 60, 63, 95, 58, 61, 70, 55, 85, 78, 51

Which box and whisker plot matches the data in the list?

A


B


C


D


Answer Key: page 298

Objective 10

The student will demonstrate an understanding of the mathematical processes and tools used in problem solving.

For this objective you should be able to

- apply mathematics to everyday experiences and activities;
- communicate about mathematics; and
- use logical reasoning.

How Do You Apply Math to Everyday Experiences?

Suppose you want to predict the likelihood of a team winning a game based on its past record. Or suppose you need to find the amount of money you could earn working at a certain hourly rate. Finding the solution to problems such as these often requires the use of math.

Solving problems involves more than just arithmetic; logical reasoning and careful planning also play very important roles. The steps in problem solving include understanding the problem, making a plan, carrying out the plan, and evaluating the solution to determine whether it is reasonable.

Museum tickets cost $5.00 for adults and $2.00 for children. A group of adults and children spent a total of $54.00 on museum tickets. If there were a total of 12 people in the group, what percent of the total amount spent was used to buy the children's tickets?

- What information is given in the problem?
  Cost of an adult's ticket = $5.00
  Cost of a child's ticket = $2.00
  Number of people in group = 12
  Cost of tickets for the group = $54.00
- What do you need to find?
  The number of children in the group
  The amount of money spent on children's tickets
  The percent spent on the children's tickets
- Represent the number of adults with a and the number of children with c.

- Write two equations in two unknowns.

  The number of adults plus the number of children equals the number of people in the group.

$$a + c = 12$$

  The number of adults times the adult ticket price plus the number of children times the child ticket price equals the total cost.

$$(5.00)a + (2.00)c = 54.00$$

$$5a + 2c = 54$$

- Solve the system of equations. Multiply the first equation by -5.

$$\begin{array}{r} -5a - 5c = -60 \\ \underline{5a + 2c = \quad 54} \\ -3c = \quad -6 \end{array}$$

$$\frac{-3c}{-3} = \frac{-6}{-3}$$

$$c = 2$$

There were 2 children in the group.

- Find the cost of the tickets for the children.

$$2 \text{ children} \cdot \$2/\text{ticket} = \$4$$

- Find the percent of the total amount spent, \$54, that was used to buy the children's tickets.

  Write and solve a proportion.

$$\frac{x}{100} = \frac{4}{54}$$

$$54x = 100(4)$$

$$54x = 400$$

$$\frac{54x}{54} = \frac{400}{54}$$

$$x \approx 7.4$$

So 7.4% of the money spent was used to buy the children's tickets.

Try It

Maria paid Robert a total of $600 to paint her house. This amount included the cost of the paint he used and his fee for labor. Robert's fee was three times the cost of the paint. If Robert charged Maria a base fee of $50 plus $20 per hour, find the number of hours Robert spent painting Maria's house.

What information are you given in the problem?

Maria paid $________ to have her house painted.

Maria paid Robert ______________ times the cost of the paint for his fee.

What do you need to find?

The amount of money spent on paint and for Robert's labor

The number of hours Robert spent painting Maria's house

Write and solve an equation to find how much Robert charged for the time he spent painting.

Write and solve an equation to find the number of hours he painted.

Let x represent the amount of money spent on paint.

Maria paid Robert three times as much money for his labor as for the paint.

Let ________ represent Robert's fee.

$$\text{Total cost} = \text{paint} + \text{Robert's fee}$$

$$600 = \_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_$$

$$600 = \_\_\_\_\_\_\_\_ x$$

$$\_\_\_\_\_\_\_\_ = x$$

Maria spent $________ on paint. Robert's fee was $3x$, or

3 • $________ = $________.

Let h represent the number of hours Robert painted the house.

Robert charged Maria a fee of \$450 for his time. This included a base fee of \$50 plus \$20 per hour.

Total cost = base fee + \$20 per hour

$$450 = \_\_\_\_\_\_\_ + \_\_\_\_\_\_\_ h$$

$$\_\_\_\_\_\_\_ = 20h$$

$$\_\_\_\_\_\_\_ = h$$

Robert painted Maria's house for _______ hours.

Maria paid \$600 to have her house painted.

Maria paid Robert three times the cost of the paint for his fee.

Let $3x$ represent Robert's fee.

Total cost = paint + Robert's fee

$$600 = x + 3x$$

$$600 = 4x$$

$$150 = x$$

Maria spent \$150 on paint. Robert's fee was $3x$, or $3 \cdot \$150 = \$450$.

Total cost = base fee + \$20 per hour

$$450 = 50 + 20h$$

$$400 = 20h$$

$$20 = h$$

Robert painted Maria's house for 20 hours.

What Is a Problem-Solving Strategy?

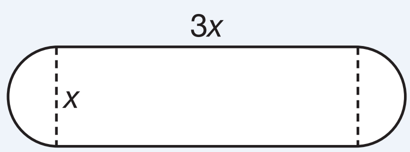
A **problem-solving strategy** is a plan for solving a problem. Different strategies work better for different types of problems. Sometimes you can use more than one strategy to solve a problem. As you practice solving problems, you will discover which strategies you prefer and which work best in various situations.

Some problem-solving strategies include

- drawing a picture;
- looking for a pattern;
- guessing and checking;
- acting it out;
- making a table;
- working a simpler problem; and
- working backwards.

The middle school is building a new quarter-mile track in the shape of a rectangle with semicircles on each end. The diameter of the semicircles equals the width of the track field. The length of the rectangular portion of the track field is to be 3 times its width. What will be the length of the rectangular portion of the field, to the nearest foot?

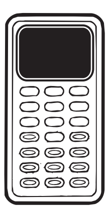
- Draw a picture and label the length and width of the field and the diameter of the semicircles.



- Since the two semicircles form one complete circle, use the formula for the circumference of a circle to find the distance around them. The diameter of the circle is x.

$$C = \pi d$$

$$C = \pi x$$



- The distance around the track is $\frac{1}{4}$ mile. Convert the measurement to feet.

$$1 \text{ mile} = 5280 \text{ feet.}$$

$$\frac{1}{4}(1 \text{ mile}) = \frac{1}{4}(5280 \text{ feet}) = 1320 \text{ feet}$$

- The distance around the track, 1320 feet, is twice the length of the rectangle plus the circumference of the two semicircles.

$$3x + 3x + \pi x = 1320$$
$$6x + \pi x = 1320$$
$$9.14x \approx 1320$$
$$x \approx 144.4$$

- The width of the rectangle is approximately 144.4 feet. The expression $3x$ represents the length of the rectangle.

$$3x \approx 3(144.4) \approx 433.2$$

If it is to be a $\frac{1}{4}$-mile track, the rectangular portion must have a length of approximately 433.2 feet.

Try It

There are 50 candies of different colors in a bag. The percent of candies of each color is shown below.

Color	Percent
Red	24
Blue	12
Yellow	8
Pink	16
Purple	16
Green	24

If you randomly draw two candies from the bag without replacing the first candy drawn, what is the probability of drawing two red candies?

The outcome of the first draw affects the likelihood of drawing a red candy on the second draw. This probability problem involves two ____ events.

The favorable outcome is drawing a ________________ candy. First find the number of red candies in the bag.

There are a total of ________ candies in the bag.

________% of the candies are red.

$$\_\_\_\_\_\_\_\_ \cdot 50 = \_\_\_\_\_\_\_\_$$

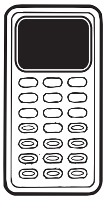
________ candies are red.

See Objective 9, page 231, for more information about probability.

Find the probability of drawing a red candy on the first draw. Express your answer as a decimal.

$$P(\text{red}_{\text{first}}) = \frac{\square}{\square} = \underline{\hspace{2cm}}$$

Find the probability of drawing a red candy on the second draw.



There are now a total of only ________ candies in the bag.

Assuming the first draw was red, there are ________ red candies still in the bag.

$$P(\text{red}_{\text{second}}) = \frac{\square}{\square} \approx \underline{\hspace{2cm}}$$

To find the probability of a compound event, ________________ the probabilities.

$$P(\text{red}_{\text{first}} \text{ and } \text{red}_{\text{second}}) = P(\text{red}_{\text{first}}) \cdot P(\text{red}_{\text{second}})$$

$$\approx \underline{\hspace{2cm}} \cdot \underline{\hspace{2cm}}$$

$$\approx \underline{\hspace{2cm}}$$

The probability of drawing two red candies from the bag is approximately ________.

This probability problem involves two dependent events. The favorable outcome is drawing a red candy. There are a total of 50 candies in the bag. 24% of the candies are red.

$$0.24 \cdot 50 = 12$$

12 candies are red.

$$P(\text{red}_{\text{first}}) = \frac{12}{50} = 0.24$$

There are now a total of only 49 candies in the bag. Assuming the first draw was red, there are 11 red candies still in the bag.

$$P(\text{red}_{\text{second}}) = \frac{11}{49} \approx 0.2245$$

To find the probability of a compound event, multiply the probabilities.

$$P(\text{red}_{\text{first}} \text{ and } \text{red}_{\text{second}}) = P(\text{red}_{\text{first}}) \cdot P(\text{red}_{\text{second}})$$

$$\approx 0.24 \cdot 0.2245$$

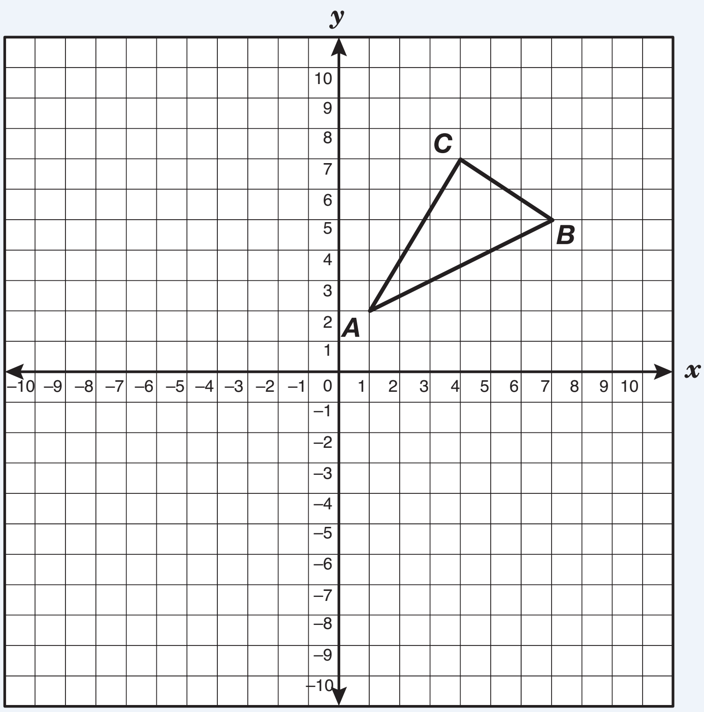
$$\approx 0.054$$

The probability of drawing two red candies from the bag is approximately 0.054.

How Do You Communicate About Mathematics?

It is important to be able to rewrite a problem using mathematical language and symbols. The words in the problem will give clues about the operations that you will need in order to solve the problem. In some problems it may be necessary to use algebraic symbols to represent quantities and then use equations to express the relationships between the quantities. In other problems you may need to represent the given information using a table or graph.

Triangle ABC is graphed on the coordinate grid below.



Find the equation of the line parallel to the segment formed by joining the midpoints of $\overline{AC}$ and $\overline{BC}$ and passing through the point $(-4, -3)$.

Find the midpoints of $\overline{AC}$ and $\overline{BC}$. Use the midpoint formula.

Midpoint of $\overline{AC}$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{1 + 4}{2}, \frac{2 + 7}{2}\right)$$
$$= \left(\frac{5}{2}, \frac{9}{2}\right)$$
$$= (2.5, 4.5)$$

Midpoint of $\overline{BC}$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{7 + 4}{2}, \frac{5 + 7}{2}\right)$$
$$= \left(\frac{11}{2}, \frac{12}{2}\right)$$
$$= (5.5, 6)$$

Find the slope of the line segment connecting the two midpoints. Use the slope formula.

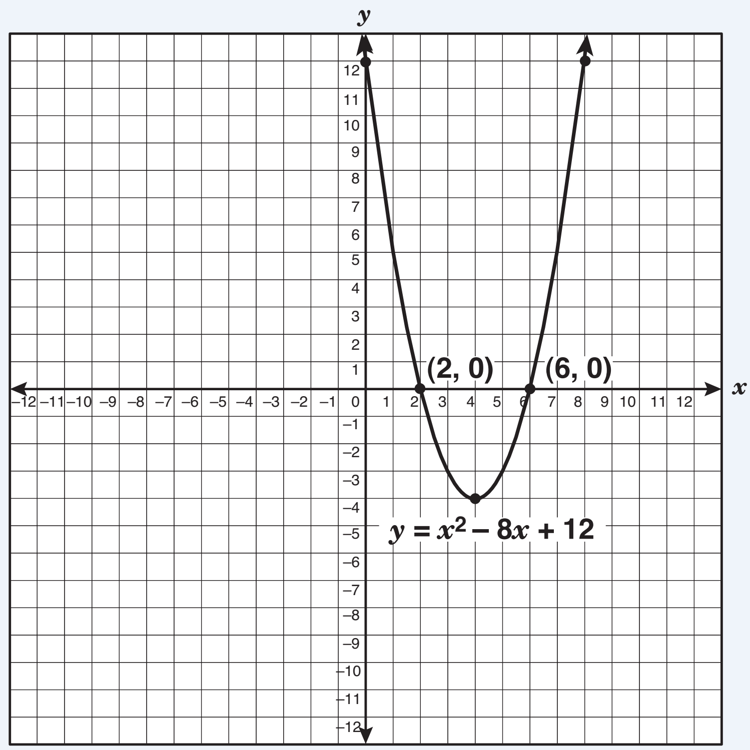
$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 4.5}{5.5 - 2.5} = \frac{1.5}{3} = 0.5$$

Find the equation of the line with a slope, m, of 0.5 passing through the point $(-4, -3)$. You can use the point-slope form of the equation.

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = 0.5(x - (-4))$$
$$y + 3 = 0.5(x + 4)$$
$$y + 3 = 0.5x + 2$$
$$y = 0.5x - 1$$

The equation of the line is $y = 0.5x - 1$, or $y = \frac{1}{2}x - 1$.

By how many units, and in what direction, must the graph of the quadratic function $y = x^2 - 8x + 12$ be translated so that the new equation has only 1 root?

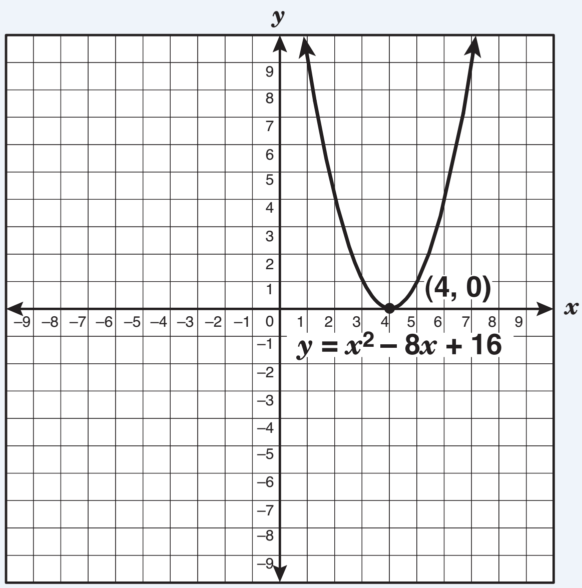


See Objective 5, page 117, for more information about roots of a quadratic function.

If the translated function is to have only one root, then its graph must touch the x-axis at only one point: its minimum value. The graph must be tangent to the x-axis. The graph must be translated up. To determine the number of units up, find the coordinates of the minimum point of the given function.

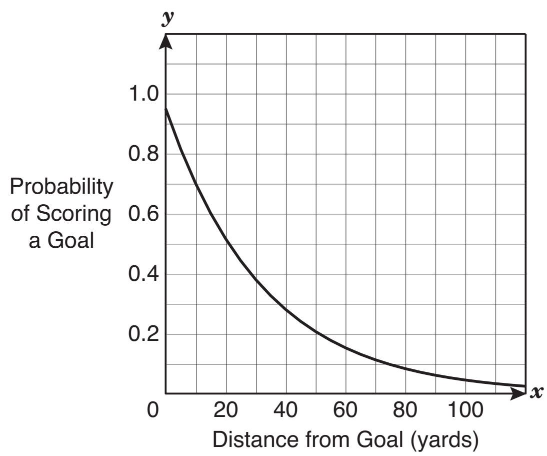
The minimum value occurs midway between the roots $x = 2$ and $x = 6$. Look at the graph. The given function has its minimum value at the point $(4, -4)$. The translated graph must have its minimum value 4 units higher, on the same axis of symmetry, $x = 4$, but at the point $(4, 0)$.

The graph must be translated up 4 units if the function is to have only one root.



Try It

The high school math club determined that the probability of scoring a goal in a soccer game can be approximated by the function drawn below.



Write a statement that compares the probability of scoring a goal from less than 10 yards away to the probability of scoring one from more than 40 yards away.

Look at the graph.

The probability of scoring a goal from 10 yards away is approximately ________. From distances of less than 10 yards, the probability ________________. A player has a better than ________% chance of scoring a goal.

The probability of scoring a goal from 40 yards away is approximately _______. From distances of more than 40 yards, the probability _______________. A player has a less than _______% chance of scoring a goal.

A player's chance of scoring a goal from less than 10 yards away is at least _______________ as good as his or her chance of scoring from more than 40 yards away.

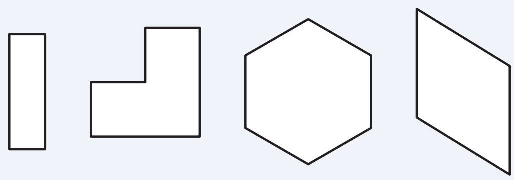
The probability of scoring a goal from 10 yards away is approximately 0.7. From distances of less than 10 yards, the probability increases. A player has a better than 70% chance of scoring a goal. The probability of scoring a goal from 40 yards away is approximately 0.3. From distances of more than 40 yards, the probability decreases. A player has a less than 30% chance of scoring a goal. A player's chance of scoring a goal from less than 10 yards away is at least twice as good as his or her chance of scoring from more than 40 yards away.

How Do You Use Logical Reasoning as a Problem-Solving Tool?

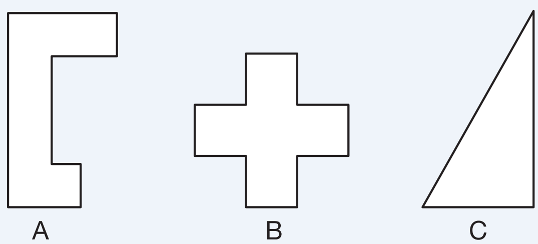
You can use logical reasoning to find patterns in a set of data. You can then use those patterns to draw conclusions about the data.

Finding patterns involves identifying characteristics that objects or numbers have in common. Look for the pattern in different ways. A sequence of geometric objects may have some property in common. For example, they may all be rectangular prisms, or they may all be dilations of the same object.

Look at the following shapes.



Which of the following shapes does not belong to this group?



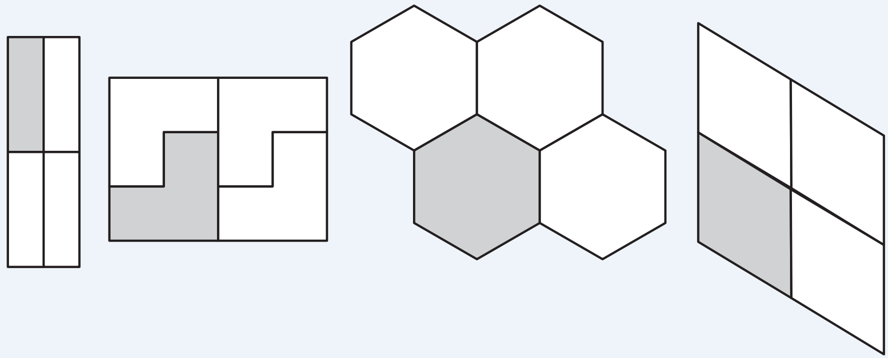
To find out which shape does not belong to the group, identify a geometric property that the first four shapes have in common.

Look at the angles. Two of the shapes appear to have right angles, but two do not. There is no property related to angles that the first four shapes seem to have in common.

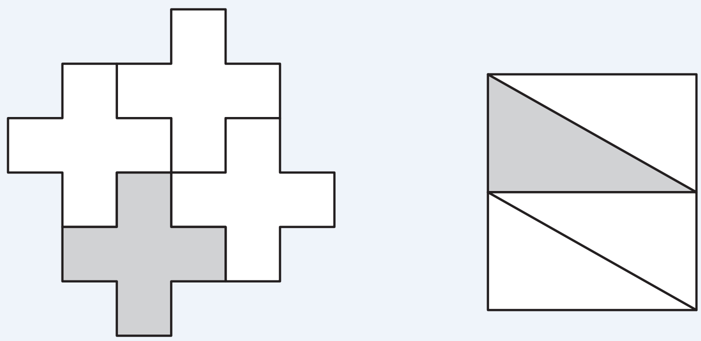
Look at the sides. Two of the shapes have four sides, and two have six sides. There is no property related to sides that the first four shapes have in common.

Look at the shapes. All the shapes are polygons but of different types. Again, there is no property related to shape that the first four have in common.

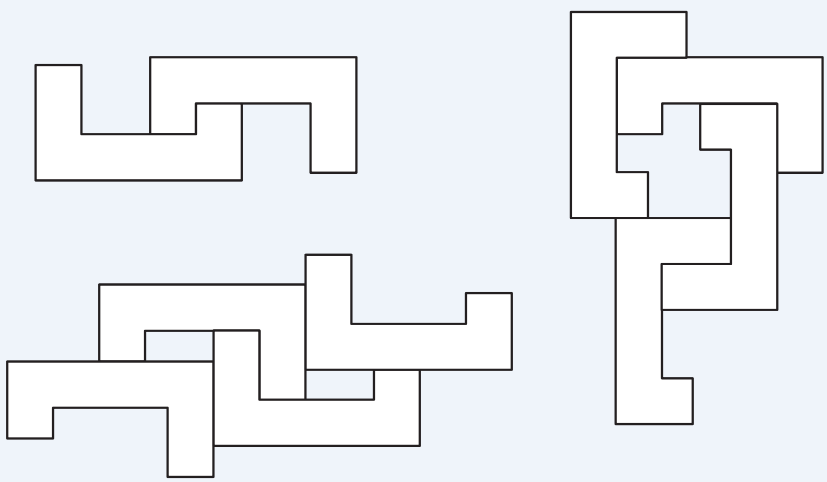
What about their ability to tessellate—to form a repeating pattern that fills a plane completely? Each of the first four shapes can tessellate.



What about the three shapes A, B, and C? Shapes B and C can tessellate.



But shape A does not tessellate.



Shape A does not belong to the group.

Try It

Write the equation that would be next in the following series of equations if the pattern continued.

Equation #1: $y + 4 = 2x + 1$

Equation #2: $y + 1 = 2x$

Equation #3: $2y = 4x + 2$

Equation #4: $y - 2x = 3$

They are all ______________ equations. One way to look for a pattern in a series of linear equations is to write them all in slope-______________ form, $y =$ _______$x +$ _______.

Equation #1	$y + 4 = 2x + 1$ $- \square = - \square$ $y =$ _______ $x -$ _______
Equation #2	$y + 1 = 2x$ $- \square = - \square$ $y =$ _______ $x -$ _______
Equation #3	$2y = 4x + 2$ $\frac{2y}{\square} = \frac{4x}{\square} + \frac{2}{\square}$ $y =$ _______ $x +$ _______
Equation #4	$y - 2x = 3$ $+ \square = + \square$ $y =$ _______ $x +$ _______

Now look for a pattern in the four equations.

Equation #1: $y = 2x -$ ________

Equation #2: $y = 2x -$ ________

Equation #3: $y = 2x +$ ________

Equation #4: $y = 2x +$ ________

The four equations are ______________ lines; all of them have a slope of ________, with their y-intercepts increasing by ________ each time: $-3, -1, +1, +3$.

The next equation in the series should be a line with a slope of ________ and a y-intercept of $3 +$ ________ $=$ ________.

If the pattern continues, the next equation in the series should be $y =$ ________$x +$ ________.

They are all linear equations. One way to look for a pattern in a series of linear equations is to write them all in slope-intercept form, $y = mx + b$.

Equation #1	$\begin{array}{r} y + 4 = 2x + 1 \\ \underline{-4 = \quad -4} \\ y = 2x - 3 \end{array}$
Equation #2	$\begin{array}{r} y + 1 = 2x \\ \underline{-1 = \quad -1} \\ y = 2x - 1 \end{array}$
Equation #3	$2y = 4x + 2$ $\dfrac{2y}{2} = \dfrac{4x}{2} + \dfrac{2}{2}$ $y = 2x + 1$
Equation #4	$\begin{array}{r} y - 2x = 3 \\ \underline{+2x = \quad +2x} \\ y = 2x + 3 \end{array}$

Now look for a pattern in the four equations.

Equation #1: $y = 2x - 3$

Equation #2: $y = 2x - 1$

Equation #3: $y = 2x + 1$

Equation #4: $y = 2x + 3$

The four equations are parallel lines; all of them have a slope of 2, with their y-intercepts increasing by 2 each time: $-3, -1, +1, +3$. The next equation in the series should be a line with a slope of 2 and a y-intercept of $3 + 2 = 5$. If the pattern continues, the next equation in the series should be $y = 2x + 5$.

The solution to a problem can be justified by identifying the mathematical properties or relationships that produced the answer. You should have a reason for drawing a conclusion, and you should be able to explain that reason.

The first angle of a triangle is twice as large as the second angle. The measure of the third angle is 78°.

Can you conclude that the measure of the first angle is 68°?

- It is given that the first angle of a triangle is twice as large as the second angle.

  Let x represent the measure of the second angle. The expression $2x$ would represent the measure of the first angle.

- Write an equation showing that the sum of the measures of the angles of a triangle is 180°.

$$x + 2x + 78 = 180$$
$$3x + 78 = 180$$
$$3x = 102$$
$$x = 34$$

  The measure of the second angle is 34°.

  The measure of the first angle is $34° \cdot 2 = 68°$.

Yes, you can conclude that the measure of the first angle is 68°.

Now practice what you've learned.

Question 98

A CD player is on sale for 30% off the original price. A 6.5% sales tax is added to the discounted price. The final cost of the CD player, including tax, is $178.92. Which equation can be used to determine the original price of the CD player?

A $(x - 0.3x) - 0.065(x - 0.3x) = 178.92$

B $(x - 0.3x) + 0.065x = 178.92$

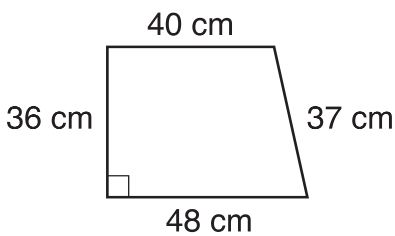
C $x - (0.3x + 0.065x) = 178.92$

D $(x - 0.3x) + 0.065(x - 0.3x) = 178.92$

Answer Key: page 298

Question 99

An automaker manufactures minivans with a side window in the shape of a trapezoid. The window's dimensions to the nearest centimeter are shown below.



What is the minimum number of square meters of glass that will be needed to manufacture 2,500 windows for these minivans?

A 39,600 m^2

B 432 m^2

C 396 m^2

D 43,200 m^2

Answer Key: page 298

Question 100

Erik plans to invest some money for college. He hopes to have between $650 and $700 in the account after 6 years. If the simple interest rate on the account is 4% annually, which of the following initial investments would meet, but not exceed, Erik's goal?

A $500

B $475

C $550

D $575

Answer Key: page 298

Question 101

The spinner used for a game is divided into four unequal areas, each a different color. The table below summarizes the results of 250 spins of the spinner.

Spinner Trials

Red	48
Blue	54
Green	122
White	26

Which statement is supported by the information in the table?

A The theoretical probability of spinning green is less than $\frac{122}{250}$.

B The theoretical probability of spinning green is greater than $\frac{122}{250}$.

C The theoretical probability of spinning green is near $\frac{122}{250}$.

D The theoretical probability of spinning green is unrelated to $\frac{122}{250}$.

Answer Key: page 299

Question 102

Which equation represents the missing step in the solution process?

Step 1: $2(s + 4) - 4 = 10$

Step 2:

Step 3: $2s + 4 = 10$

Step 4: $2s = 6$

Step 5: $s = 3$

A $2s + 8 - 4 = 10$

B $2s + 4 - 4 = 10$

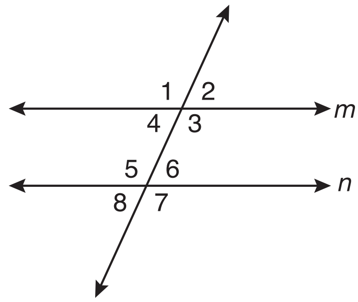
C $4s + 8 - 4 = 10$

D $2s + 24 - 4 = 10$

Answer Key: page 299

Question 103

In the diagram below, lines m and n are parallel.



Which of the following can you correctly conclude from the diagram?

A $\angle 1$ and $\angle 2$ are complementary because their sum is 90°.

B $\angle 1$ is congruent to $\angle 7$ because corresponding and vertical angles are congruent.

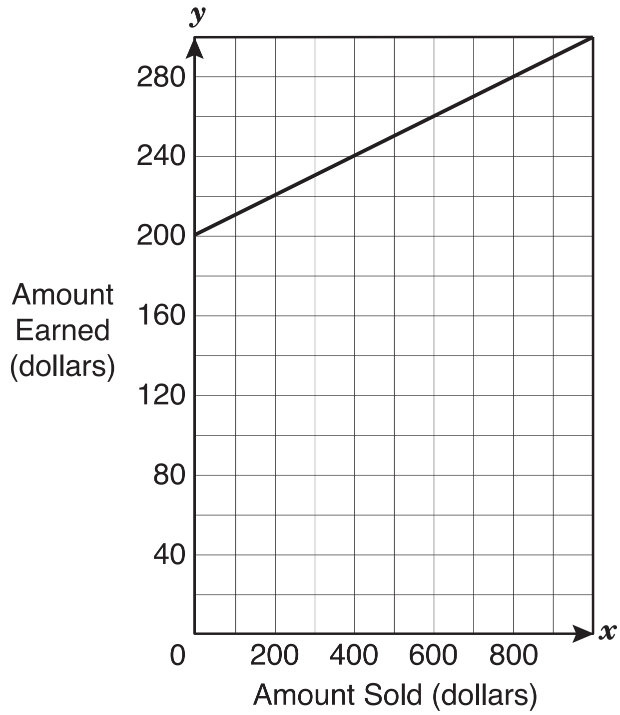
C $\angle 2$ and $\angle 8$ are supplementary because their sum is 180°.

D $\angle 4$ is congruent to $\angle 5$ because corresponding angles are congruent.

Answer Key: page 299

Question 104

Gina works at a clothing store where she earns a weekly salary plus a percent of her total weekly sales as a commission. The graph below shows the amount, y, that Gina will earn in one week if she sells x dollars' worth of clothing.



The table below shows Gina's sales for four weeks.

Gina's Clothing Sales

Week 1	$750
Week 2	$600
Week 3	$900
Week 4	$850

Which number best reflects the total amount of money she earned in sales commission for this four-week period?

A $310

B $3100

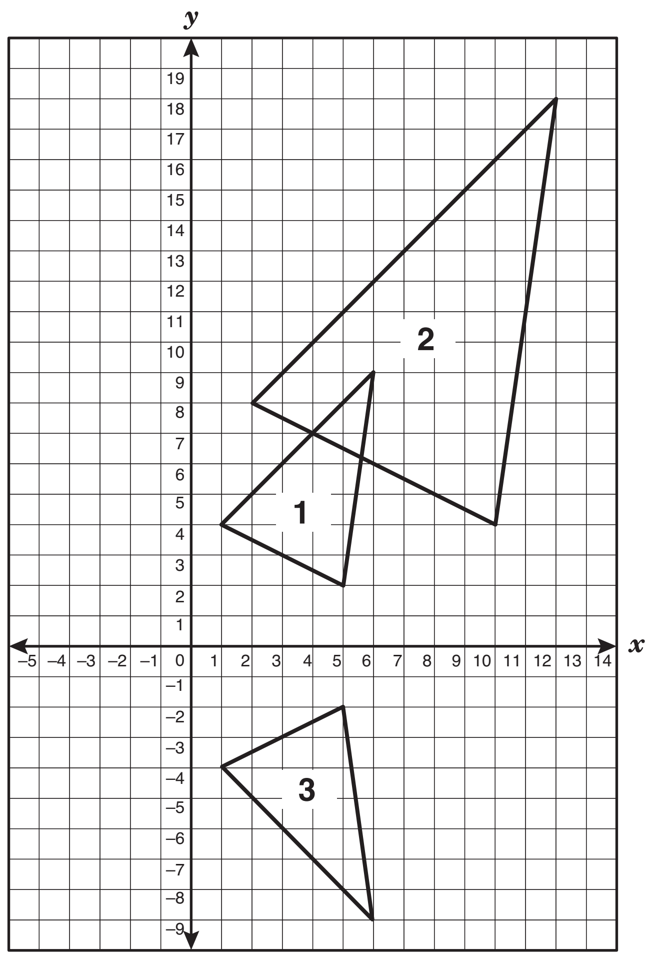
C $800

D $1110

Answer Key: page 299

Question 105

Three triangles are graphed below.



Which of the following triangles does not belong to this group?

A Triangle W: (−1, 4), (−5, 2), (−6, 9)

B Triangle X: (1, 2), (5, 0), (6, 7)

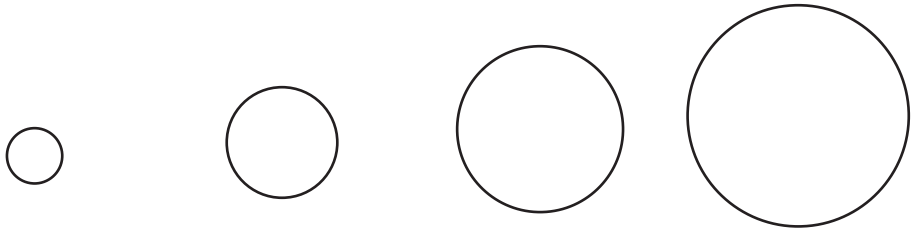
C Triangle Y: (4, −1), (2, −5), (12, −5)

D Triangle Z: (−2, 4), (2, 2), (3, 9)

Answer Key: page 299

Question 106

Look at the series of circles below with their areas shown.



$A = 3.14\text{ cm}^2$ $A = 12.57\text{ cm}^2$ $A = 28.27\text{ cm}^2$ $A = 50.27\text{ cm}^2$

If the pattern continues, what would be the approximate circumference of the sixth circle in this series?

A 18.85 centimeters

B 37.70 centimeters

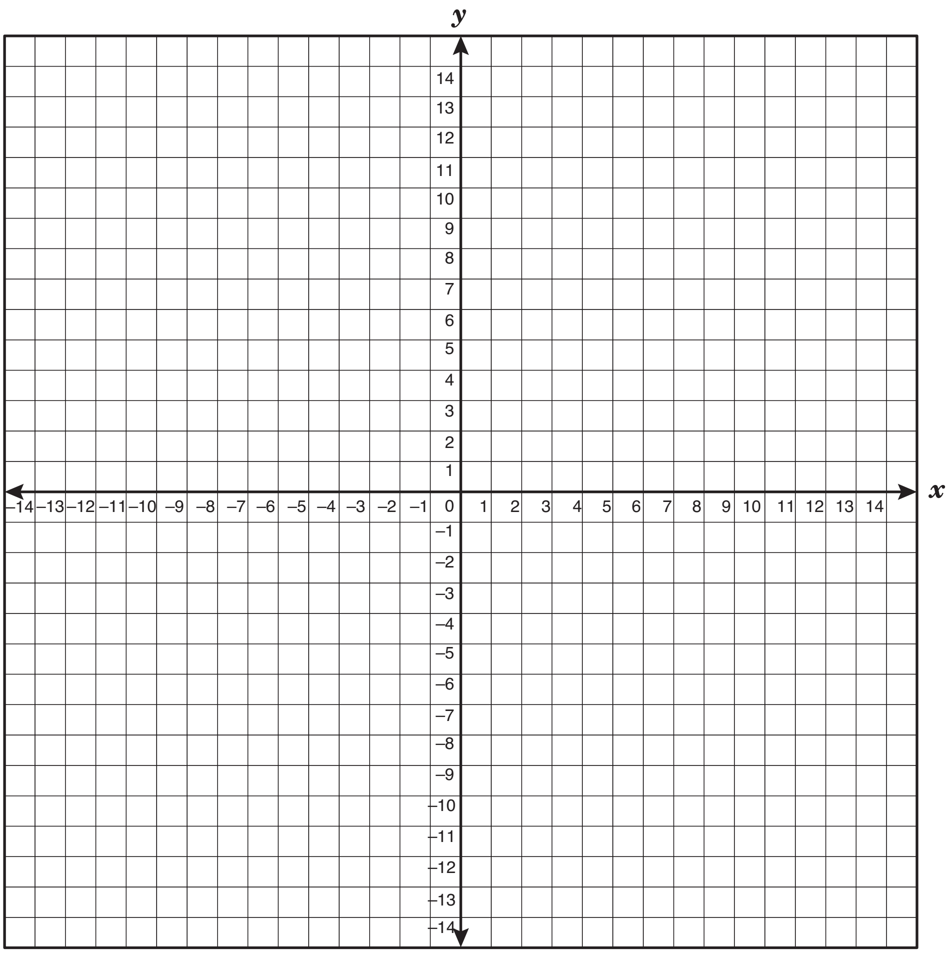
C 31.42 centimeters

D 25.13 centimeters

Answer Key: page 300

Question 107

A triangle has coordinates A (3, 7), B (12, −5), and C (2, 0). The altitude drawn to side AB intersects side AB at point D (6, 3). If triangle ABC is dilated by a scale factor of 5, what is the area of the enlarged triangle?



A 73.5 square units

B 37.5 square units

C 937.5 square units

D 187.5 square units

Answer Key: page 300

Mathematics Answer Key

Objective 1

Question 1 (page 30)

A Incorrect. The number of shirts sold is the independent quantity.

B Incorrect. The amount the team paid for the shirts is a constant; it does not change.

C Correct. The dependent quantity in a functional relationship is the quantity whose value depends on another value. The team's profit depends on the number of shirts sold; the profit is the dependent quantity.

D Incorrect. The selling price of the shirts is a constant; it does not change.

Question 2 (page 30)

A Correct. The cost of the popcorn and soda is \$12. The cost of the DVDs is \$4 times the number of DVDs rented. The total cost of the DVDs is represented by $4n$. The amount spent on popcorn, soda, and DVDs must be less than or equal to \$35. The inequality that expresses this relationship is $4n + 12 \leq 35$.

Question 3 (page 30)

B Correct. In a functional relationship no x-value repeats in a list of the ordered pairs. The ordered pairs in this table all have different x-values. The ordered pairs in Table B represent a functional relationship.

Question 4 (page 31)

C Correct. The total cost of the item is a function of the price of the item. The price of the item is represented by x. Calculate the sales tax by multiplying the cost of the item by the tax rate, $6.5\% = 0.065$. The sales tax is represented by $0.065x$. To find the total cost of the item, add the sales tax to the price of the item.

$$x + 0.065x = 1x + 0.065x = 1.065x$$

The total cost of the item is modeled by the function $c = 1.065x$.

Question 5 (page 31)

B Correct. Look for a pattern in the ordered pairs.

$$s = 50, f = 0.5$$
$$s = 100, f = 1$$

The value of f in both cases is the value of s divided by 100. This is the rule in answer choice B. Check to see whether this rule works for the remaining ordered pairs.

$\underline{s = 200}$	$\underline{s = 250}$	$\underline{s = 300}$
$f = \frac{s}{100}$	$f = \frac{s}{100}$	$f = \frac{s}{100}$
$f = \frac{200}{100}$	$f = \frac{250}{100}$	$f = \frac{300}{100}$
$f = 2$	$f = 2.5$	$f = 3$

Only the rule in answer choice B matches the ordered pairs in the table.

Question 6 (page 31)

A Incorrect. Use the function to test the data points. Test (2, 4). Substitute $x = 2$ into the function $f(x) = 4 - 2x$ and check that $f(2) = 4$.

$$f(x) = 4 - 2x$$
$$f(2) = 4 - 2(2)$$
$$f(2) = 4 - 4$$
$$f(2) = 0$$

For the function $f(x) = 4 - 2x$, if $x = 2$, then $y = 0$. The ordered pair (2, 0) belongs to the function. The point (2, 4) is not a point on the graph of the function. Table A is not correct.

B Incorrect. Use the function to test the data points. Test $(-3, -2)$. Substitute $x = -3$ into the function $f(x) = 4 - 2x$ and check that $f(-3) = -2$.

$$f(x) = 4 - 2x$$
$$f(-3) = 4 - 2(-3)$$
$$f(-3) = 4 + 6$$
$$f(-3) = 10$$

For the function $f(x) = 4 - 2x$, if $x = -3$, then $y = 10$. The ordered pair $(-3, 10)$ belongs to the function. The point $(-3, -2)$ is not a point on the graph of the function. Table B is not correct.

C Incorrect. Use the function to test the data points. Test $(3, -1)$. Substitute $x = 3$ into the function $f(x) = 4 - 2x$ and check that $f(3) = -1$.

$$f(x) = 4 - 2x$$
$$f(3) = 4 - 2(3)$$
$$f(3) = 4 - 6$$
$$f(3) = -2$$

For the function $f(x) = 4 - 2x$, if $x = 3$, then $y = -2$. The ordered pair $(3, -2)$ belongs to the function. The point $(3, -1)$ is not a point on the graph of the function. Table C is not correct.

D Correct. Use the function to test the data points. Test $(-2, 8)$. Substitute $x = -2$ into the function $f(x) = 4 - 2x$ and check that $f(-2) = 8$.

$$f(x) = 4 - 2x$$
$$f(-2) = 4 - 2(-2)$$
$$f(-2) = 4 + 4$$
$$f(-2) = 8$$

For the function $f(x) = 4 - 2x$, if $x = -2$, then $y = 8$. The ordered pair $(-2, 8)$ belongs to the function. The point $(-2, 8)$ is a point on the graph of the function.

When you check the remaining points, they all satisfy the function rule. When you substitute $x = 0$ into the function $f(x) = 4 - 2x$, you find that $f(0) = 4$. The point $(0, 4)$ is a point on the graph of the function. When you substitute $x = 2$ into the function $f(x) = 4 - 2x$, you find that $f(2) = 0$. The point $(2, 0)$ is a point on the graph of the function. When you substitute $x = 4$ into the function $f(x) = 4 - 2x$, you find that $f(4) = -4$. The point $(4, -4)$ is a point on the graph of the function. All the points in Table D are points in the function $f(x) = 4 - 2x$.

Question 7 (page 32)

C Correct.

To determine which function is represented by the graph, find several points that satisfy the equation and verify that they are on the graph. For example, substitute $x = 0$ into the equation $y = 3x^2 + 1$.

$$y = 3(0)^2 + 1$$
$$y = 1$$

The point $(0, 1)$ should be on the graph of $y = 3x^2 + 1$. The point $(0, 1)$ is on only Graphs B and C.

Substitute $x = 1$ into the equation.

$$y = 3(1)^2 + 1$$
$$y = 4$$

The point $(1, 4)$ should also be on the graph of $y = 3x^2 + 1$. Only Graph C also contains the point $(1, 4)$.

Finally, choose a point on the graph other than $(0, 3)$ or $(1, 4)$ and verify that its coordinates satisfy the equation. The point $(-1, 4)$ is on the graph. Substitute $x = -1$ and $y = 4$ into the equation.

$$4 \stackrel{?}{=} (3)(-1)^2 + 1$$
$$4 \stackrel{?}{=} (3)(1) + 1$$
$$4 \stackrel{?}{=} 3 + 1$$
$$4 = 4$$

The point $(-1, 4)$ satisfies the equation. Only Graph C represents the quadratic function $y = 3x^2 + 1$.

Question 8 (page 33)

D Correct. The inequality represents all pairs of numbers (x, y) such that $\frac{3}{2}x + y < 9$. Since the inequality does not include pairs that make the expression equal to 9, the graph should not include the line $\frac{3}{2}x + y = 9$. This line that forms the boundary of the shaded region should be dashed, not solid. Only graphs C and D have a dashed line.

Choose a point inside the shaded region in graph D and verify that its coordinates satisfy the inequality. For example, the point $(0, 8)$ is inside the shaded region. Substitute $x = 0$ and $y = 8$ into the inequality.

$$\frac{3}{2}x + y < 9$$
$$\frac{3}{2}(0) + 8 < 9$$
$$8 < 9$$

The point $(0, 8)$ satisfies the inequality. Since the point $(0, 8)$ satisfies the inequality, all the points on that side of the line will also satisfy the inequality.

Question 9 (page 34)

B Correct.

Use the function's rule to determine which ordered pairs belong to it. Evaluate the function $f(x) = x^2 + 5$ for the independent values in the table.

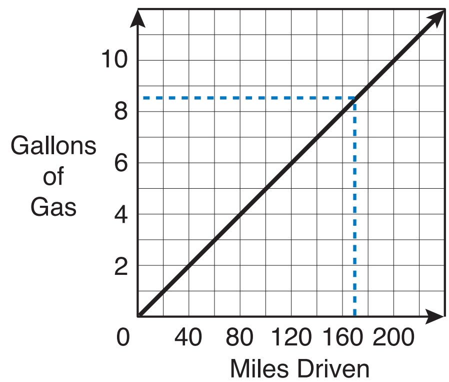
$\underline{x = 0}$	$\underline{x = 1}$
$f(0) = (0)^2 + 5$	$f(1) = 1^2 + 5$
$f(0) = 0 + 5$	$f(1) = 1 + 5$
$f(0) = 5$	$f(1) = 6$
$(0, 5)$ belongs to $f(x)$	$(1, 6)$ belongs to $f(x)$

$\underline{x = 2}$	$\underline{x = 3}$
$f(2) = (2)^2 + 5$	$f(3) = (3)^2 + 5$
$f(2) = 4 + 5$	$f(3) = 9 + 5$
$f(2) = 9$	$f(3) = 14$
$(2, 9)$ belongs to $f(x)$	$(3, 14)$ belongs to $f(x)$

Only the rule in answer choice B produces the correct ordered pairs.

Question 10 (page 34)

B Correct. To find how many gallons of gas Maria will need for a 170-mile trip, locate 170 on the horizontal axis and find the corresponding value on the vertical axis. Draw a vertical line extending from 170 and intersecting the graph. Now draw a horizontal line extending from this point to the vertical axis. The line crosses the axis between 8 and 9. Maria's car will use about 8.5 gallons of gas on a 170-mile trip.



Question 11 (page 35)

B Correct. The first part of the graph shows the height of water in the bucket increasing from a height of 0 at a constant rate. Then the height of the water remains the same while Paul carries the bucket. This is represented by the first horizontal portion of the graph. The height of the water decreases quickly when Paul trips and spills it. This is represented by the nearly vertical decrease in the graph. The graph's value at this point is halfway between 0 and the highest point on the graph, since Paul spilled half the water.

The second horizontal section of the graph reflects Paul returning to the faucet. The height increases at a constant rate again as he refills the bucket.

Objective 2

Question 12 (page 63)

C Correct. The graph is a parabola. If a graph is a parabola, then it is a quadratic function. The parent function is the quadratic function $y = x^2$.

Question 13 (page 63)

C Correct. The domain of this function is represented by x, the number of tickets Sam purchases. In the context of this problem, he is buying at least 1 ticket but is not allowed to buy more than 6 tickets. Furthermore, it makes no sense in this context to buy half a ticket, so the domain must be whole numbers from 1 to 6.

Question 14 (page 63)

D Correct. The vertical axis represents weekly sales, and the horizontal axis represents weeks since the store opened. For the first 5 weeks, for increasing values of x, the graph gets higher on the vertical axis. This means the sales increased for the first 5 weeks. For the next week the graph is horizontal. This means the sales were constant for that week. For the last week shown, the graph goes down. The sales decreased for the last week.

Question 15 (page 64)

C Correct. As the age of a car increases, its value decreases. This means there is a negative correlation between the age of a car and its value. Graph C shows a negative correlation. For increasing values of x, the points get lower on the vertical axis.

Question 16 (page 65)

A Correct. The average tuition increased by \$125 between 1980 and 1985, \$95 between 1985 and 1990, \$125 between 1990 and 1995, \$95 between 1995 and 2000, and \$125 between 2000 and 2005.

If this pattern continues, the next increase will be \$95.

$$\$950 + \$95 = \$1045$$

Question 17 (page 65)

D Correct. Represent the number of pounds of almonds Nancy bought with x. She bought 5 more pounds of pecans than almonds, so represent the number of pounds of pecans with the expression $x + 5$.

The cost of the almonds is equal to the price per pound, \$5.50, times the number of pounds she bought, x.

$$\text{Cost of almonds} = 5.5x$$

The cost of the pecans is equal to the price per pound, \$4.25, times the number of pounds she bought, $x + 5$.

$$\text{Cost of pecans} = 4.25(x + 5)$$

The total cost of the nuts she bought is equal to the sum of these two expressions.

$$5.5x + 4.25(x + 5) =$$
$$5.5x + 4.25x + 4.25 \bullet 5 =$$
$$9.75x + 21.25$$

Question 18 (page 65)

C Correct. To verify that the equation $c = 4n + 16$ works for each of the data points, substitute each value of n into the equation and verify that it gives the correct value for c.

Number (n)	$c = 4n + 16$	Value
1	$4(1) + 16 = 4 + 16 = 20$	20
2	$4(2) + 16 = 8 + 16 = 24$	24
3	$4(3) + 16 = 12 + 16 = 28$	28
4	$4(4) + 16 = 16 + 16 = 32$	32

The rule $c = 4n + 16$ best represents the relationship between n, the number of people, and c, the cost.

Question 19 (page 65)

B Correct. One way to find the relationship between the terms in a sequence and their position in the sequence is to build a table. Test the values in the table against the rule in choice B.

Sequence 0, 6, 16, 30, 48, …

Position (n)	$2n^2 - 2$	Value
1	$2(1)^2 - 2 = 2(1) - 2 = 2 - 2 = 0$	0
2	$2(2)^2 - 2 = 2(4) - 2 = 8 - 2 = 6$	6
3	$2(3)^2 - 2 = 2(9) - 2 = 18 - 2 = 16$	16
4	$2(4)^2 - 2 = 2(16) - 2 = 32 - 2 = 30$	30
5	$2(5)^2 - 2 = 2(25) - 2 = 50 - 2 = 48$	48

The rule $2n^2 - 2$ correctly represents each value in the sequence.

Question 20 (page 66)

C Correct. Let x represent Mark's age. Represent Barbara's age with $3x$. If the sum of their ages is 52, then $x + 3x = 52$. Simplify by combining like terms: $4x = 52$. The equation $4x = 52$ can be used to find their ages.

Question 21 (page 66)

A Correct. First multiply $n - 3$ by $n + 2$.

$$(n - 3)(n + 2) =$$
$$n^2 + 2n - 3n - 6 =$$
$$n^2 - n - 6$$

Then multiply by 4.

$$4(n^2 - n - 6) = 4n^2 - 4n - 24$$

Objective 3

Question 22 (page 96)

A Incorrect. The amount spent on n shirts that cost \$20 each can be represented by the function $s = 20n$, where s represents the amount spent. This is a linear function.

B Incorrect. The number of miles driven for h hours at a constant speed of 60 miles per hour can be represented by the function $m = 60h$, where m represents the number of miles driven. This is a linear function.

C Incorrect. The total amount saved after depositing \$100, and then \$30 a month for n months thereafter, can be represented by the function $s = 100 + 30n$, where s represents the total amount saved. This is a linear function.

D Correct. The area of a rectangular garden that is x feet wide and has a length equal to twice its width can be represented by the function $y = x(2x) = 2x^2$, where y represents the area. This function contains a term, $2x^2$, in which the variable is squared. The function is not linear.

Question 23 (page 96)

D Correct. To confirm that the linear equation $y = -\frac{1}{2}x + \frac{3}{2}$ represents the same linear function as the data in the table, substitute at least two ordered pairs from the table into the equation.

You could choose the ordered pairs (3, 0) and (1, 1).

x	Substituted into the equation $y = -\frac{1}{2}x + \frac{3}{2}$	y	Yes/No
3	$-\frac{1}{2}(3) + \frac{3}{2} =$ $-\frac{3}{2} + \frac{3}{2} =$ 0	0	Yes
1	$-\frac{1}{2}(1) + \frac{3}{2} =$ $-\frac{1}{2} + \frac{3}{2} =$ $\frac{2}{2} =$ 1	1	Yes

The two representations are equivalent.

Question 24 (page 96)

B Correct. The rate of change is the slope of the line graphed. Use any two points on the graph to find the slope. You could choose the points (0, −4) and (2, −1).

Slope $(m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - (-4)}{2 - 0} = \frac{-1 + 4}{2} = \frac{3}{2}$

Question 25 (page 97)

C Correct. The y-intercept of the graph is (0, 60). This represents the number of tons of food in the storage bin immediately after the last purchase of food was placed in the bin.

Question 26 (page 97)

B Correct.

Since it costs \$25 for each case of balloons, and \$125 for each case of streamers, the equation is $25x + 125y = 500$. To find the x-intercept, substitute zero in for y, since for any x-intercept the y-value is zero. $25x + 125(0) = 500$ gives $25x = 500$, and dividing both sides by 25 gives $x = 20$. The x-intercept is (20, 0).

Question 27 (page 97)

C Correct. Parallel lines must have equal slopes. When a linear equation is written in the form $y = mx + b$, the value of m is equal to the slope of the graph. The equations $y = x + 1$ and $y = x - 25$ both have a slope value of $m = 1$; therefore, both have a slope of 1. If two equations have the same slope, then the graphs of these equations are parallel lines.

Question 28 (page 97)

B Correct. The slope of the new line is the negative reciprocal of the slope of the original line; the product of the slopes is −1.

$$5 \cdot -\frac{1}{5} = -1$$

Perpendicular lines have slopes that are negative reciprocals of each other. The y-intercept is not changed. Both lines have a y-intercept of −2 and will cross the y-axis at the point (0, −2).

Question 29 (page 98)

A Correct. Use the coordinates of the given points to calculate m, the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-5)}{2 - (-1)} = \frac{1 + 5}{2 + 1} = \frac{6}{3} = 2$$

Substitute the value of m and the values of x and y from one of the given points into the slope-intercept form of a linear equation, $y = mx + b$. Solve for the value of b.

Use (2, 1).

$$\begin{aligned} y &= mx + b \\ 1 &= 2(2) + b \\ 1 &= 4 + b \\ -3 &= b \end{aligned}$$

Substitute the values of m and b into the slope-intercept form of the equation.

$$\begin{aligned} y &= mx + b \\ y &= 2x - 3 \end{aligned}$$

Question 30 (page 98)

C Correct. The slope of the graph is the rate of change of the equation. The rate of change in the original equation is 2000 gallons per hour. The rate of change in the second equation is twice as fast, or $2 \cdot 2000 = 4000$ gallons per hour. The slope of the line increases from 2000 to 4000; this makes the line steeper.

Question 31 (page 98)

D Correct. Let c represent the cost of the jelly beans and n represent the number of pounds purchased. Write a direct variation equation of the form $c = kn$.

Substitute $c = 12$ and $n = 5$ into the equation to find the value of k.

$$\begin{aligned} 12 &= k(5) \\ k &= 2.4 \end{aligned}$$

Jelly beans cost \$2.40 per pound. The direct variation equation is $c = 2.4n$.

Substitute $c = 7200$ into the equation and solve for n.

$$7200 = 2.4n$$
$$3000 = n$$

So 3000 pounds of jelly beans were purchased during the past month.

Objective 4

Question 32 (page 112)

B Correct. Represent the number of calories from d cups of dry food. The number of calories is equal to 200 times the number of cups of dry food.

$$200d$$

Represent the number of calories from b biscuits. The number of calories is 400 times the number of biscuits.

$$400b$$

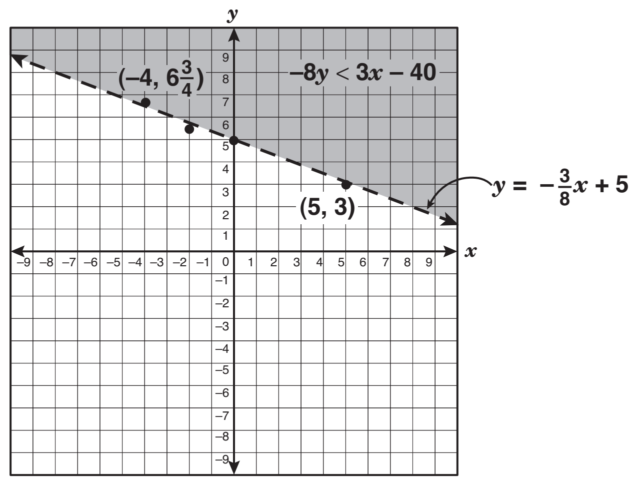
The total number of calories from dry food plus biscuits must be less than or equal to 1000.

Use the inequality $200d + 400b \leq 1000$ to represent the amount of dry food and the number of biscuits Fido is allowed to eat each day.

Question 33 (page 112)

A Correct. This can be solved by substituting each of the ordered pairs to see which one satisfies the inequality, but the graphical solution is shown here.

One way to graph this inequality is to put it in slope-intercept form, that is, solve for y. The first step is to divide by -8, but when dividing by a negative number, remember to reverse the inequality. So $-8y < 3x - 40$ then becomes $y > -\frac{3}{8}x + 5$. Therefore m is $-\frac{3}{8}$, and b is 5. Since the inequality is strictly "greater than" (as opposed to "greater than or equal to"), draw $y = -\frac{3}{8}x + 5$ with a dashed line. This indicates that any points on $y = -\frac{3}{8}x + 5$ are not solutions for $y > -\frac{3}{8}x + 5$. Finally, since the inequality is "greater than," shade above the line.



As shown in the graph above, $(-4, 6\frac{3}{4})$ is the only answer choice in the solution for $-8y < 3x - 40$.

Question 34 (page 113)

D Correct. Represent the charge for a small room with s and the charge for a large room with l.

Write and solve an equation that shows that the total amount of money collected from 20 large rooms is \$1800.

$$20l = 1800$$
$$\frac{20l}{20} = \frac{1800}{20}$$
$$l = 90$$

Write and solve an equation that shows when 10 large rooms and 21 small rooms are occupied, the hotel collects \$2370.

$$\begin{aligned} 10l + 21s &= 2370 \\ 10(90) + 21s &= 2370 \\ 900 + 21s &= 2370 \\ -900 \quad\quad &= -900 \\ \hline 21s &= 1470 \\ \frac{21s}{21} &= \frac{1470}{21} \\ s &= 70 \end{aligned}$$

The hotel charges \$70 per small room.

Question 35 (page 113)

A Correct. Paperback books cost \$6 each. The expression $6p$ represents the cost of ordering p paperback books. If Jamal orders one hardback, the total cost will be $6p + 12(1)$, or $6p + 12$.

Jamal wants to spend no more than \$72. The inequality $6p + 12 \leq 72$ represents the amount he can spend.

Solve the inequality for p to determine the maximum number of paperbacks he can order.

$$\begin{aligned} 6p + 12 &\leq 72 \\ 6p &\leq 60 \\ p &\leq 10 \end{aligned}$$

Jamal can order no more than 10 paperback books if he is to spend no more than \$72. The least number of paperbacks Jamal can order is 0. Jamal can order from 0 to 10 paperbacks. So the inequality $0 \leq p \leq 10$ represents the number of paperback books Jamal can order.

Question 36 (page 113)

A Incorrect. Substitute $x = 11$ and $y = 12$ into the inequality.

$$\begin{aligned} 3x + 2y &\geq 60 \\ 3(11) + 2(12) &\overset{?}{\geq} 60 \\ 33 + 24 &\overset{?}{\geq} 60 \\ 57 &\geq 60 \quad \text{False.} \end{aligned}$$

These values of x and y do not satisfy the inequality.

B Correct. Substitute $x = 12$ and $y = 14$ into the inequality.

$$\begin{aligned} 3x + 2y &\geq 60 \\ 3(12) + 2(14) &\overset{?}{\geq} 60 \\ 36 + 28 &\overset{?}{\geq} 60 \\ 64 &\geq 60 \quad \text{True.} \end{aligned}$$

These values of x and y satisfy the inequality.

C Incorrect. Substitute $x = 14$ and $y = 8$ into the inequality.

$$\begin{aligned} 3x + 2y &\geq 60 \\ 3(14) + 2(8) &\overset{?}{\geq} 60 \\ 42 + 16 &\overset{?}{\geq} 60 \\ 58 &\geq 60 \quad \text{False.} \end{aligned}$$

These values of x and y do not satisfy the inequality.

D Incorrect. Substitute $x = 10$ and $y = 14$ into the inequality.

$$\begin{aligned} 3x + 2y &\geq 60 \\ 3(10) + 2(14) &\overset{?}{\geq} 60 \\ 30 + 28 &\overset{?}{\geq} 60 \\ 58 &\geq 60 \quad \text{False.} \end{aligned}$$

These values of x and y do not satisfy the inequality.

Question 37 (page 113)

A Correct. Write an equation that shows that the number of ounces of chocolate chips plus the number of ounces of peanut butter chips is 16.

$$c + p = 16$$

There are 8 chocolate chips per ounce and 12 peanut butter chips per ounce.

The expression $8c$ represents the number of chocolate chips in c ounces.

The expression $12p$ represents the number of peanut butter chips in p ounces.

Write an equation that shows there are a total of 300 chips.

$$8c + 12p = 300$$

The system of linear equations

$$\begin{aligned} c + p &= 16 \\ 8c + 12p &= 300 \end{aligned}$$

could be used to find the number of ounces of both chocolate chips and peanut butter chips Andrea used to make the cookies.

Question 38 (page 114)

C Correct. Represent the width of the garden with w and the length of the garden with l. Julie is using 36 feet of fencing around the edge of the garden, so the garden has a perimeter of 36 feet. Use the formula for perimeter.

$$2l + 2w = 36$$

The length of the garden is 3 feet more than 1.5 times the width. Write the length in terms of the width.

$$l = 1.5w + 3$$

Solve the system of equations.

$$\begin{aligned} 2l + 2w &= 36 \\ l &= 1.5w + 3 \end{aligned}$$

Use the substitution method. Substitute $1.5w + 3$ for l in the first equation and simplify.

$$\begin{aligned} 2(1.5w + 3) + 2w &= 36 \\ 3w + 6 + 2w &= 36 \\ 5w + 6 &= 36 \\ 5w &= 30 \\ w &= 6 \end{aligned}$$

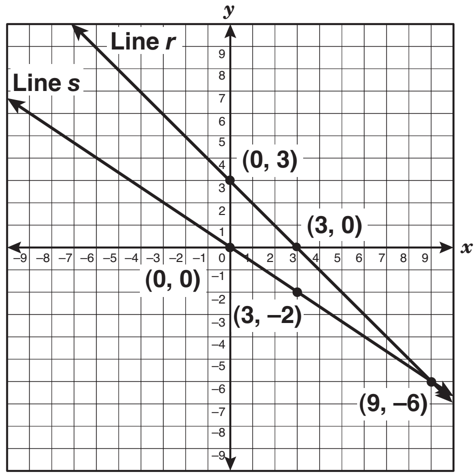
The width of the garden is 6 feet. Substitute $w = 6$ into the second equation to find l.

$$\begin{aligned} l &= 1.5w + 3 \\ l &= 1.5 \cdot 6 + 3 \\ l &= 9 + 3 \\ l &= 12 \end{aligned}$$

The length of the garden is 12 feet.

Question 39 (page 114)

D Correct. One way to solve this system is graphically. Draw line r by plotting (3, 0) and (0, 3) and drawing the line intersecting those points. Draw line s by plotting a point at the origin and using the slope to find another point. Since the slope for s is $-\frac{2}{3}$, start at the origin, go down two units, then right three units, and plot a point.



The point (9, −6) is the solution to this system of linear equations.

Question 40 (page 115)

C Correct. Write each equation in slope-intercept form.

$$\begin{aligned} 2x - 4y &= 6 \\ -4y &= -2x + 6 \\ y &= \tfrac{1}{2}x - \tfrac{3}{2} \end{aligned} \qquad \begin{aligned} 6x &= 12y + 18 \\ -12y &= -6x + 18 \\ y &= \tfrac{1}{2}x - \tfrac{3}{2} \end{aligned}$$

Both equations have a slope of $\frac{1}{2}$ and a y-intercept of $-\frac{3}{2}$. Since the slope and y-intercept are the same for both lines, the graphs of both equations will be the same line. The solution set of the system is a line; there are infinitely many solutions.

Question 41 (page 115)

D Correct. Write both equations in slope-intercept form.

$$\begin{aligned} -3x + y &= 2 \\ y &= 3x + 2 \end{aligned} \qquad \begin{aligned} 6x - 2y &= 2 \\ -2y &= -6x + 2 \\ y &= 3x - 1 \end{aligned}$$

Both equations have a slope of 3. The y-intercept of the first equation is 2, and the y-intercept of the second equation is −1.

Since the slopes are the same and the y-intercepts are different, the graphs of the equations will be parallel lines, which will not intersect. The system has no solution.

Objective 5

Question 42 (page 138)

D Correct. The graph of $y = ax^2$ is always symmetrical about the y-axis. The inequality $a < 0$ means that a is negative. Therefore, the graph opens downward, which indicates that the graph is in Quadrants III and IV.

Question 43 (page 138)

C Correct. In the given equation, $y = x^2 - 3$, $c = -3$. The vertex of Graph B is 7 units higher than the vertex of the graph of the given equation. Add 7 to the value of c to find the value of c for Equation B.

$$c = -3 + 7 = 4$$

In the equation of Graph B, $c = 4$. Graph B opens in the same direction and is the same size as the given graph. Therefore, a, the coefficient of x^2, should be the same as it is in the given equation. The equation of Graph B is $y = x^2 + 4$.

Question 44 (page 138)

A Incorrect. The absolute value of the coefficient of x^2 in Equation A is 2. The absolute value of the coefficient of x^2 in the given equation is 2. When the absolute values of the coefficients of x^2 are equal, the graphs are congruent.

B Correct. The absolute value of the coefficient of x^2 in Equation B is 1. The absolute value of the coefficient of x^2 in the given equation is 2. When the size of the coefficient is decreased, the graph becomes wider. The graph of $y = x^2 + 1$ is not congruent to the graph of $y = 2x^2 + 1$.

C Incorrect. The absolute value of the coefficient of x^2 in Equation C is 2. The absolute value of the coefficient of x^2 in the given equation is 2. When the absolute values of the coefficients of x^2 are equal, the graphs are congruent.

D Incorrect. The absolute value of the coefficient of x^2 in Equation D is 2. The absolute value of the coefficient of x^2 in the given equation is 2. When the absolute values of the coefficients of x^2 are equal, the graphs are congruent.

Question 45 (page 138)

D Correct.
The vertical axis represents the height, and the horizontal axis represents the number of seconds since the ball was projected. Find the point on the parabola that has a value of $h = 56$. Find the corresponding value of t on the horizontal axis: $t = 3.5$.

Question 46 (page 139)

B Correct. The x-axis represents the selling price of the cakes, and the y-axis represents the profits earned. The lowest value of y shown on the graph is approximately \$165. This value corresponds to the selling prices of \$2.00 and \$9.00.

The maximum value of y is at the vertex of the parabola (5.5, 275). The maximum profit is \$275, which corresponds to a selling price of \$5.50.

Question 47 (page 139)

A Correct. Set the equation equal to zero by adding $-x$ to both sides.

$$\begin{aligned} x &= x^2 - 42 \\ -x &= \quad\ -x \\ \hline 0 &= x^2 - x - 42 \end{aligned}$$

The expression $x^2 - x - 42$ can be factored. Solve the equation by factoring.

$$\begin{aligned} x^2 - x - 42 &= 0 \\ (x - 7)(x + 6) &= 0 \end{aligned}$$

Set each factor equal to 0.

$$\begin{aligned} x - 7 &= 0 \qquad & x + 6 &= 0 \\ x &= 7 & x &= -6 \end{aligned}$$

The two solutions are -6 and 7. Therefore, the solution set is $\{-6, 7\}$.

Question 48 (page 139)

C Correct.
Use the formula for the total surface area of a cylinder to find the radius.

$$A = 2\pi rh + 2\pi r^2$$

Substitute the known values of A and h to solve for r.

$$\begin{aligned} 4\pi &= 2\pi r(1.5) + 2\pi r^2 \\ 4\pi &= 3\pi r + 2\pi r^2 \\ \frac{4\pi}{\pi} &= \frac{3\pi r}{\pi} + \frac{2\pi r^2}{\pi} \qquad \text{Divide both sides by } \pi. \\ 4 &= 3r + 2r^2 \\ -4 &= \qquad\quad -4 \\ \hline 0 &= 2r^2 + 3r - 4 \end{aligned}$$

One way to solve this equation is to use the quadratic formula.

$a = 2, b = 3, c = -4$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$r = \frac{-3 \pm \sqrt{3^2 - 4(2)(-4)}}{2(2)} = \frac{-3 \pm \sqrt{9 + 32}}{4}$$

$$r = \frac{-3 \pm \sqrt{41}}{4}$$

$$r \approx \frac{-3 \pm 6.40}{4}$$

$$r \approx \frac{-3 + 6.4}{4} \text{ and } r \approx \frac{-3 - 6.4}{4}$$

$r \approx 0.85$ and $r \approx -2.35$

Since the radius of a cylinder cannot be negative, the answer is 0.85 inch.

Question 49 (page 139)

D Correct. Let h represent the height of the triangle. The base is 6 inches less than twice the height. The expression $2h - 6$ represents the length of the base.

Substitute the expressions for the height and the length of the base and the value of the area given in the problem into the formula for the area of a triangle.

$$A = \frac{1}{2}bh$$

$$28 = \frac{1}{2}(2h - 6)(h)$$

Simplify and write the equation in standard form.

$$28 = \frac{1}{2}(h)(2h - 6) = \frac{1}{2}h(2h) - \frac{1}{2}h(6)$$

$$28 = h^2 - 3h$$

Set the equation equal to zero by subtracting 28 from both sides.

$$0 = h^2 - 3h - 28$$

Then solve for h by factoring.

$$0 = h^2 - 3h - 28$$

$$0 = (h - 7)(h + 4)$$

Set each factor equal to 0.

$$h - 7 = 0 \qquad h + 4 = 0$$
$$h = 7 \qquad h = -4$$

Since the height cannot be negative, the solution $h = -4$ is not used.

The height of the triangle is 7 inches. Substitute $h = 7$ into the expression $2h - 6$ to find the length of the base.

Length of the base:

$$2h - 6 = 2(7) - 6 = 14 - 6 = 8 \text{ inches}$$

Question 50 (page 140)

C Correct. The related quadratic function of $x^2 - 5x + 2 = 0$ is the function $y = x^2 - 5x + 2$ shown on the graph. The solutions of the equation are the roots of the function—the x-coordinates of the points where the graph crosses the x-axis.

The graph crosses the x-axis between 0 and 1 and between 4 and 5. The solutions lie between these numbers. Only the pair of integers 4 and 5 is given as an answer choice.

Question 51 (page 140)

A Correct. Substitute the expression for the radius, $r = 5x^4yz^3$, into the formula for surface area of a sphere.

$$S = 4\pi r^2$$
$$S = 4\pi(5x^4yz^3)^2$$

Simplify the expression.

$$S = 4\pi(5^2x^{4 \cdot 2}y^{1 \cdot 2}z^{3 \cdot 2})$$
$$S = 4\pi \cdot 25x^8y^2z^6$$
$$S = 4 \cdot 25\pi x^8y^2z^6$$
$$S = 100\pi x^8y^2z^6$$

Question 52 (page 140)

B Correct. First simplify the expressions in parentheses that are raised to a power.

$$\frac{(2a^3b)^4}{(3ac^4)^2(4a^5b)} = \frac{(2^4a^{3 \cdot 4}b^4)}{(3^2a^2c^{4 \cdot 2})(4a^5b)} = \frac{16a^{12}b^4}{9a^2c^8 \cdot 4a^5b}$$

Combine any like bases in the numerator or denominator.

$$\frac{16a^{12}b^4}{9a^2c^8 \cdot 4a^5b} = \frac{16a^{12}b^4}{36a^7bc^8}$$

Divide variables that have like bases.

$$\frac{16a^{12}b^4}{36a^7bc^8} = \frac{16}{36} \cdot \frac{a^{12}}{a^7} \cdot \frac{b^4}{b} \cdot \frac{1}{c^8} =$$
$$\frac{4}{9} \cdot a^{12-7} \cdot b^{4-1} \cdot \frac{1}{c^8} = \frac{4a^5b^3}{9c^8}$$

Question 53 (page 140)

B Correct. The axis of symmetry is a vertical line that passes through the vertex and divides the parabola into two symmetrical halves.

Objective 6

Question 54 (page 164)

D Correct. The total area of the sidewalk is equal to the total area of the large rectangle, which includes the sidewalk around the pool, minus the area of the pool.

The pool has a width of 30 ft and a length of 40 ft.

The area of the pool is $30(40)$ ft^2.

The sidewalk adds x feet onto both sides of the length and the width of the small rectangle formed by the pool.

The width of the large rectangle is $x + 30 + x = 30 + 2x$.

The length of the large rectangle is $x + 40 + x = 40 + 2x$.

The area of the large rectangle is its length multiplied by its width.

$$(30 + 2x)(40 + 2x)$$

The area of the sidewalk is the area of the large rectangle minus the area of the pool.

$$(30 + 2x)(40 + 2x) - 30(40)$$

Question 55 (page 164)

C Correct. $\overline{AB}$ and $\overline{AD}$ are tangents to circle C from the same external point. They are equal in length; $AB = AD$. Since $\triangle ABD$ has two equal sides, it is an isosceles triangle. Therefore, $m\angle ABD = m\angle ADB$.

$$m\angle ABD + m\angle ADB + m\angle A = 180°$$

Since $m\angle A = 52°$, then $m\angle ABD + m\angle ADB = 180° - 52° = 128°$.

Since $m\angle ABD$ is equal to $m\angle ADB$, $m\angle ABD$ is one-half their sum.

$$m\angle ABD = \frac{128°}{2} = 64°$$

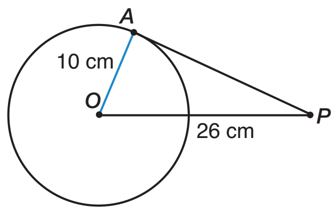
Since $\overline{CB}$ is a radius and $\overline{AB}$ is a tangent at point B, $\angle ABC$ is a right angle.

Thus, $\angle CBD$ and $\angle ABD$ are complementary. Their sum is 90°.

$$m\angle CBD = 90° - m\angle ABD$$
$$m\angle CBD = 90° - 64°$$
$$m\angle CBD = 26°$$

Question 56 (page 164)

B Correct. The tangent from point P forms a right angle with the radius OA drawn to point A.



Triangle PAO is a right triangle. Since (5, 12, 13) is a Pythagorean triple, its double is also a Pythagorean triple: (10, 24, 26). Thus, $\overline{AP}$ is 24 centimeters long.

Question 57 (page 165)

A Correct. The pattern in Figure I is formed by a shape, a rotation of the shape by 180°, and a translation of the shape and its rotated image to the right.

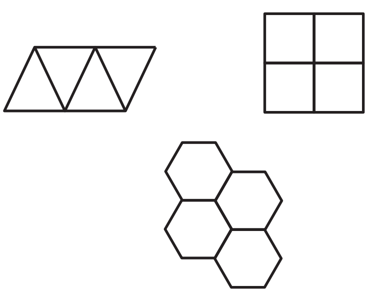
The pattern in Figure II is formed by a shape and three translated images of the shape.

The pattern in Figure III is formed by a shape, and a rotation of the shape by 180°, and a reflection of the shape and its rotation across a vertical line.

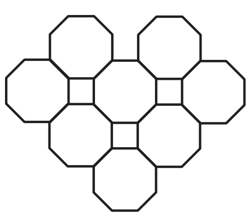
Only Figures I and II can be made using only translations and rotations.

Question 58 (page 165)

B Correct. Equilateral triangles, squares, and regular hexagons can all tessellate a plane.



Regular octagons cannot tessellate a plane by themselves.



Question 59 (page 165)

A Correct. The sidewalk and the shortcut form a right triangle with leg lengths of 60 yards and 80 yards. The Pythagorean triple (6, 8, 10) is multiplied by 10 to get (60, 80, 100).

The shortcut forms the hypotenuse of the right triangle. The shortcut is 100 yards long. The two sides of the rectangle add up to 140 yards. Cutting across the grass would save Gina $140 - 100 = 40$ yards.

Question 60 (page 166)

B Correct.

The pole, the ground, and the wire form a 45°-45°-90° triangle. The wire is the hypotenuse of the triangle. The length of one leg, 8 feet, is given. The length of the hypotenuse of a 45°-45°-90° triangle is the length of a leg times $\sqrt{2}$.

$$8\sqrt{2} \approx 11.3 \text{ feet}$$

Question 61 (page 166)

D Correct.

Dolores's horizontal distance from the boat is the distance along the ground from the base of the bridge to the boat.

The angle of depression from the top of the bridge to the boat is 30°. The top angle of the triangle is the complement of the angle of depression, 60°. The right triangle is a 30°-60°-90° triangle.

The height of the bridge, 45 feet, is the length of the shorter leg. The distance from the bridge to the boat is the longer leg. The length of the longer leg is the length of the shorter leg times $\sqrt{3}$.

$$45\sqrt{3} \approx 78$$

The horizontal distance is approximately 78 feet.

Question 62 (page 166)

B Correct. Use the formula for the volume of a cube to find $V = \left(\frac{1}{2}x\right)^3 = \frac{1}{8}x^3$. Use the formula for the volume of a pyramid to find $V = \frac{1}{3}\left(\frac{1}{2}x\right)^2(x) = \frac{1}{12}x^3$. The sum of the volumes of the two figures will be $\frac{1}{8}x^3 + \frac{1}{12}x^3 = \frac{5}{24}x^3$.

Question 63 (page 167)

C Correct. Use the graph to find the coordinates of each vertex of the translation.

Vertex M has coordinates (1, 1). Moving the point 4 units left and 2 units up will move the vertex to (−3, 3).

Vertex N has coordinates (2, −1). Moving the point 4 units left and 2 units up will move the vertex to (−2, 1).

Vertex P has coordinates (6, −1). Moving the point 4 units left and 2 units up will move the vertex to (2, 1).

Vertex Q has coordinates (5, 1). Moving the point 4 units left and 2 units up will move the vertex to (1, 3).

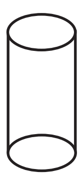
Question 64 (page 167)

A Correct. A reflection of $\triangle TRS$ across the y-axis changes the vertices to (3, 3), (3, −1), and (1, −1). Translating the figure down 3 units changes the vertices to (3, 0), (3, −4), and (1, −4). $\triangle TRS$ has been reflected across the y-axis and then translated 3 units down.

Objective 7

Question 65 (page 186)

C Correct. The circles form the bases of the figure. Wrapping the rectangle around the circular bases will form a cylinder.



Question 66 (page 186)

A Correct. This view represents the figure as seen from the top.

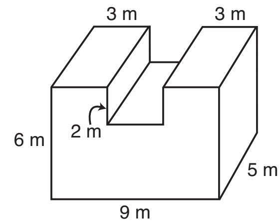
B Incorrect. This view represents the figure as seen from the front.

C Incorrect. This view is similar to the top view. However, the square formed by the cube at the top of the figure is not centered in this view.

D Incorrect. This view represents the figure as seen from the left side.

Question 67 (page 187)

C Correct. The three-dimensional figure shown by the three views is a rectangular prism with a smaller rectangular prism removed from the top.



To find the volume of the figure, calculate the volume of the large rectangular prism; then subtract the volume of the removed prism.

Use the formula $V = Bh$ to find the volume of a prism. The base of the large prism has a length of 9 meters and a width of 5 meters.

The area of the base, B, is $9 \cdot 5 = 45\text{ m}^2$. The height of the prism is 6 meters.

$$V = Bh$$
$$V = 45 \cdot 6$$
$$V = 270\text{ m}^3$$

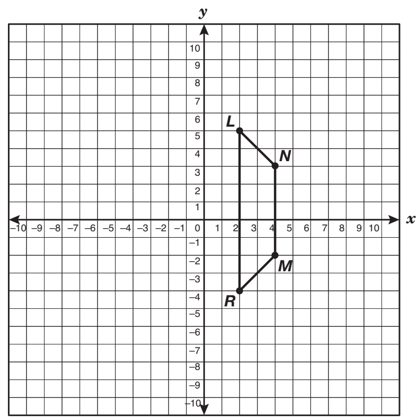
The removed prism has a rectangular base with a length of 3 meters and a width of 5 meters. The area of the base, B, is $3 \cdot 5 = 15\text{ m}^2$. The height is 2 meters.

$$V = Bh$$
$$V = 15 \cdot 2$$
$$V = 30\text{ m}^3$$

Volume of large prism − volume of removed prism = 270 − 30 = 240 cubic meters.

Question 68 (page 188)

D Correct. A trapezoid is a quadrilateral with exactly one pair of parallel sides. In an isosceles trapezoid the two nonparallel sides, its legs, are congruent. $\overline{MN}$ and $\overline{LR}$ form the parallel bases of the trapezoid. $\overline{NL}$ and $\overline{MR}$ form the congruent legs of the trapezoid.



Question 69 (page 188)

B Correct. Two of the sides of the triangle must be perpendicular to form a right triangle. The line segment with endpoints $(-1, -2)$ and $(-1, 3)$ is vertical. The line segment with endpoints $(-1, -2)$ and $(4, -2)$ is horizontal. These two segments are perpendicular. The segment with endpoints $(-1, 3)$ and $(4, -2)$ forms the hypotenuse of the right triangle.

Question 70 (page 188)

The correct answer is 13. Use the distance formula to find the distance between the two endpoints of the segment, $(7, -3)$ and $(-5, 2)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(-5 - 7)^2 + (2 - (-3))^2}$$
$$d = \sqrt{(-12)^2 + (5)^2}$$
$$d = \sqrt{144 + 25}$$
$$d = \sqrt{169}$$
$$d = 13$$

The length of the segment is 13 units.

		1	3	.			
0	0	0	0		0	0	0
1	1	●	1		1	1	1
2	2	2	2		2	2	2
3	3	3	●		3	3	3
4	4	4	4		4	4	4
5	5	5	5		5	5	5
6	6	6	6		6	6	6
7	7	7	7		7	7	7
8	8	8	8		8	8	8
9	9	9	9		9	9	9

Question 71 (page 189)

B Correct. The points $(0, 2)$ and $(3, 0)$ are on line k. Calculate the slope of line k.

$$m = \frac{2 - 0}{0 - 3} = -\frac{2}{3}$$

The line that is perpendicular to line k will have a slope of $\frac{3}{2}$, the negative reciprocal of $-\frac{2}{3}$. In the equation $y = \frac{3}{2}x - 4$, the slope, m, is $\frac{3}{2}$.

The equation $y = \frac{3}{2}x - 4$ represents a line perpendicular to line k.

Question 72 (page 189)

B Correct. The endpoints of the segment are A and B. The coordinates of point A are $(-5.1, 8.3)$. Let (x_2, y_2) represent the coordinates of point B. The coordinates of the midpoint, M, are $(-1.5, -3.2)$. Substitute these values into the midpoint formula.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

First substitute the values of the x-coordinates of the midpoint, M, and endpoint A into the formula. Solve for x_2.

$$-1.5 = \frac{-5.1 + x_2}{2}$$
$$-3 = -5.1 + x_2$$
$$2.1 = x_2$$

Now substitute the values of the y-coordinates of the midpoint, M, and endpoint A into the formula. Solve for y_2.

$$-3.2 = \frac{8.3 + y_2}{2}$$
$$-6.4 = 8.3 + y_2$$
$$-14.7 = y_2$$

The coordinates of point B are $(2.1, -14.7)$.

Question 73 (page 189)

A Correct. The parallel segments $\overline{AD}$ and $\overline{BC}$ form the bases of the trapezoid. Use the points $A\,(-2, 6)$ and $D\,(4, -3)$ to find the slope of the line containing $\overline{AD}$.

$$m = \frac{-3 - 6}{4 - (-2)} = \frac{-9}{6} = -\frac{3}{2}$$

The line containing $\overline{AD}$ crosses the y-axis at 3.

Substitute $m = -\frac{3}{2}$ and $b = 3$ into $y = mx + b$.

The equation of the line containing $\overline{AD}$ is

$y = -\frac{3}{2}x + 3$.

Since $\overline{BC}$ is parallel to $\overline{AD}$, the line containing $\overline{BC}$ also has a slope of $-\frac{3}{2}$.

Point B (2, 4) is also on this line.

Substitute $m = -\frac{3}{2}$, $x = 2$, and $y = 4$ into the slope-intercept form of the equation of a line and solve for b.

$$y = mx + b$$
$$4 = -\frac{3}{2}(2) + b$$
$$4 = -3 + b$$
$$7 = b$$

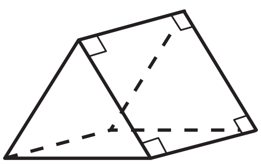
Substitute $m = -\frac{3}{2}$ and $b = 7$ into $y = mx + b$.

The equation of the line containing $\overline{BC}$ is

$y = -\frac{3}{2}x + 7$.

Question 74 (page 189)

B Correct. A prism with bases that are equilateral triangles could have three rectangular faces.



Objective 8

Question 75 (page 223)

A Correct.

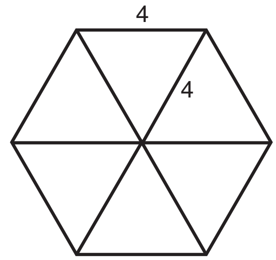
Use the formula $A = \frac{1}{2}bh$ to find the area of one triangle.

$$A = \frac{1}{2}(4)(9)$$
$$A = 18 \text{ in.}^2$$

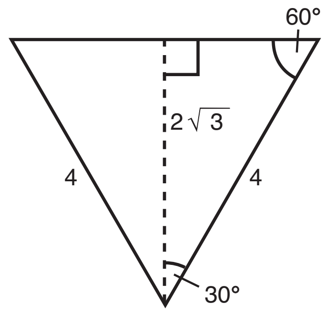
Multiply the area of one triangle by 6 to find the area of the six triangles.

$$A = 6 \cdot 18$$
$$A = 108 \text{ in.}^2$$

To find the area of the regular hexagon, divide it into 6 congruent regions.



Each triangle in the hexagon above is an equilateral triangle with a side length of 4 inches. Draw the height on one of the triangles as shown below.



The triangles formed by the altitude are 30°-60°-90° triangles. The length of the hypotenuse is 4 inches. The altitude, the longer leg, is equal to $2\sqrt{3}$ inches.

Use the formula $A = \frac{1}{2}bh$ to find the area of the triangle.

$$A = \frac{1}{2}(4)(2\sqrt{3})$$
$$A = 4\sqrt{3}$$
$$A \approx 6.93 \text{ in.}^2$$

Multiply by 6 to obtain the area of the hexagon.

$$A \approx 6 \cdot 6.93$$
$$A \approx 41.58 \text{ in.}^2$$

The total area of the figure is the sum of the areas of the hexagon and the triangles.

$$A \approx 108 + 41.58$$
$$A \approx 150 \text{ in.}^2$$

Elroy needs at least 150 square inches of cardboard to make the figure.

Question 76 (page 223)

A Correct.

To find the area of the sector, use the proportion

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{measure of central angle}}{360°}$$

The measure of the central angle is 170°.

The ropes are radii of the circle. The length of the radius is 9 yards. Find the area of the circle.

$$A = \pi(9)^2 = 81\pi \approx 254.47 \text{ yd}^2$$

Substitute these values into the proportion. Let x represent the area of the sector.

$$\frac{x}{254.47} = \frac{170}{360}$$

$$360x = 43{,}259.9$$

$$x = 120.166 \text{ yd}^2$$

The area of the sector is approximately 120 square yards.

Question 77 (page 224)

B Correct.

The length of the radius is 5.4 centimeters. Use the proportion

$$\frac{\text{length of arc}}{\text{circumference of circle}} = \frac{\text{degree measure of arc}}{360°}$$

to find the arc length.

Calculate the circumference of the circle.

$$C = 2\pi(5.4) \approx 33.929 \text{ cm}$$

The degree measure of the arc is 65°.

Let x represent the length of the arc. Substitute these values into the proportion and solve for x.

$$\frac{x}{33.929} = \frac{65}{360}$$

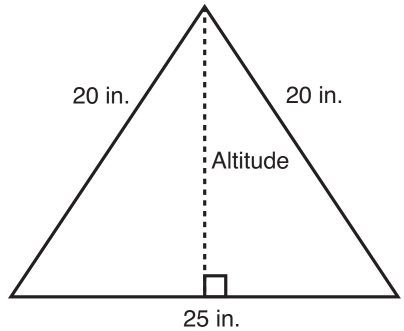
$$360x = 2205.385$$

$$x \approx 6.126 \text{ cm}$$

The length of the arc is approximately 6.1 cm.

Question 78 (page 224)

D Correct. To determine the height of the truck with the sign, find the height of the triangle that forms the base of the prism. The lengths of two sides of the triangle are equal. The triangle is isosceles.



The height is equal to the length of the altitude of the triangle. The altitude divides the triangle into two right triangles with a hypotenuse of 20 inches.

The altitude bisects the base. The length of one leg is $25 \div 2 = 12.5$ inches.

The altitude is the missing leg of the right triangle. Let a represent the length of the altitude. Use the Pythagorean Theorem to find a.

$$a^2 + 12.5^2 = 20^2$$

$$a^2 + 156.25 = 400$$

$$a^2 = 243.75$$

$$a = \sqrt{243.75}$$

$$a \approx 15.612$$

$$a \approx 16 \text{ inches}$$

$$a \approx 1 \text{ foot } 4 \text{ inches}$$

Add the approximate height of the sign to the height of the truck.

6 feet 10 inches + 1 foot 4 inches =
7 feet 14 inches = 8 feet 2 inches

Their total height is about 8 feet 2 inches. The truck with the sign on it will not fit in the garage.

Question 79 (page 225)

B Correct. Calculate the volume of the prism and the volume of the cylinder. Subtract the volume of the cylinder from the volume of the prism to find the volume of the machine part.

Use the formula $V = Bh$ for the volume of a prism.

The rectangular base of the prism has a length of 55 mm and a width of 30 mm. Its area is $55 \cdot 30 = 1{,}650$.

$$V = Bh = 1{,}650 \cdot 23 = 37{,}950 \text{ mm}^3$$

Use the formula $V = Bh$ for the volume of a cylinder.

The radius of the base of the cylinder is $20 \div 2 = 10$ mm. The area of the base is $\pi(10)^2 = 100\pi \approx 314$.

The height of the cylinder is equal to the length of the prism, 55 mm.

$$V = Bh \approx 314 \cdot 55 \approx 17{,}279 \text{ mm}^3$$

$$\text{Volume of the prism} - \text{Volume of the cylinder} \approx 37{,}950 - 17{,}279 \approx 20{,}671 \text{ mm}^3$$

The volume of metal used in each machine part is approximately 20,671 cubic millimeters.

Question 80 (page 225)

A Correct. Parallelogram *ABCD* has a base of 4 units and a height of 2 units. The parallelogram with coordinates (0, 2), (2, 2), (1, 1), and (−1, 1) has a base of 2 units and a height of 1 unit. Since $4 \cdot \frac{1}{2} = 2$ and $2 \cdot \frac{1}{2} = 1$, this parallelogram is a dilation of parallelogram *ABCD* by a scale factor of $\frac{1}{2}$. The figures are similar.

Question 81 (page 226)

B Correct. Since the stripe and the base of the sail are parallel lines, their corresponding angles are all equal. Thus, $\triangle ABC$ is similar to $\triangle DBE$ because their angles are equal in measure. Set up a proportion that relates $\overline{BC}$ to three known measurements. $\overline{DE}$ corresponds to $\overline{AC}$, and $\overline{BE}$ corresponds to $\overline{BC}$.

$$\frac{DE}{AC} = \frac{BE}{BC}$$

$$\frac{4.8}{12} = \frac{10}{BC}$$

Use cross products to solve for *BC*.

$$4.8 \cdot BC = 12 \cdot 10$$

$$4.8 \cdot BC = 120$$

$$BC = 25$$

The length of $\overline{BC}$ is 25 feet. Subtract to find the length of $\overline{EC}$.

$$BC - BE = EC$$

$$25 - 10 = EC$$

$$15 = EC$$

The distance from the bottom of the sail to the stripe is 15 feet.

Question 82 (page 226)

C Correct. The larger cube is a dilation of the smaller cube. The surface area of the dilated cube equals the square of the scale factor multiplied by the original surface area. The scale factor of the dilation is 3, so the square of the scale factor is $(3)^2 = 9$. The surface area of the cube increases by a factor of 9.

Question 83 (page 226)

C Correct. $\triangle ABC$ is similar to $\triangle EDC$. To find the length of $\overline{DC}$, write a proportion and solve for *DC*.

In similar figures, corresponding sides are opposite congruent angles.

$\overline{AB}$ corresponds to $\overline{ED}$, and $\overline{BC}$ corresponds to $\overline{DC}$.

$$\frac{AB}{ED} = \frac{BC}{DC}$$

$$\frac{20}{12} = \frac{15}{x}$$

$$20x = 12 \cdot 15$$

$$20x = 180$$

$$\frac{20x}{20} = \frac{180}{20}$$

$$x = 9$$

$\overline{DC}$ is 9 inches long.

Question 84 (page 226)

A Correct.

The formula for the volume of a cone is $V = \frac{1}{3}Bh$, where B is the area of the base.

The base of a cone is a circle, and the area of a circle is πr^2. Substitute this expression for *B* in the volume formula.

$$V = \frac{1}{3}\pi r^2 h$$

Find the volume of the sugar cone.

Its radius, *r*, is 2 centimeters, and its height, *h*, is 15 centimeters.

$$V = \frac{1}{3}\pi(2)^2(15)$$

$$V = 20\pi$$

$$V \approx 62.83 \text{ cm}^3$$

Find the volume of the waffle cone. Its radius is 3 times longer than the radius of the sugar cone. Since the radius of the sugar cone is 2 centimeters, the radius of the waffle cone is $3 \cdot 2 = 6$ centimeters.

Its height is 15 centimeters.

$$V = \frac{1}{3}\pi(6)^2(15)$$

$$V = 180\pi$$

$$V \approx 565.49 \text{ cm}^3$$

Find the ratio of the two volumes.

$$\frac{565.49}{62.83} \approx 9$$

The volume of the waffle cone is about 9, or 3^2, times greater than the volume of the sugar cone.

Question 85 (page 227)

A Correct.

The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. The diameter is given in this question, so divide 1.5 by 2 to get a radius of 0.75 inch. Substituting this value in for r gives $V = \frac{4}{3}\pi(0.75)^3$. Evaluating this gives a volume approximately equal to 1.8 cubic inches.

Question 86 (page 227)

B Correct.

Use the formula for the lateral surface area of a pyramid, $S = \frac{1}{2}Pl$. The perimeter of the base of the pyramid, P, is $4 \cdot 12 = 48$. Substituting this and the slant height into the formula gives $S = \frac{1}{2}(48)(15)$. Computing this gives a lateral surface area of 360 cm^2.

Objective 9

Question 87 (page 252)

B Correct. Find Dither's weight at the end of the first three-month period, when it increased by 10%.

New weight = old weight + 10% of old weight

$$x = 1200 + 0.10(1200)$$
$$x = 1200 + 120$$
$$x = 1320$$

Then find Dither's weight at the end of the next two-month period, when it decreased by 10%.

New weight = old weight − 10% of old weight

$$x = 1320 - 0.10(1320)$$
$$x = 1320 - 132$$
$$x = 1188$$

At the end of 5 months, Dither weighed 1188 pounds.

Question 88 (page 252)

A Correct. Let x equal the number of hours it will take to drive 10 miles at an average speed of 30 miles per hour. Set up and solve a proportion to find x.

$$\frac{\text{miles}}{\text{hours}} \quad \frac{10}{x} = \frac{30}{1}$$
$$30x = 10 \cdot 1$$
$$30x = 10$$
$$x = \frac{1}{3}$$

The drive will take $\frac{1}{3}$ hour at 30 miles per hour.

One-third of an hour is equal to 20 minutes.

Let y equal the number of hours it will take to drive 10 miles at an average speed of 40 miles per hour. Set up and solve a proportion to find y.

$$\frac{\text{miles}}{\text{hours}} \quad \frac{10}{y} = \frac{40}{1}$$
$$40y = 10 \cdot 1$$
$$40y = 10$$
$$y = \frac{1}{4}$$

The drive will take $\frac{1}{4}$ hour at 40 miles per hour.

One-fourth of an hour is equal to 15 minutes. The drive will take 20 minutes at 30 miles per hour and 15 minutes at 40 miles per hour. It will take $20 - 15 = 5$ minutes longer to complete the drive at 30 miles per hour than at 40 miles per hour.

Question 89 (page 252)

A Correct. On each of the three days, the probability that it will rain is 20%, so the probability that it will not rain on any one of these days is 100% − 20% = 80%. Not raining on all three days is a compound event—its probability can be found using the product rule for the probability of compound events.

P(no rain 1st day and no rain 2nd day and no rain 3rd day)

$= P$(no rain 1st day) $\cdot$ P(no rain 2nd day) $\cdot$ P(no rain 3rd day)

$= 0.80 \cdot 0.80 \cdot 0.80$

$= 0.512 \approx 51\%$

The probability that it will not rain during the next three-day period is about 51%.

Question 90 (page 252)

C Correct.

Since there are 5 quarters out of a total of 15 coins, the probability of the first coin being a quarter is $\frac{5}{15}$. Since he is not putting the first quarter back (dependent events), there are now 4 quarters out of a total of 14 coins remaining in his pocket, for a probability of $\frac{4}{14}$. Multiplying these probabilities gives $\frac{20}{210}$, which simplifies to $\frac{2}{21}$.

Question 91 (page 253)

B Correct. Of the 50 students surveyed, $11 + 22 = 33$ watch less than 2 hours of television per day, since the 11 students who watch less than 1 hour per day must also be included in the total. The experimental probability that a student watches less than 2 hours of television per day is $\frac{33}{50}$. Let s equal the number of students at Amelia's school who watch less than 2 hours of television per day. Write a ratio comparing s to the total number of students at the school: $\frac{s}{460}$. Use the ratios to write a proportion.

$$\frac{33}{50} = \frac{s}{460}$$

Solve by using cross products.

$$\begin{aligned} 50s &= 33 \cdot 460 \\ 50s &= 15{,}180 \\ s &\approx 304 \end{aligned}$$

Based on the survey, about 304 students at Amelia's school watch an average of less than 2 hours of television per day.

Question 92 (page 254)

C Correct. Range is used to indicate the variation in a set of data. The range of a set of numbers is the difference between the largest value and the smallest value. A set of data with little variation, such as lake temperatures that remain almost constant, will have a very small range.

Question 93 (page 254)

A Incorrect. To find the mean, find the sum of the mileages and then divide by the number of items.

$$\text{Mean} = \frac{90 + 35 + 110 + 15 + 32 + 150}{6} = 72$$

The mean mileage on the minivans is 72,000 miles. The mean is greater than the median.

B Correct. To find the median, first list the mileages in order from least to greatest.

15, 32, 35, 90, 110, 150

Then find the average of the two middle numbers.

$$\text{Median} = \frac{35 + 90}{2} = 62.5$$

The median mileage on the minivans is 62.5 thousand miles, which is the same as 62,500 miles. The median is less than the other data measures, so it would be most likely to convince readers that the minivans have low mileage.

C Incorrect. To find the range, subtract the lowest mileage from the highest mileage.

$$\text{Range} = 150 - 15 = 135$$

The range is 135,000 miles. The range does not describe the vehicles' typical mileages; it describes the difference between the lowest and highest mileage values.

D Incorrect. Each mileage value appears only once, so there is no mode.

Question 94 (page 255)

C Correct. Use the percentages given in the graph to find the frequency of each number rolled.

One: 5% of 20 $= 0.05 \cdot 20 = 1$

Two: 15% of 20 $= 0.15 \cdot 20 = 3$

Three: 25% of 20 $= 0.25 \cdot 20 = 5$

Four: 25% of 20 $= 5$

Five: 15% of 20 $= 3$

Six: 15% of 20 $= 3$

The number one was rolled 1 time; the numbers two, five, and six were each rolled 3 times; and the numbers three and four were each rolled 5 times. These frequencies match those given in choice C.

Question 95 (page 256)

D Correct. The graph shows that 40 red carnations were sold. Estimate the total number of carnations sold.

20 yellow + 32 pink + 40 red + 15 white + 28 blue = 135

About 135 carnations were sold in all. The ratio of red carnations to total carnations sold was about $\frac{40}{135}$. Let x equal the percentage of carnations that were red. Write a proportion that can be used to find x.

$$\begin{aligned} \frac{40}{135} &= \frac{x}{100} \\ 135x &= 40 \cdot 100 \\ 135x &= 4000 \\ x &\approx 30 \end{aligned}$$

About 30% of the carnations sold were red. Therefore, about 30% of the circle graph should represent red carnations.

Question 96 (page 256)

A Incorrect. Fifteen students received a B. Add to find the total number of students who took the test.

$$6 + 15 + 9 + 3 = 33$$

The fraction who received a B is $\frac{15}{33}$. Change the fraction to a percent.

$$\frac{15}{33} \approx 0.45 = 45\%$$

About 45% of the students received a B.

B Correct. Make a list of the grades in order; there were 6 A's, 15 B's, 9 C's, and 3 D's.

AAAAAABBBBBBBBBB<u>B</u>BBBBCCCCCCCCCDDD

The middle letter in this list is a B. Therefore, the median grade is a B.

C Incorrect. Add the frequencies to find the number of students who took the test.

$$6 + 15 + 9 + 3 = 33$$

Thirty-three students took the test.

D Incorrect. The highest letter grade was an A, and the lowest letter grade was a D. The lowest A grade is 90, and the highest D grade is 69. The difference between these two values is $90 - 69 = 21$. Therefore, the range of the test scores cannot be less than 21.

Question 97 (page 257)

A Correct. The minimum score is 51. The maximum score is 95. The lower quartile is 59. The upper quartile is 75, and the median score is 64.

Objective 10

Question 98 (page 274)

D Correct. Let x represent the original price of the CD player. To find the 30% discount, multiply the original price, x, by 0.30. The expression $0.30x$ represents the amount of money discounted. Since the expressions $0.30x$ and $0.3x$ are equivalent, the expression $(x - 0.3x)$ can be used to represent the sale price after the discount.

To find the 6.5% tax, multiply the sale price, $(x - 0.3x)$, by 0.065.

The expression $0.065(x - 0.3x)$ represents the amount of the sales tax.

The final price equals the discounted price plus the sales tax.

The expression $(x - 0.3x) + 0.065(x - 0.3x)$ represents the final price of the item.

The final price equals $178.92.

The equation $(x - 0.3x) + 0.065(x - 0.3x) = \$178.92$ can be used to find x, the original price of the CD player.

Question 99 (page 274)

C Correct. Find the area of one window using the formula for the area of a trapezoid.

$$A = \frac{1}{2}(b_1 + b_2)h$$

The height, h, is 36 cm. The bases, b_1 and b_2, are 40 cm and 48 cm. Substitute these values into the formula.

$$A = \frac{1}{2}(b_1 + b_2)h$$
$$A = \frac{1}{2}(40 + 48)36$$
$$= \frac{1}{2}(88)36$$
$$= 1{,}584$$

Each window has an area of 1,584 cm^2.

Convert the area to square meters. There are 100 centimeters in 1 meter. Therefore, there are $(100)^2 = 10{,}000\ cm^2$ in 1 square meter. Square meters is the larger unit, so divide by 10,000 to convert square centimeters to square meters.

$$1{,}584\ cm^2 \div \frac{10{,}000\ cm^2}{m^2} = 0.1584\ m^2$$

To manufacture each window requires 0.1584 m^2 of glass. Multiply by 2,500 to find the number of square meters of glass needed to manufacture 2,500 windows.

$$(0.1584)(2{,}500) = 396$$

A total of 396 m^2 of glass is required to manufacture 2,500 windows.

Question 100 (page 274)

C Correct. Use the simple interest formula, $I = prt$, to determine the amount of interest each investment amount will earn in 6 years.

I is the amount of interest earned, p is the principal or amount initially invested, r is the interest rate (4%, or 0.04), and t is the time in years that the money is invested (6 years).

Investment of \$550:
$I = prt$
$I = 550(0.04)(6)$
$I = \$132$
Total amount = \$550 + \$132 = \$682

After 6 years an investment of \$550 produces a balance of \$682; this is an amount between \$650 and \$700.

Of the answer choices, the only initial investment that produces a balance between \$650 and \$700 after 6 years is \$550.

Question 101 (page 274)

C Correct. The theoretical probability of an event occurring is related to the experimental probability of that event occurring. The greater the number of trials of the experiment, the closer the experimental results should come to the theoretical results. After 250 trials the theoretical probability of spinning green should be close to the experimental probability obtained in this experiment, $\frac{122}{250}$.

Question 102 (page 275)

A Correct. Step 1 has an expression in parentheses, but Step 3 does not. Simplify by using the distributive property to multiply each of the terms in parentheses by 2.

$$2(s + 4) - 4 = 10$$
$$2(s) + 2(4) - 4 = 10$$
$$2s + 8 - 4 = 10$$

Question 103 (page 275)

B Correct. If two parallel lines are cut by a transversal, then corresponding pairs of angles are congruent. $\angle 1$ and $\angle 5$ are corresponding angles; therefore, $\angle 1 \cong \angle 5$.

If two lines intersect, then vertical angles are congruent. $\angle 5$ and $\angle 7$ are vertical angles; therefore, $\angle 5 \cong \angle 7$.

If $\angle 1 \cong \angle 5$ and $\angle 5 \cong \angle 7$, it follows that $\angle 1 \cong \angle 7$ because they are both congruent to $\angle 5$.

Thus, $\angle 1$ is congruent to $\angle 7$ because the corresponding and vertical angles are congruent.

Question 104 (page 276)

A Correct. One way to solve the problem is to use the graph to determine Gina's weekly salary for each week in the month.

	Sales	Salary
Week 1	\$750	\$275
Week 2	\$600	\$260
Week 3	\$900	\$290
Week 4	\$850	\$285

Gina's total salary for the four-week period is the sum of her weekly salaries, or \$1110. The part of this salary that was commission is \$1110 minus her base salary for the four weeks. Look at the graph. It begins at the point (0, 200). That means if Gina's total sales for a week were \$0, then her base salary for the week would be \$200. In four weeks she earned 4 • \$200 = \$800 in base salary. Thus, the commission Gina earned for the four weeks was \$1110 − \$800 = \$310.

Question 105 (page 277)

A Incorrect. Triangles 2 and 3 are transformations of triangle 1. Triangle W is a reflection of triangle 1 across the y-axis. Since triangle W is a transformation of triangle 1, it belongs in the group.

B Incorrect. Triangles 2 and 3 are transformations of triangle 1. Triangle X is a translation 2 units down of triangle 1. Since triangle X is a transformation of triangle 1, it belongs in the group.

C Correct. Triangles 2 and 3 are transformations of triangle 1. Triangle 2 is a dilation of triangle 1, and triangle 3 is a reflection of triangle 1 across the x-axis. Since triangle Y is not a transformation of triangle 1, it does not belong in the group.

D Incorrect. Triangles 2 and 3 are transformations of triangle 1. Triangle Z is a translation 3 units to the left of triangle 1. Since triangle Z is a transformation of triangle 1, it belongs in the group.

Question 106 (page 278)

B Correct.

Use the areas given to find the radius of each circle in the pattern.

Circle	Area (cm^2)	$A = \pi r^2$	Radius (cm)
1	3.14	$3.14 = \pi r^2$ $1 \approx r^2$	$r \approx 1$
2	12.57	$12.57 = \pi r^2$ $4 \approx r^2$	$r \approx 2$
3	28.27	$28.27 = \pi r^2$ $9 \approx r^2$	$r \approx 3$
4	50.27	$50.27 = \pi r^2$ $16 \approx r^2$	$r \approx 4$

The radius increases by 1 cm for each successive circle in the pattern.

The sixth circle will have a radius of 6 cm. Use the formula for the circumference of a circle.

$$C = 2\pi r$$
$$C = 2\pi(6)$$
$$C \approx 37.70 \text{ cm}$$

The circumference of the sixth circle in this series is approximately 37.70 centimeters.

Question 107 (page 279)

C Correct.

One way to find the area of the enlarged triangle is to find the area of the original triangle and then multiply it by the square of the scale factor. Use the formula $A = \frac{1}{2}bh$ to find the area of the original triangle. Use the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, to find the length of $\overline{AB}$, the base of the triangle, and $\overline{CD}$, its height.

Base (length of $\overline{AB}$)
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $d = \sqrt{(3 - 12)^2 + (7 - -5)^2}$ $d = \sqrt{(-9)^2 + (12)^2}$ $d = \sqrt{81 + 144}$ $d = \sqrt{225}$ $d = 15$

Height (length of $\overline{CD}$)
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $d = \sqrt{(6 - 2)^2 + (3 - 0)^2}$ $d = \sqrt{(4)^2 + (3)^2}$ $d = \sqrt{16 + 9}$ $d = \sqrt{25}$ $d = 5$

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(15)(5)$$
$$A = \frac{1}{2}(75)$$
$$A = 37.5$$

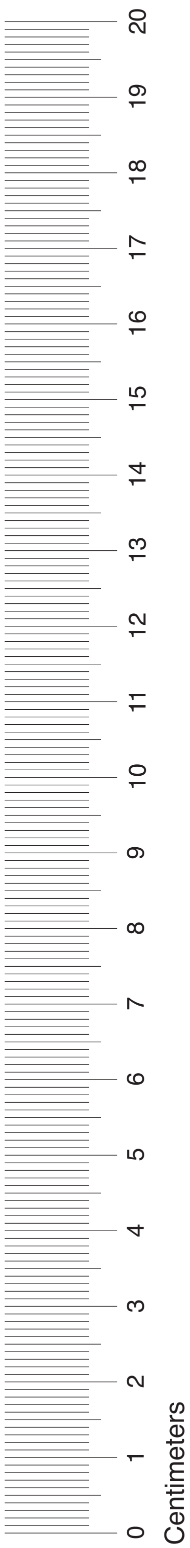
The area of triangle ABC is 37.5 square units. The area of the enlarged triangle will equal that area times the square of the scale factor, $5^2 = 25$.

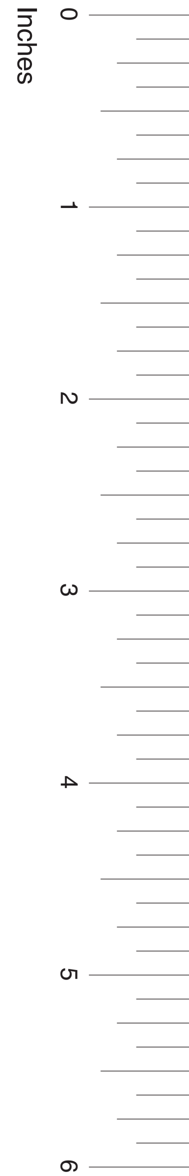
$$A = 37.5(25)$$
$$A = 937.5$$

The area of the enlarged triangle is 937.5 square units.

Grades 9, 10, and Exit Level Mathematics Chart

TAKS Texas Assessment of Knowledge and Skills





LENGTH

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

CAPACITY AND VOLUME

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 fluid ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 fluid ounces

MASS AND WEIGHT

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

TIME

1 year = 365 days

1 year = 12 months

1 year = 52 weeks

1 week = 7 days

1 day = 24 hours

1 hour = 60 minutes

1 minute = 60 seconds

Continued on the next side

Grades 9, 10, and Exit Level Mathematics Chart

Perimeter	rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	circle	$C = 2\pi r$ or $C = \pi d$
Area	rectangle	$A = lw$ or $A = bh$
	triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	regular polygon	$A = \frac{1}{2}aP$
	circle	$A = \pi r^2$
P represents the Perimeter of the Base of a three-dimensional figure.		
B represents the Area of the Base of a three-dimensional figure.		
Surface Area	cube (total)	$S = 6s^2$
	prism (lateral)	$S = Ph$
	prism (total)	$S = Ph + 2B$
	pyramid (lateral)	$S = \frac{1}{2}Pl$
	pyramid (total)	$S = \frac{1}{2}Pl + B$
	cylinder (lateral)	$S = 2\pi rh$
	cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	cone (lateral)	$S = \pi rl$
	cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	sphere	$S = 4\pi r^2$
Volume	prism or cylinder	$V = Bh$
	pyramid or cone	$V = \frac{1}{3}Bh$
	sphere	$V = \frac{4}{3}\pi r^3$
Special Right Triangles	$30°, 60°, 90°$	$x,\ x\sqrt{3},\ 2x$
	$45°, 45°, 90°$	$x,\ x,\ x\sqrt{2}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$